1. **(25 Points)** Consider a random experiment in which a point is selected at random from the unit square (sample space $\mathcal{S} = [0,1] \times [0,1]$). Assume that all points in $\mathcal{S}$ are equally likely to be selected. Let the random variable $X(\omega)$ be the distance from the outcome ω to the nearest edge of (i.e., the nearest point on one of the four sides) of the unit square.
   (a) Find the cumulative distribution function (c.d.f.) of X. Draw a graph of the c.d.f.
   (b) Find the probability density function (p.d.f.) of X. Draw a graph of the p.d.f.
   (c) What is the probability that X is less than 1/8 ?

2. **(25 Points)** State and prove the *Chebyshev inequality* for random variable X with mean μ and variance σ^2. In constructing your proof, keep in mind that X may be either a discrete or continuous random variable.

3. **(25 Points)** Let $X_1, \ldots, X_n, \ldots$ be a sequence of independent, identically distributed random variables, each uniformly distributed on the interval $[0,1]$, an hence having pdf

$$f_X(x) = 1_{[0,1]}(x) = \begin{cases} 1, & \text{for } 0 \le x \le 1; \\ 0, & \text{elsewhere.} \end{cases}$$

   Let Y_n be a new random variable defined by

$$Y_n = \min\{X_1, \ldots, X_n\}.$$

   (a) Find the pdf of Y_n.
   (b) Does the sequence $\{Y_n\}$ converge in probability? Justify your answer.
   (c) Does the sequence $\{Y_n\}$ converge in distribution? If it does, specify the cumulative distribution function of the random variable it converges to.

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4. (25 Points) Let $\mathbb{X}(t)$ and $\mathbb{Y}(t)$ be two independent Gaussian random processes, both having zero-mean and both having the identical autocovariance function $C(t_1, t_2)$. Define the new random process $\mathbb{Z}(t)$ as

$$\mathbb{Z}(t) = \mathbb{X}(t) \cos \omega_o t + \mathbb{Y}(t) \sin \omega_o t,$$

where ω_o is a constant radian frequency.

(a) Find the mean of $\mathbb{Z}(t)$.

(b) Find the autocovariance function of $\mathbb{Z}(t)$.

(c) Under what, if any, conditions on $C(t_1, t_2)$ is $\mathbb{Z}(t)$ a wide-sense stationary random process?

(d) Under what, if any, conditions on $C(t_1, t_2)$ is $\mathbb{Z}(t)$ a (strict-sense) stationary random process?

(e) Write an expression for the joint characteristic function of the random variables $\mathbb{Z}(t_1), \ldots, \mathbb{Z}(t_n)$ obtained by sampling the random process $\mathbb{Z}(t)$ at arbitrary time instants $t_1, \ldots, t_n$. Express your answer in terms of the common autocovarince function $C(t_j, t_k)$ of $\mathbb{X}(t)$ and $\mathbb{Y}(t)$.