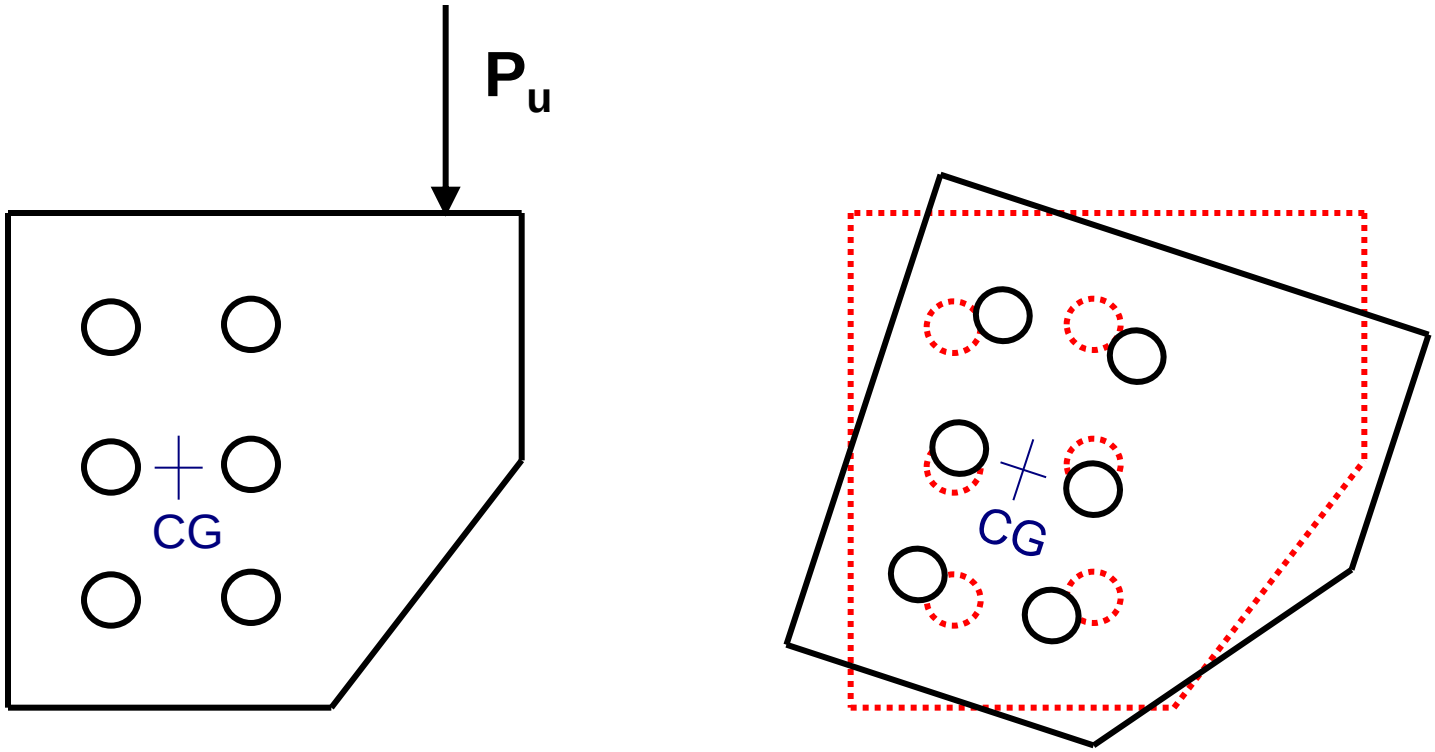
Eccentric Connections – Shear

- Eccentrically loaded bolt groups
- Ultimate Strength (Plastic Analysis)
  - Uses Instantaneous Center of Rotation (I.C.)
  - “most rational method”

