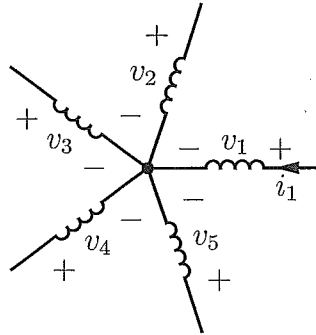


Problem 1. [33 points] Five identical magnetically coupled inductors are connected and positioned in air as shown below. The magnetic axes are lying on the same plane equally spaced apart, and they are all pointing outwards. (Note: this is *not* an electric machine, so these inductors should not be interpreted as distributed windings. You may think of them as solenoidal inductors wrapped around plastic cores.) There is a return path for the current through the neutral, which is not shown. You may neglect the resistance of the wires, and assume linear magnetics.



When you answer the below questions, please provide a brief justification.

- (a) A test is conducted where all coils are open-circuited, except for coil 1 that carries a current

$$i_1(t) = 10 \cos(100t)$$

in A, where  $t$  is in s, and the argument of the cosine is in rad. Measurements on some of the coils indicate that

$$v_1(t) = 40 \cos(\omega_1 t + \phi_1)$$

$$v_2(t) = 10 \cos(\omega_2 t + \phi_2)$$

$$v_4(t) = 1 \cos(\omega_4 t + \phi_4)$$

in V. Provide values for  $\omega_j$  and  $\phi_j$ ,  $j = 1, 2, 4$ .

- (b) Calculate the coupling field energy when  $i_3(t) = 10$  A,  $i_5(t) = 2$  A, i.e., both are dc, and all other currents are zero.

Problem 2. [33 points] The winding function of the  $b$ -phase in the stator of a 3-phase symmetric induction machine is

$$w_{bs}(\phi_{sm}) = 100 \sin(2\phi_{sm}) - 10 \sin(6\phi_{sm})$$

where  $\phi_{sm}$  denotes mechanical angle.

- (a) Express  $w_{as}(\phi_{sm})$ .  
(b) Express the conductor density  $n_{bs}(\phi_{sm})$ .

Write in Exam Book Only

Problem 3. [34 points] The line-to-neutral voltages (in V) in a Y-connected three-phase stationary circuit are

$$\mathbf{v}_{abc}^T(t) = [1.5e^{-10t} \quad 0 \quad 3]$$

where  $t$  is measured in s. The voltages are transformed to an arbitrary reference frame, using an angle  $\theta(t)$  that is a continuous function of time, where

$$\theta(t) = \begin{cases} \theta_1(t) & \text{for } 0 \leq t < 1 \\ 4\pi(t-1)^\alpha & \text{for } 1 \leq t < 2 \\ \theta_3(t) & \text{for } 2 \leq t \leq 3 \end{cases}$$

The parameter  $\alpha$  is a positive integer. The obtained  $qd$ -axes voltage waveforms are shown below. Determine: (i)  $\alpha$ , and (ii) the speed  $\omega(t)$  of the reference frame for  $0 \leq t \leq 3$  s. Justify your answer.

Recall:

$$\mathbf{K}_s(\theta) = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

