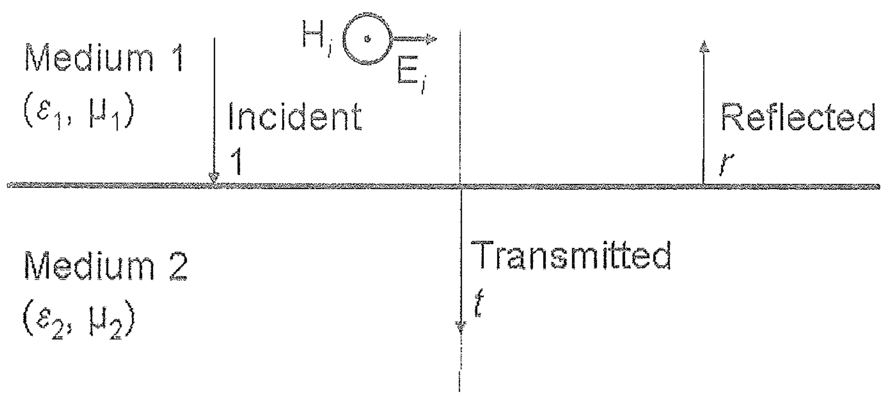
FO-2
August 2012 QE

A plane electromagnetic wave is normally incident on the interface between two media labelled 1 (with ε_1 and μ_1) and 2 (with ε_2 and μ_2), respectively. A wave of unit amplitude in medium 1 ($|E_i| = E_{i0} = 1$), incident on the interface, has reflected and transmitted amplitudes r and t respectively.



1. (25 Points)
Prove the following for the reflection and transmission coefficients r and t:

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$t = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Where $\eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}$, $i = 1,2$

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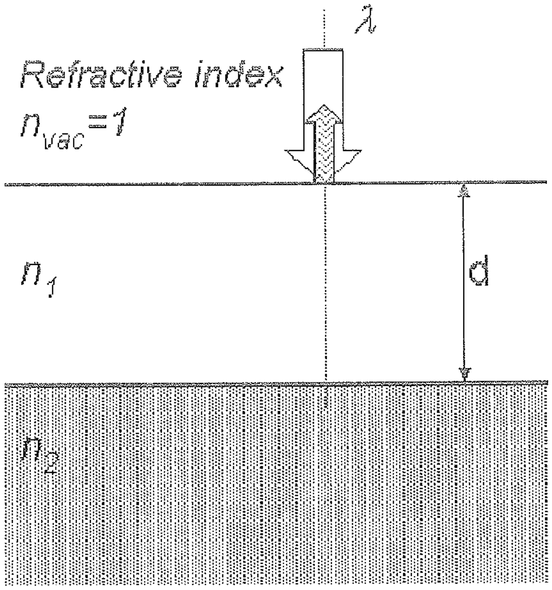
A plane electromagnetic wave of arbitrary amplitude is now normally incident from vacuum on a plane surface film of uniform thickness d covering a semi-infinite dielectric. The film and the substrate have indices of refraction n_1 and n_2, respectively. Assume that $\mu = 1$ for both media.



2. (25 Points)
Find an expression for the amplitude of the wave reflected into the vacuum ($E_{reflected}$) in terms of n_1, n_2 and the vacuum wavelength λ by postulating a standing wave electromagnetic field in the dielectric n_1.
(Hint: Use system of equations required by matching boundary conditions at the interfaces).

3. (25 Points)
Find the same expression as in 2) for the wave reflected into the vacuum by considering multiple reflections of the transmitted light inside the dielectric slab.
(Hint: Represent a total reflected amplitude as a sum of all reflected fractions when light undergoes multiple reflections).

4. (10 Points)
Under what condition will the reflected wave vanish?

5. (5 Points)
The condition you found in sub-question (4) is the principle of operation of the simplest interference anti-reflective (AR) coating consisting of a single layer of transparent material. Formulate requirements (4) in terms of thickness of AR layer and the ratio between the AR layer's and the substrate's refractive indices.

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6. (10 Points)
You are designing an AR coating for an optical component for experiments with HeNe laser (633 nm). The most common type of optical glass that can be considered as a substrate n_2 is crown glass, which has an index of refraction of about 1.52.

A) (3 Points) What is the reflection of this glass if used without any coatings? Assume that the glass plate is thick. Give the answer for fraction of the incident *power* that is reflected from the interface - in other words find the reflectance R.

B) (2 Points) What is the refractive index of an optimum single layer AR coating in this case? Can a solid material have this optimum refractive index?

C) (2 Points) What is the min thickness of an AR layer?

D) (3 Points) A typical AR single-layer coating is made of magnesium fluoride, MgF_2, that has a refractive index of 1.38 at 633 nm. How much it reduces the reflection compared to bare glass?

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