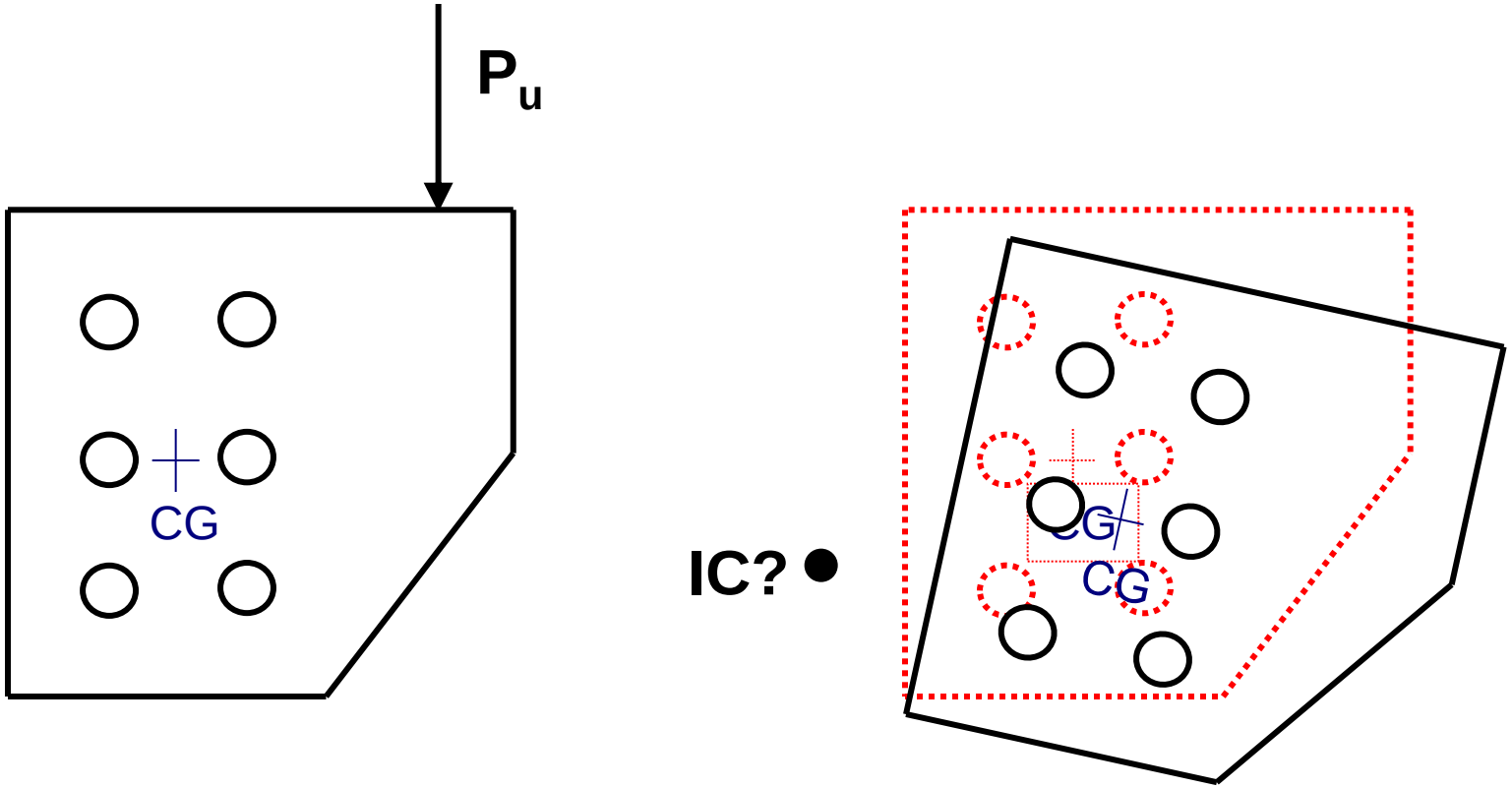
Elastic (Vector) Analysis

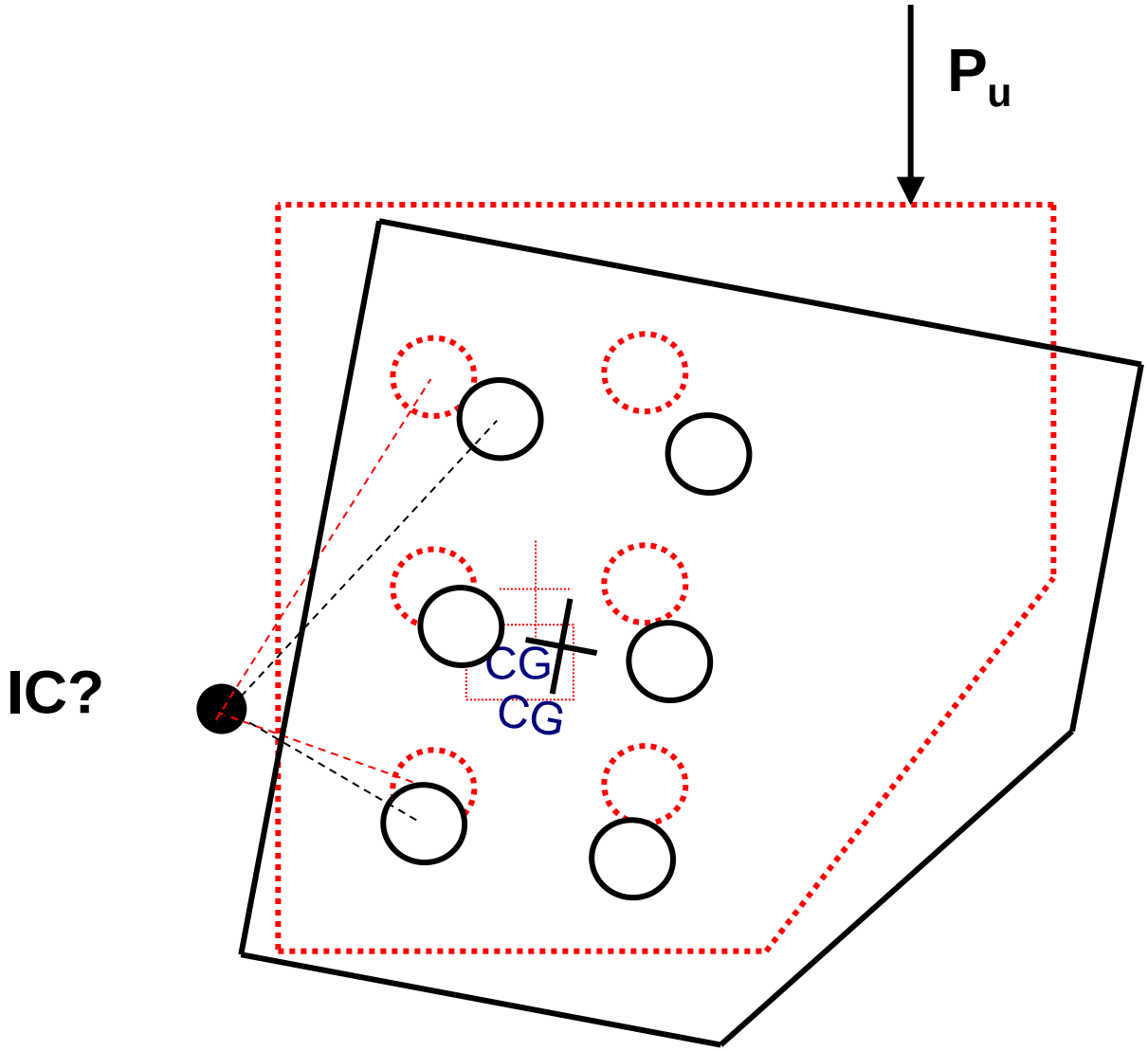
- Assumed rotation about the C.G. of bolt group

