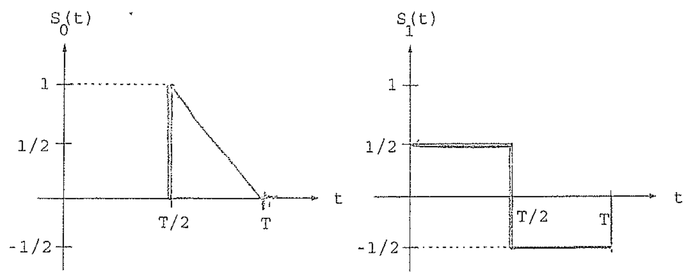
CS-3
August 2012 QE

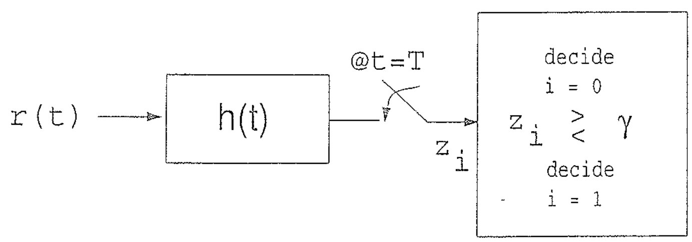
Problem 1. (40 points) The real baseband received signal $r(t)$ is given by

$$r(t) = s_i(t) + n(t); i = 0, 1,$$

where $\Pr\{i = 0\} = \Pr\{i = 1\} = 1/2$, and $n(t)$ is an additive white Gaussian noise process with two-sided power spectral density $N_0/2$. The two real signals $s_0(t)$ and $s_1(t)$ are illustrated in the following figure.



Suppose that the receiver structure is given as shown below. The received signal is processed



with a linear time-invariant filter with impulse response $h(t)$, and the output of that filter is sampled at time t=T to produce the statistic z_i. If $z_i > \gamma$, i=0 is decided. If $z_i < \gamma$, i=1 is decided.

(When necessary, express answers in terms of $\Phi(x)$, the cumulative distribution function of a zero-mean, unit-variance Gaussian random variable, or in terms of $Q(x) = 1 - \Phi(x)$.)

(a) First, suppose $h(t) = p_T(t)$, that is, $h(t) = 1$ for $0 \le t < T$, and $h(t) = 0$, elsewhere. Find the minimax threshold, that is, the threshold that minimizes the maximum of the probability of error given that $s_0(t)$ is sent and the probability of error given that $s_1(t)$ is sent. Also, find the corresponding average probability of decision error in terms of N_0 and T.

(b) Now, suppose the impulse response $h(t)$ can be adjusted. Find and sketch the impulse response of the matched filter for this signal set. Label key values in your sketch.

(c) Find the average error probability when the matched filter and the minimax threshold are used in terms of N_0 and T.

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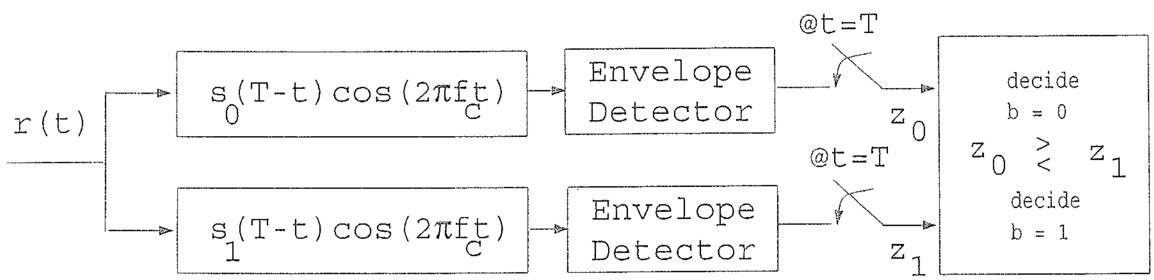
Problem 2. (30 points) Suppose that $s_0(t)$ and $s_1(t)$ are real baseband, orthonormal signals on the interval $[0, T]$. These signals are used to transmit an equi-probable data bit $b \in \{0, 1\}$ through an RF channel.

The received signal is given by

$$r(t) = \begin{cases} \sqrt{\mathrm{A}} s_0(t)\cos(2\pi f_c t + \theta_0) + n(t), & \text{when } b = 0, \\ \sqrt{\mathrm{B}} s_1(t)\cos(2\pi f_c t + \theta_1) + n(t), & \text{when } b = 1, \end{cases}$$

where $n(t)$ is a real AWGN process with two-sided power spectral density $N_0/2$ and f_c is the carrier frequency, which is much larger than the bandwidth of $s_i(t)$. The parameters θ_0 and θ_1 are unknown carrier phases.

The receiver block diagram is given below. In one branch, the received signal is processed with a filter with impulse response $s_0(T-t)cos(2\pi f_c t)$ and passed through an envelope detector. The output of that envelope detector is sampled at t=T to produce the statistic z_0. In another branch, the received signal is processed with a filter with impulse response $s_1(T-t)cos(2\pi f_c t)$ and passed through an envelope detector. The output of that envelope detector is sampled at t=T to produce the statistic z_1. If $z_0 > z_1$, b=0 is decided. If $z_1 > z_0$, b=1 is decided.



Find the average probability of bit error as a function of A, B, and N_0.

Problem 3. (30 points) Thermal noise with two-sided spectral density $N_0/2 = 7 \times 10^{-6}$ W/Hz is the input to a linear time-invariant filter with transfer function $H(f) = f^2$ for $|f| \leq 10$ Hz.

(a) Find the power spectral density at the output of the filter.

(b) Determine how much of the output power resides in the frequency range described by $1\ Hz \leq |f| \leq 2\ Hz$.

(c) What is the total output power?

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