CS-1
August 2011 QE

1. **(25 Points)** Let Let $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ be three jointly distributed random variables with joint pdf

$$f_{\mathbf{XYZ}}(x,y,z) = \frac{3z^2}{7\sqrt{2\pi}} e^{-zy} \exp\left[-\frac{1}{2}\left(\frac{x-y}{z}\right)^2\right] \cdot 1_{[0,\infty)}(y) \cdot 1_{[1,2]}(z).$$

   (a) Find the joint probability density function $f_{\mathbf{YZ}}(y,z)$.
   (b) Find the conditional probability density function $f_{\mathbf{X}}(x|y,z)$.
   (c) Find the probability density function $f_{\mathbf{Z}}(z)$.
   (d) Find the conditional probability density function $f_{\mathbf{Y}}(y|z)$.
   (e) Find the conditional probability density function $f_{\mathbf{XY}}(x,y|z)$.

2. **(25 Points)** Show that if a continuous-time Gaussian random process $\mathbf{X}(t)$ is wide-sense stationary, it is also strict-sense stationary.

3. **(25 Points)** Show that the sum of two jointly distributed Gaussian random variables that are not necessarily statistically independent is a Gaussian random variable.

4. **(25 Points)** Assume that $\mathbf{X}(t)$ is a zero-mean continuous-time Gaussian white noise process with autocorrelation function

$$R_{\mathbf{XX}}(t_1,t_2) = \delta(t_1 - t_2).$$

   Let $\mathbf{Y}(t)$ be a new random process obtained by passing $\mathbf{X}(t)$ through alinear time-invariant system with impulse response $h(t)$ whose Fourier transform $H(\omega)$ has the ideal low-pass characteristic

$$H(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \Omega, \\ 0, & \text{elsewhere,} \end{cases}$$

   where $\Omega > 0$.
   (a) Find the mean of $\mathbf{Y}(t)$.
   (b) Find the autocorrelation function of $\mathbf{Y}(t)$.
   (c) Find the joint pdf of $\mathbf{Y}(t_1)$ and $\mathbf{Y}(t_2)$ for any two arbitrary sample times t_1 and t_2.
   (d) what is the minimum time difference $t_1 - t_2$ such that $\mathbf{Y}(t_1)$ and $\mathbf{Y}(t_2)$ are statistically independent?

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