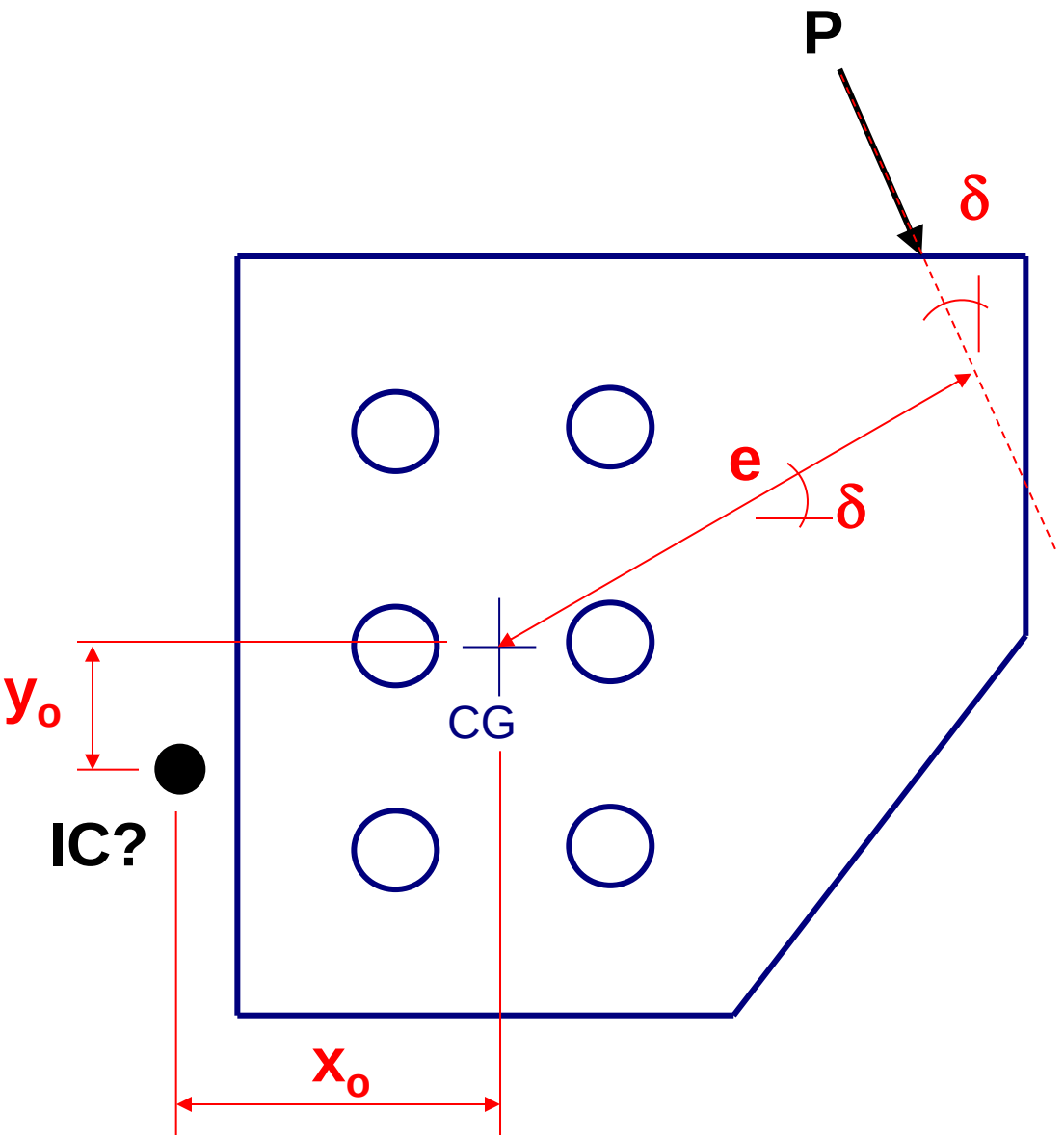


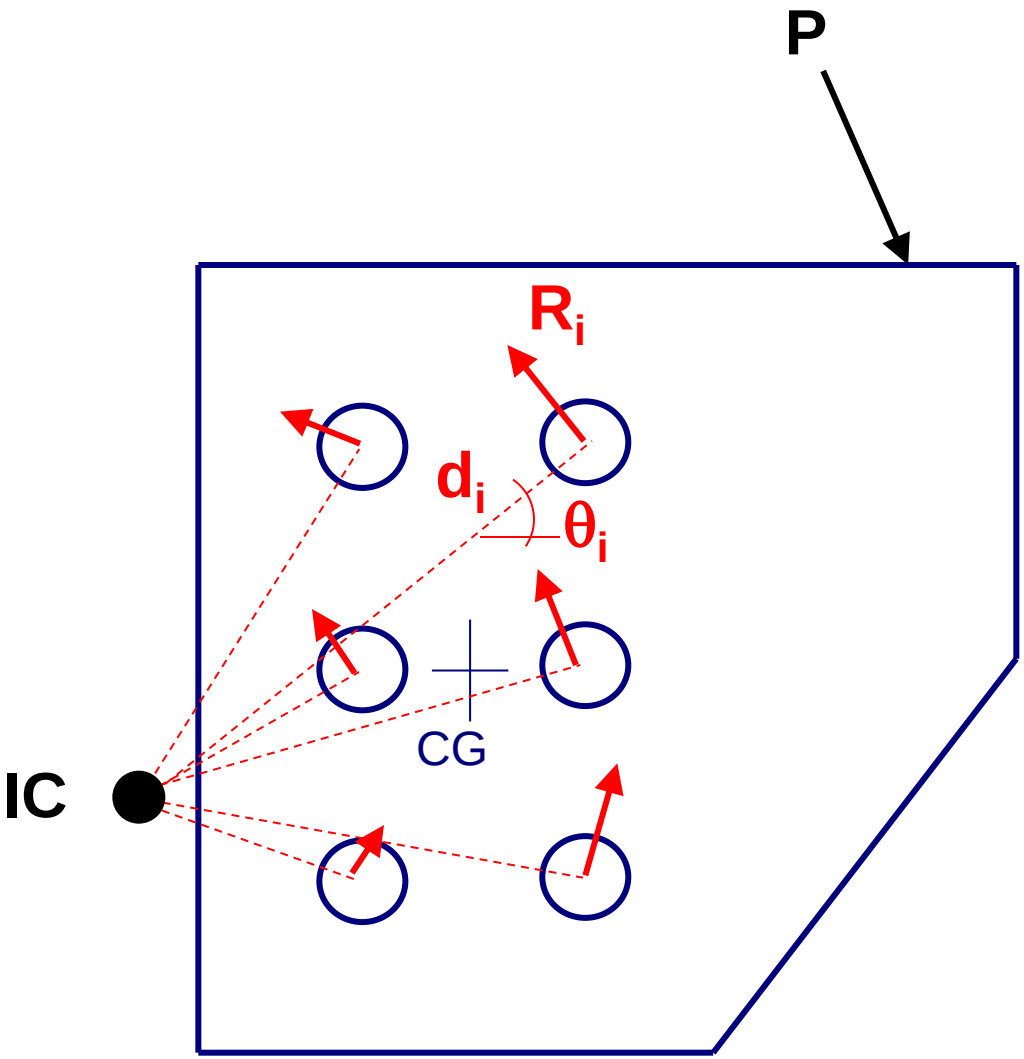
Instantaneous Center (I.C.)

