This is a 3 hour, closed book, written examination. The exam will have nine (9) subject areas with two (2) problems in each subject area. You will have to solve eight (8) of the eighteen (18) problems, but you must chose problems in at least six (6) different subject areas. The nine subject areas are:

1. Calculus
2. Determinants and Matrix Algebra
3. Ordinary Differential Equations
4. Complex Variables
5. Vector Analysis
6. Fourier Analysis
7. Laplace Transforms
8. Partial Differential Equations
9. Numerical Methods

NOTE:
Calculators can be used only for simple arithmetic calculations and you will be expected to abide by the Mechanical Engineering Code of Honor for ME Graduate Students taking the Applied Mathematics Area Exam. A copy of the code of honor is in Attachment A of this document.

As noted above, this is a closed book exam and no crib sheets are allowed. A table of Laplace Transforms will be provided. A copy of this table is given in Attachment B of this document.

The questions in subject areas 1 through 8 are set by professors in the Mathematics Department and the questions in the 9th subject area are set by Mechanical Engineering professors who teach ME 581. The people who set the questions also grade your solutions to those questions.

On the following pages are lists of topics that may be covered in each of the nine subject area questions. Also given are the Purdue classes in which the material is covered and sample references. Of course, there are many other texts that cover similar material, which may also be suitable as references.

The emphasis in the exam is on solving problems as opposed to doing proofs.
Section 1 Calculus

Topics
The topics covered in the calculus questions include:

- Finding maxima and minima of functions
- Methods of integration of definite and indefinite integrals including integration by parts and change of variables
- Graphical interpretation of functions
- Partial derivatives and total differentials
- Taylor series
- Double and triple integrals
- Transformation of coordinates
- The chain rule

Relevant Purdue Courses and References
MA161, MA162, MA165, MA166, & MA261, Text: [1, 2].

Section 2 Determinants and Matrix Algebra

Topics
The topics covered in the determinants and matrix algebra questions include:

- Solution of linear algebraic equations
- Algebraic manipulation of rectangular or square matrices such as sums, products, transpose, etc.
- Computation of a matrix inverse
- Evaluation of determinants
- Eigenvalues of a matrix
- Matrix concepts: rank, null space, row space, column space
- Vector space concepts: linear dependence, linear independence, dimension, basis
- Inner product space
- Properties of symmetric, skew-symmetric, orthogonal, hermitian, skew-hermitian and unitary matrices
- Geometry of linear transformations

Relevant Purdue Courses and References
MA262, Text: [3]; MA265, Text: [4]; MA511, Text: [5]; MA527, Text: [6].
Section 3 Ordinary Differential Equations

Topics
The topics covered in the ordinary differential equations questions include:

- Series solutions: ordinary points, regular singular points
- Determination of general and particular solutions of first order and second order differential equations
- Formulation and solution of differential equations for physical problems, e.g., falling or sliding bodies, rate equations, diffusion, vibrating systems.
- Solving systems of linear differential equations
- $2 \times 2$ autonomous systems. Stability, phase plane analysis

Relevant Purdue Courses and References
MA262, Text: [3]; MA266, MA303 & MA304, Text: [7]; MA527, Text: [6, 8].

Section 4 Complex Variables

Topics
The topics covered in the complex variables questions include:

- Algebra of complex numbers
- Polar notation and phasor diagrams
- Extraction of roots
- Functions of complex variables
- Power series
- Cauchy integral formula and contour integrals
- Evaluation of real and complex integrals by residues
- Properties of conformal mapping

Relevant Purdue Courses and References
MA425, Text: [9]; MA525, Text: [10]; MA528, Text: [6, 8].
Section 5 Vector Analysis

Topics
The topics covered in the vector analysis questions include:

- Vector algebra and calculus
- Operations involving gradient, curl and divergence
- Stoke’s theorem
- Line and surface integrals
- Conservative, irrotational and solenoidal fields
- Scalar potentials
- Divergence theorem

Relevant Purdue Courses and References

Section 6 Fourier Analysis

Topics
The topics covered in the Fourier analysis questions include:

- General coefficients
- Sine and cosine series of odd and even functions
- Complex Fourier series
- Parseval’s identity
- Fourier transform and its properties

Relevant Purdue Courses and References
MA303 or MA304, Text: [7]; MA520, Text: [12]; MA527, Text: [6, 8].
Section 7 Laplace Transforms

Topics
The topics covered in the Laplace transforms questions include:

- Solution of ordinary differential equations with constant coefficients
- Method of partial fraction expansions
- Inversions of Laplace transforms: Heaviside function, delta function, convolutions

Relevant Purdue Courses and References
MA303 or MA304, Text: [7]; MA527, Text: [6, 8].