- Accounts for *translation* and *rotation*

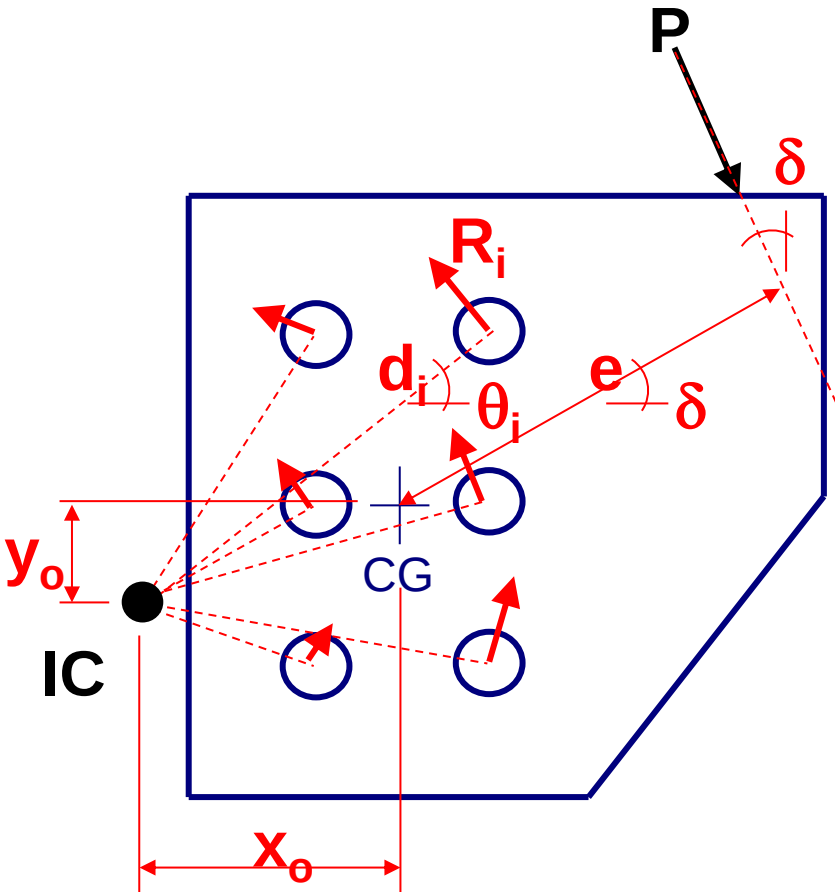



P_u
IC?
CG
CG


P
δ
e
δ
CG
y_o
IC?
x_o


P
R_i
d_i
θ_i
CG
IC

How do we know if our guess for the I.C. is correct?



$$\Sigma F_x = 0$$

$$\sum_{i=1}^{n} R_i \sin\theta_i - P\sin\delta = 0$$

$$\Sigma F_y = 0$$

$$\sum_{i=1}^{n} R_i \cos\theta_i - P\cos\delta = 0$$

$$\Sigma M = 0$$

$$\sum_{i=1}^{n} R_i d_i - P(e + x_o\cos\delta + y_o\sin\delta) = 0$$

Load-Deformation (AISC Fig. 7-3)

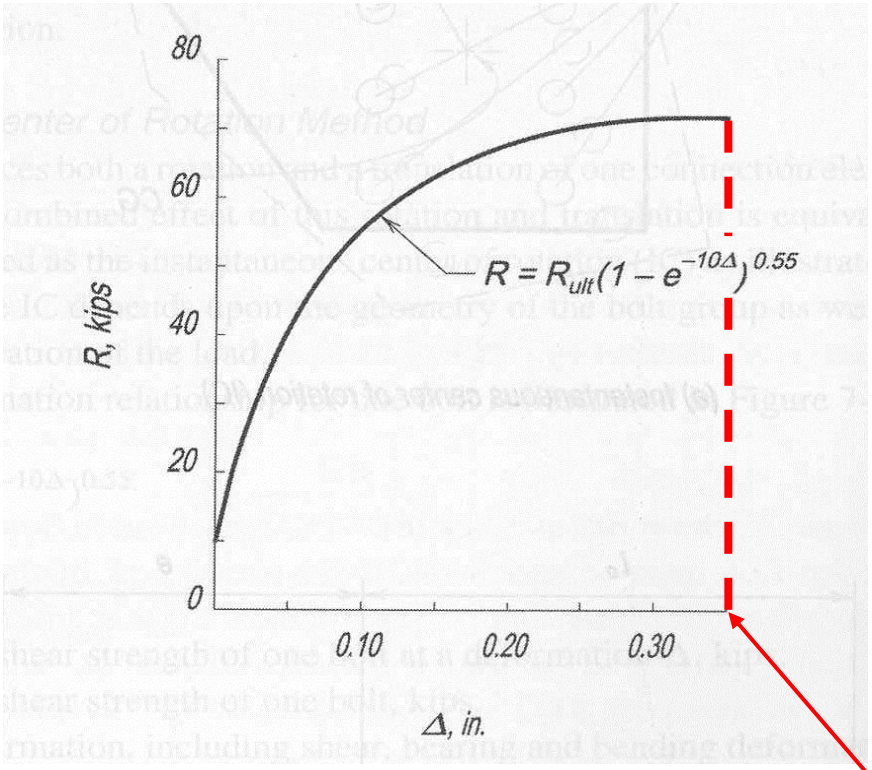


Fig. 7-3, for one ¾" ASTM A325 bolt in single shear, from tests (Fisher)

$$R_i = R_{ult}(1 - e^{-10\Delta})^{0.55}$$

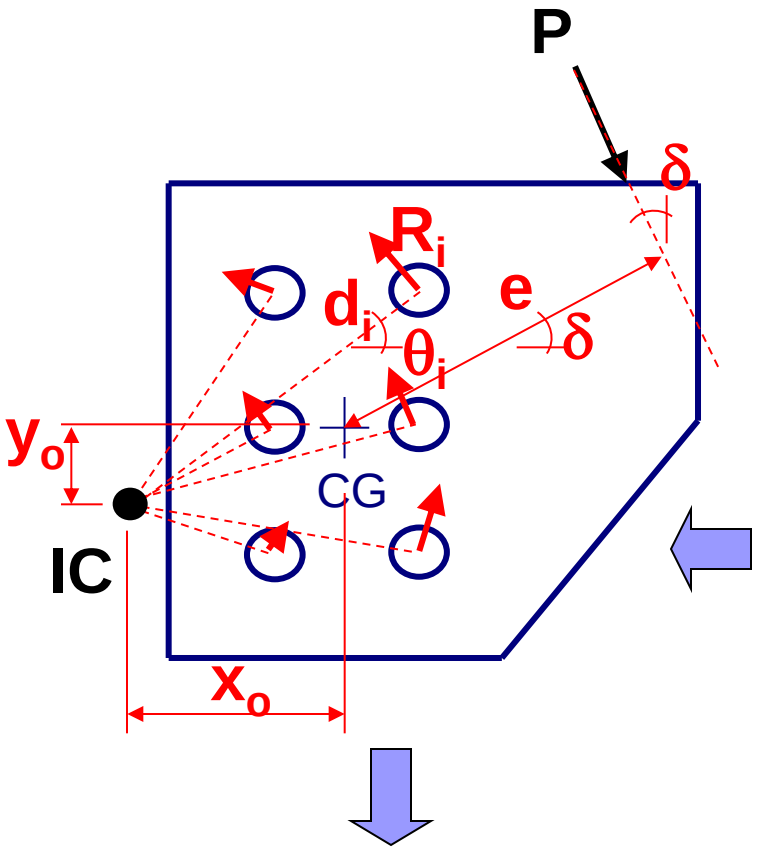
2.718

$\tau_u A_b$

where τ_u is ~70% of F_{ub}
(from experiments)

Alternatively, use shear strength of bolt as calculated by LRFD for R_{ult}

NOTE Δ_{max}



$$\Sigma F_x = 0$$

$$\sum_{i=1}^{n} R_i \sin\theta_i - P\sin\delta = 0$$

$$\Sigma F_y = 0$$

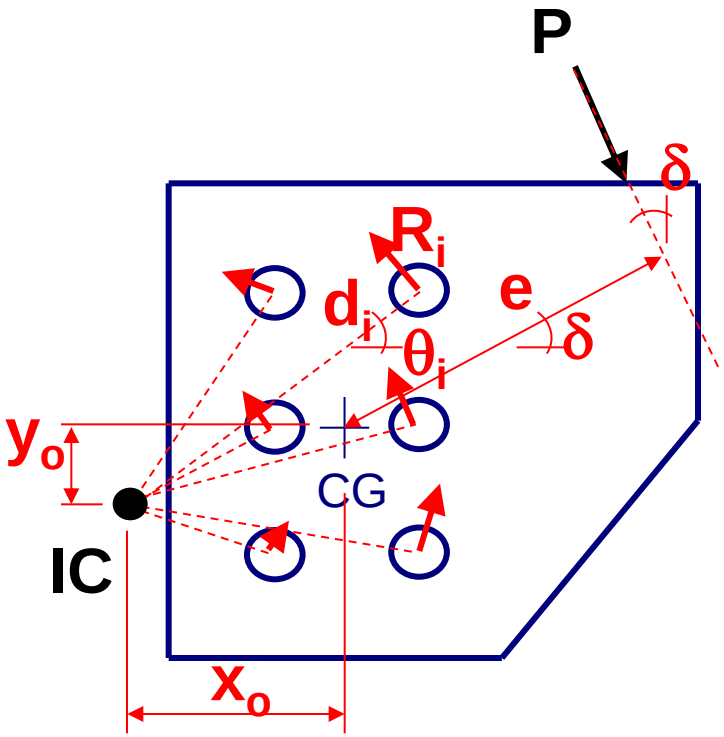
$$\sum_{i=1}^{n} R_i \cos\theta_i - P\cos\delta = 0$$

$$\Sigma M = 0$$

$$\sum_{i=1}^{n} R_i d_i - P(e + x_o\cos\delta + y_o\sin\delta) = 0$$

$$R_i = R_{ult}\left(1 - e^{-10\Delta}\right)^{0.55}$$

Iterative process required to solve!



$$\frac{d_i}{d_{max}}\Delta_{max}$$

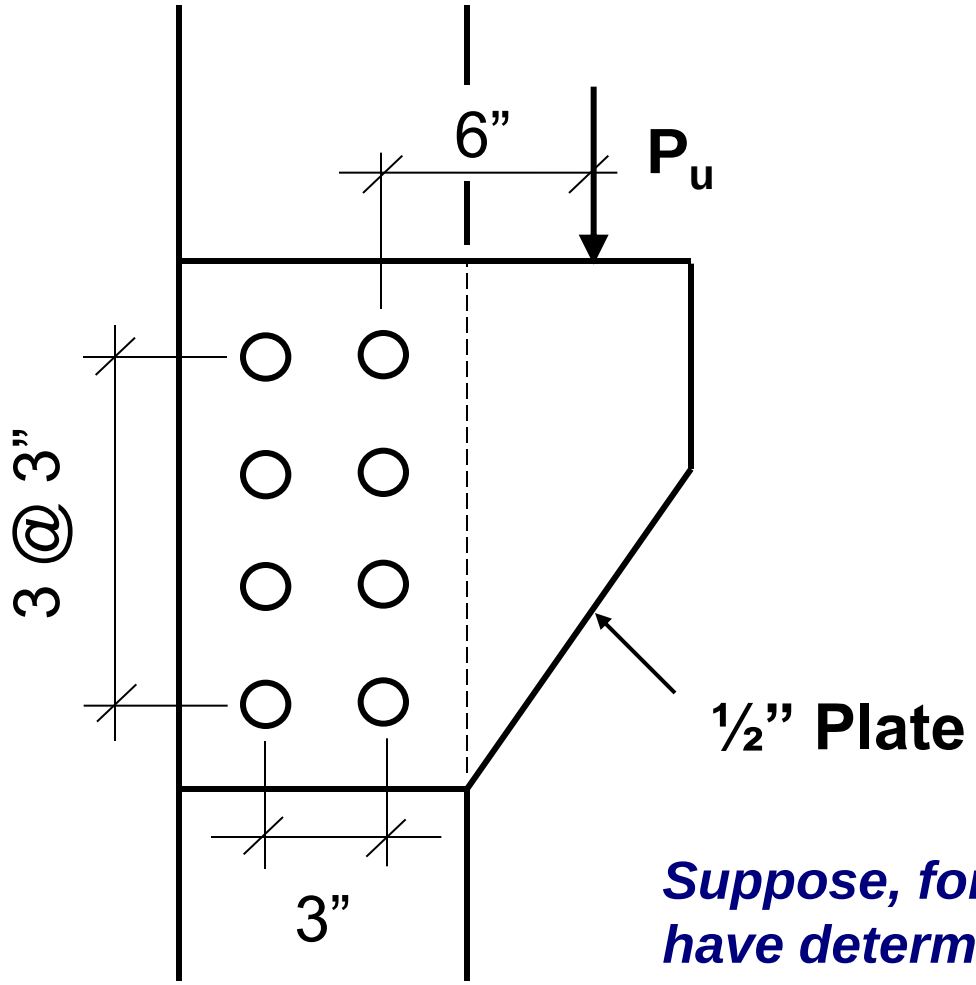
From tests
~ 0.34”

cos θ

x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$
0.3	3	3.01	0.15	25.13	1.67	75.57