Section 8 Partial Differential Equations

Topics
The topics covered in the partial differential equations questions include:

- Laplace, wave and heat equations in one, two or three variables
- Solution of initial and/or boundary value problems for these equations by the method of separation of variables
- Solution of heat equation in infinite domain using heat kernel
- D'Alembert solution for one-dimensional wave equation
- Properties of heat and wave equations

Relevant Purdue Courses and References
MA527, Text: [6, 8, 13]; MA520, Text: [12].
Section 9 Numerical Methods

Topics

The topics covered in the numerical methods questions include:

- Systems of linear equations
- Eigenproblems
- Solution of nonlinear equations
- Polynomial approximation and interpolation
- Least squares approximation
- Numerical differential and difference formulas
- Numerical integration (quadrature)
- Solution of one-dimensional initial-value problems
- Solution of one-dimensional boundary-value problems
- Elliptic partial differential equations
- Parabolic partial differential equations
- Hyperbolic partial differential equations

Relevant Purdue Courses and References

CS414, MA514, Text: [14, 15]; ME581, Text: [16, 17].
References


MECHANICAL ENGINEERING
Code of Honor for ME Graduate Students
(Prepared for students taking the Applied Mathematics Area Exam)

It has come to the attention of the School of Mechanical Engineering and the Department of Mathematics that the use of electronic calculators in the Applied Mathematics Area Exam can give an unfair advantage to some students depending upon the type of calculator used. It is also clear that for some questions on the Applied Mathematics Area Exam, the availability of an electronic calculator with basic arithmetic functions is essential.

For these reasons, the School of Mechanical Engineering is hereby instituting a uniform policy relative to the use of calculators in the Applied Math Area Exams:

The students will limit the use of the electronic calculators to simple arithmetic computations including standard mathematical functions. The students will not use any of the more advanced features of the calculators. Advanced features include, for example, pre-programmed matrix algebra capabilities and symbolic computation packages or tools.

There will not be any monitoring of these requirements but it is expected that the students will adhere to this code of honor.

I have read the Code of Honor for ME Graduate Students and I agree to abide by this Code.

Signature: ________________________
PRINT Last Name: ________________________, PRINT First Name: ________________________,
Date: _______________
## TABLE OF LAPLACE TRANSFORMS

\[ f(t) = \mathcal{L}^{-1}\{F(s)\} \quad F(s) = \int_0^\infty f(t)e^{-st}dt \]

\[ a, b \text{ and } c \text{ real} \]

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( \frac{1}{s} ), Re{s} &gt; 0</td>
</tr>
<tr>
<td>2.</td>
<td>( e^{at} )</td>
<td>( \frac{1}{s-a} ), Re{s} &gt; a</td>
</tr>
<tr>
<td>3.</td>
<td>( t^n ), ( n ) is a positive integer</td>
<td>( \frac{n!}{s^{n+1}} ), Re{s} &gt; 0</td>
</tr>
<tr>
<td>4.</td>
<td>( t^p ), ( p &gt; -1 )</td>
<td>( \frac{\Gamma(p+1)}{s^{p+1}} ), Re{s} &gt; 0</td>
</tr>
<tr>
<td>5.</td>
<td>( \sin(at) )</td>
<td>( \frac{a}{s^2+a^2} ), Re{s} &gt; 0</td>
</tr>
<tr>
<td>6.</td>
<td>( \cos(at) )</td>
<td>( \frac{s}{s^2+a^2} ), Re{s} &gt; 0</td>
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<td>7.</td>
<td>( \sinh(at) )</td>
<td>( \frac{a}{s^2-a^2} ), Re{s} &gt;</td>
</tr>
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<td>8.</td>
<td>( \cosh(at) )</td>
<td>( \frac{s}{s^2-a^2} ), Re{s} &gt;</td>
</tr>
<tr>
<td>9.</td>
<td>( e^{at} \sin(bt) )</td>
<td>( \frac{b}{(s-a)^2+b^2} ), Re{s} &gt; a</td>
</tr>
<tr>
<td>10.</td>
<td>( e^{at} \cos(bt) )</td>
<td>( \frac{(s-a)}{(s-a)^2+b^2} ), Re{s} &gt; a</td>
</tr>
<tr>
<td>11.</td>
<td>( t^n e^{at} ), ( n ) is a positive integer</td>
<td>( \frac{n!}{(s-a)^{n+1}} ), Re{s} &gt; a</td>
</tr>
<tr>
<td>12.</td>
<td>( u_c(t) )</td>
<td>( e^{-cs} \left( \frac{1}{s} \right) ), Re{s} &gt; 0</td>
</tr>
<tr>
<td>13.</td>
<td>( u_c(t)f(t-c) )</td>
<td>( e^{-cs}F(s) )</td>
</tr>
<tr>
<td>14.</td>
<td>( e^{ct}f(t) )</td>
<td>( F(s-c) )</td>
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<tr>
<td>15.</td>
<td>( f(ct) )</td>
<td>( \frac{1}{c}F\left( \frac{s}{c} \right) )</td>
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<tr>
<td>16.</td>
<td>( \int_0^t f(t-\tau)g(\tau)d\tau )</td>
<td>( F(s)G(s) )</td>
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<tr>
<td>17.</td>
<td>( \delta(t-c) )</td>
<td>( e^{-cs} )</td>
</tr>
<tr>
<td>18.</td>
<td>( f^{(n)}(t) )</td>
<td>( s^nF(s) - s^{n-1}f(0) - ... - f^{(n-1)}(0) )</td>
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<tr>
<td>19.</td>
<td>( (-t)^nf(t) )</td>
<td>( F^{(n)}(s) )</td>
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