Σ Fy and Σ M

Salmon et al., 4.11 (b)



- **A36 Plate**
- **Calculate maximum load, P_u**
- **Use ultimate strength method**
- **7/8” A325N bolts**

Suppose, for this example, that we have determined that bolt shear capacity controls over bolt bearing

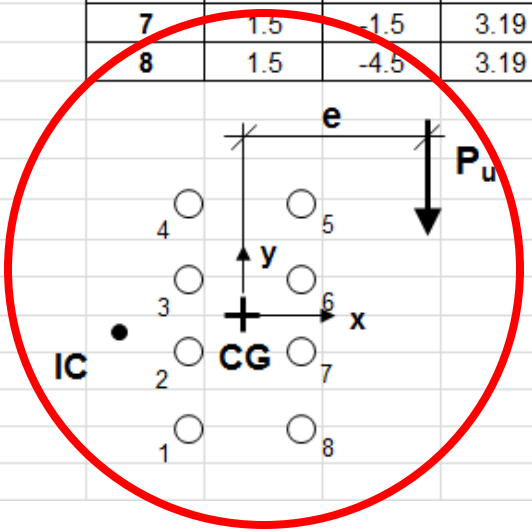
Spreadsheet

Eccentric Connections (Shear) - Ultimate Strength Method

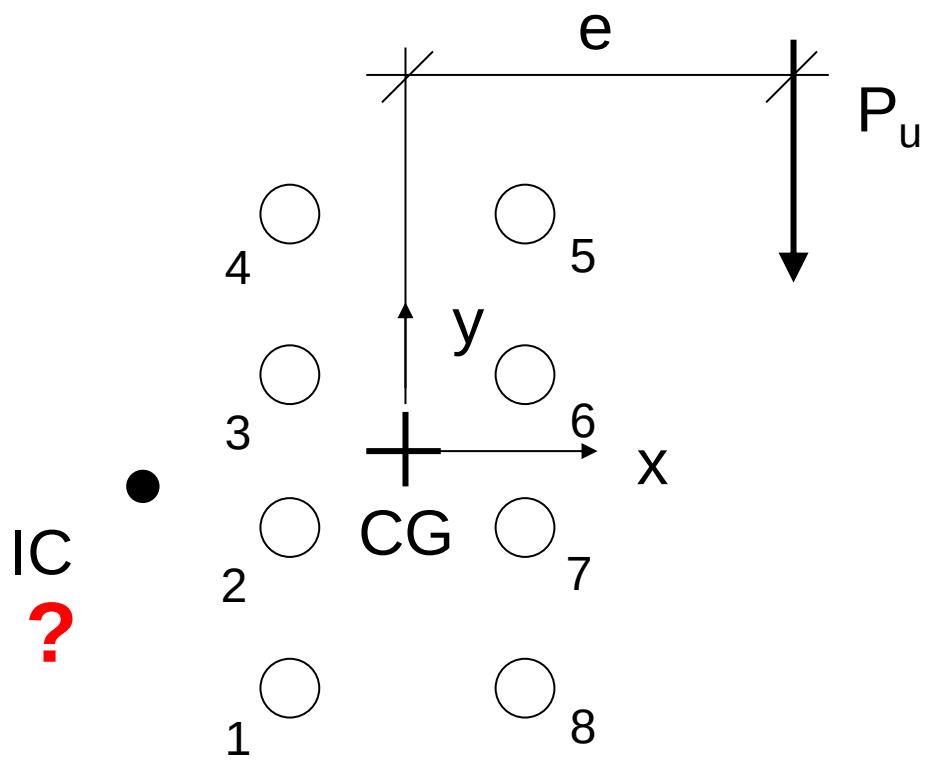
Bolt properties						Eccentricity (in)		Spacing (in)	
F_{ub}	120	*ksi*	Dia.	0.875	*in*	e	7.5	x	3
N or X	1	*N=1, X=0*						y	3

	x	y		
IC	-1.69	0	Δ_{max}	0.34
			d_{max}	5.52

	wrt CG		*wrt IC*							
Bolt	x	y	x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$	$R_i y_i/d_i$
1	-1.5	-4.5	0.19	-4.5	4.50	0.28	31.37	1.32	141.30	-31.34
2	-1.5	-1.5	0.19	-1.5	1.51	0.09	24.68	3.10	37.31	-24.48
3	-1.5	1.5	0.19	1.5	1.51	0.09	24.68	3.10	37.31	24.48
4	-1.5	4.5	0.19	4.5	4.50	0.28	31.37	1.32	141.30	31.34
5	1.5	4.5	3.19	4.5	5.52	0.34	31.90	18.45	175.95	26.02
6	1.5	1.5	3.19	1.5	3.53	0.22	30.41	27.52	107.20	12.94
7	1.5	-1.5	3.19	-1.5	3.53	0.22	30.41	27.52	107.20	-12.94
8	1.5	-4.5	3.19	-4.5	5.52	0.34	31.90	18.45	175.95	-26.02
							SUM	100.8	923.5	0.00
								ΣFy	ΣM	
							P_u =	75.59		
							P_u =		75.37	
								diff	0.29%	
								ΣFx		0



Define bolt group, etc.



Bolt Properties, define bolt group, eccentricity, etc.

Bolt properties						Eccentricity (in)		Spacing (in)	
F_{ub}	120	*ksi*	Dia.	0.875	*in*	e	7.5	x	3
N or X	1	*N=1, X=0*						y	3

	x	y
IC	-1.69	0

Δ_{max}	0.34
d_{max}	5.52

	wrt CG		*wrt IC*							
Bolt	x	y	x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$	$R_i y_i/d_i$
1	-1.5	-4.5	0.19	-4.5	4.50	0.28	31.37	1.32	141.30	-31.34
2	-1.5	-1.5	0.19	-1.5	1.51	0.09	24.68	3.10	37.31	-24.48
3	-1.5	1.5	0.19	1.5	1.51	0.09	24.68	3.10	37.31	24.48
4	-1.5	4.5	0.19	4.5	4.50	0.28	31.37	1.32	141.30	31.34
5	1.5	4.5	3.19	4.5	5.52	0.34	31.90	18.45	175.95	26.02
6	1.5	1.5	3.19	1.5	3.53	0.22	30.41	27.52	107.20	12.94
7	1.5	-1.5	3.19	-1.5	3.53	0.22	30.41	27.52	107.20	-12.94
8	1.5	-4.5	3.19	-4.5	5.52	0.34	31.90	18.45	175.95	-26.02

Locate bolts wrt C.G.

Calculate wrt *guess* for I.C.

Note, maximum Δ defined as 0.34, as shown in tests

Bolt properties						Eccentricity (in)		Spacing (in)	
F_{ub}	120	*ksi*	Dia.	0.875	*in*	e	7.5	x	3
N or X	1	*N=1, X=0*						y	3

	x	y
IC	-1.69	0

Δ_{max}	0.34
d_{max}	5.52

	wrt CG		*wrt IC*							
Bolt	x	y	x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$	$R_i y_i/d_i$
1	-1.5	-4.5	0.19	-4.5	4.50	0.28	31.37	1.32	141.30	-31.34
2	-1.5	-1.5	0.19	-1.5	1.51	0.09	24.68	3.10	37.31	-24.48
3	-1.5	1.5	0.19	1.5	1.51	0.09	24.68	3.10	37.31	24.48
4	-1.5	4.5	0.19	4.5	4.50	0.28	31.37	1.32	141.30	31.34
5	1.5	4.5	3.19	4.5	5.52	0.34	31.90	18.45	175.95	26.02
6	1.5	1.5	3.19	1.5	3.53	0.22	30.41	27.52	107.20	12.94
7	1.5	-1.5	3.19	-1.5	3.53	0.22	30.41	27.52	107.20	-12.94
8	1.5	-4.5	3.19	-4.5	5.52	0.34	31.90	18.45	175.95	-26.02

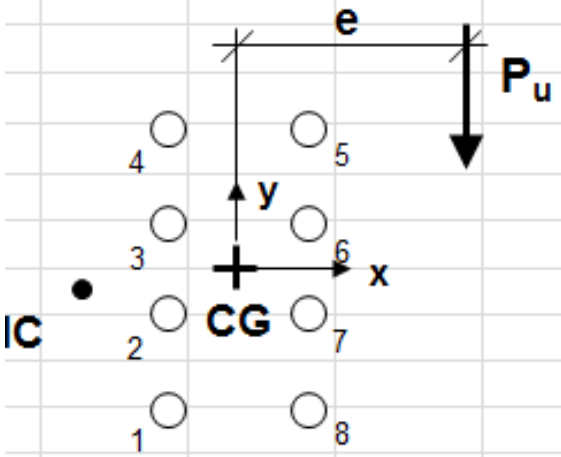
Note: R_i calculated based on R_n according to AISC → $0.563F_u^b$ for X or $0.450F_u^b$ for N (instead of τ_u)

Bolt properties						Eccentricity (in)		Spacing (in)	
F_{ub}	120	*ksi*	Dia.	0.875	*in*	e	7.5	x	3
N or X	1	*N=1, X=0*						y	3

	x	y
IC	-1.69	0

Δ_{max}	0.34
d_{max}	5.52

	wrt CG		*wrt IC*							
Bolt	x	y	x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$	$R_i y_i/d_i$
1	-1.5	-4.5	0.19	-4.5	4.50	0.28	31.37	1.32	141.30	-31.34
2	-1.5	-1.5	0.19	-1.5	1.51	0.09	24.68	3.10	37.31	-24.48
3	-1.5	1.5	0.19	1.5	1.51	0.09	24.68	3.10	37.31	24.48
4	-1.5	4.5	0.19	4.5	4.50	0.28	31.37	1.32	141.30	31.34
5	1.5	4.5	3.19	4.5	5.52	0.34	31.90	18.45	175.95	26.02
6	1.5	1.5	3.19	1.5	3.53	0.22	30.41	27.52	107.20	12.94
7	1.5	-1.5	3.19	-1.5	3.53	0.22	30.41	27.52	107.20	-12.94
8	1.5	-4.5	3.19	-4.5	5.52	0.34	31.90	18.45	175.95	-26.02
							SUM	100.8	923.5	0.00



$\Sigma Fy, \Sigma M, \Sigma Fx$

	ΣFy	ΣM	
P_u =	75.59		
P_u =		75.37	
	diff	0.29%	
	ΣFx		0

ϕ applied here

$\Sigma M = (P_u)(e+x_o\cos\delta+y_o\sin\delta)$

Note: this spreadsheet set up for $y_o = 0$ and $\delta = 0$

AISC Table 7-7

(Tables 7-6 to 7-13 for Bolt Groups)

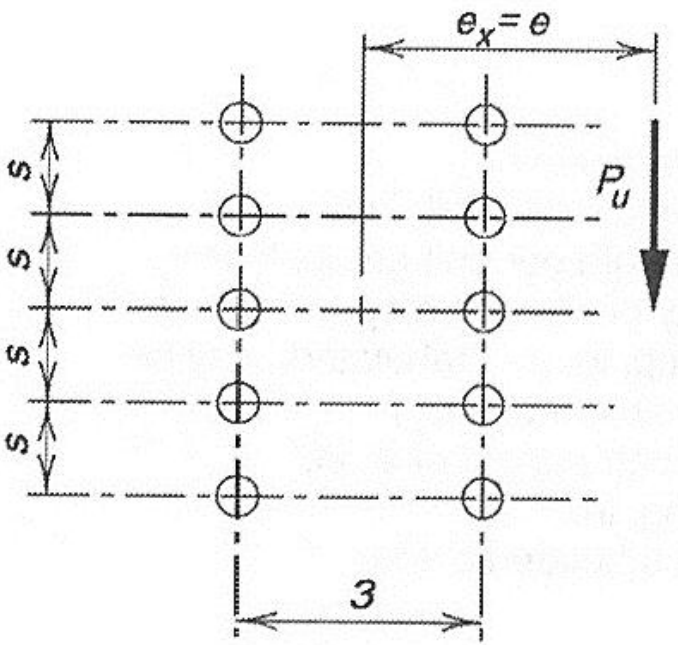


Table 7-7
Coefficients C for Eccentrically Loaded Bolt Groups
Angle = 0°

Available strength of a bolt group, ϕR_n or R_n/Ω, is determined with

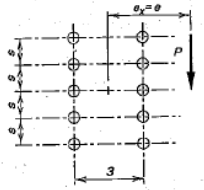
$$R_n = C \times r_n$$

or

LRFD	ASD
$C_{min} = \frac{P_u}{\phi r_n}$	$C_{min} = \frac{\Omega P_a}{r_n}$

where

P = required force, P_u or P_a, kips
r_n = nominal strength per bolt, kips
e = eccentricity of P with respect to centroid of bolt group, in. (not tabulated, may be determined by geometry)
e_x = horizontal component of e, in.
s = bolt spacing, in.
C = coefficient tabulated below

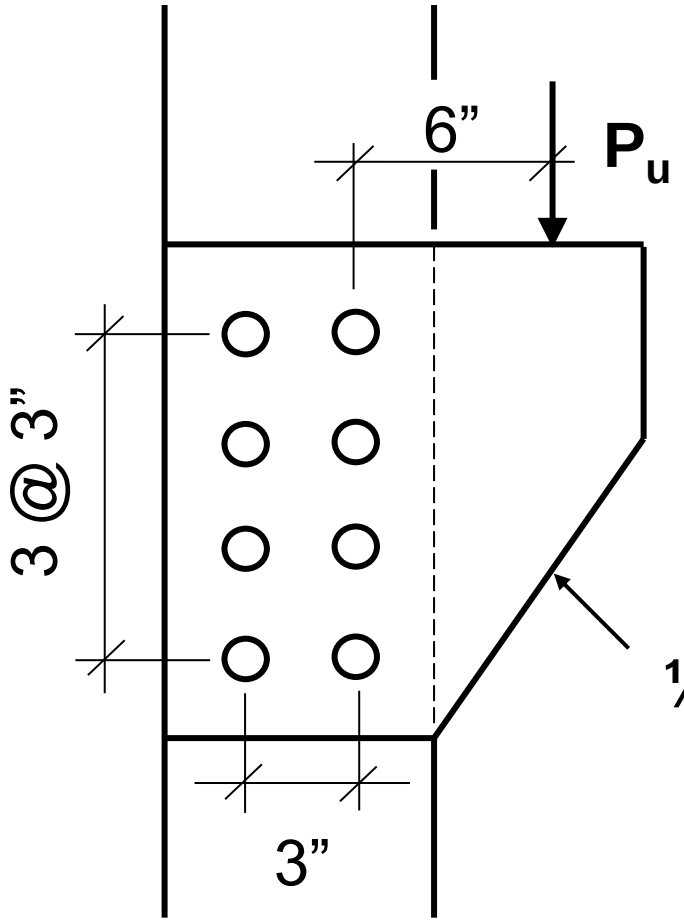


s, in.	e_x, in.	Number of Bolts in One Vertical Row, n											
		1	2	3	4	5	6	7	8	9	10	11	12
3	2	0.84	2.54	4.48	6.59	8.72	10.8	12.9	15.0	17.0	19.0	21.0	23.0
	3	0.65	2.03	3.68	5.67	7.77	9.91	12.1	14.2	16.3	18.3	20.4	22.5
	4	0.54	1.67	3.06	4.86	6.84	8.93	11.1	13.2	15.4	17.5	19.6	21.7
	5	0.45	1.42	2.59	4.21	6.01	8.00	10.1	12.2	14.4	16.5	18.7	20.8
	6	0.39	1.22	2.25	3.69	5.32	7.17	9.16	11.2	13.4	15.5	17.7	19.8
	7	0.35	1.08	1.99	3.27	4.74	6.46	8.33	10.3	12.4	14.5	16.7	18.8
	8	0.31	0.96	1.78	2.93	4.27	5.86	7.60	9.50	11.5	13.6	15.7	17.8
	9	0.28	0.86	1.60	2.65	3.87	5.34	6.97	8.75	10.7	12.7	14.7	16.8
	10	0.26	0.78	1.46	2.42	3.53	4.90	6.42	8.10	9.91	11.8	13.8	15.9
	12	0.22	0.66	1.24	2.06	3.01	4.19	5.51	7.01	8.63	10.4	12.2	14.2
	14	0.19	0.57	1.08	1.78	2.62	3.66	4.82	6.15	7.61	9.19	10.9	12.7
	16	0.17	0.51	0.95	1.57	2.32	3.24	4.27	5.47	6.79	8.23	9.78	11.4
	18	0.15	0.45	0.85	1.41	2.07	2.90	3.83	4.92	6.11	7.43	8.85	10.4
	20	0.14	0.41	0.77	1.27	1.88	2.63	3.48	4.47	5.55	6.76	8.07	9.48
	24	0.12	0.34	0.65	1.07	1.58	2.21	2.93	3.77	4.69	5.72	6.85	8.06
	28	0.10	0.29	0.56	0.92	1.36	1.90	2.53	3.25	4.05	4.95	5.93	7.00
	32	0.09	0.26	0.49	0.80	1.19	1.67	2.22	2.86	3.57	4.36	5.23	6.18
	36	0.08	0.23	0.43	0.72	1.06	1.49	1.98	2.55	3.18	3.90	4.67	5.52
	C', in.	2.94	8.33	15.8	26.0	38.7	54.2	72.2	93.1	117	143	172	204
6	2	0.84	3.24	5.39	7.47	9.51	11.5	13.5	15.5	17.5	19.5	21.5	23.4
	3	0.65	2.79	4.93	7.08	9.17	11.2	13.3	15.3	17.3	19.3	21.3	23.3
	4	0.54	2.41	4.44	6.60	8.75	10.9	12.9	15.0	17.0	19.1	21.1	23.1
	5	0.45	2.10	3.97	6.11	8.27	10.4	12.5	14.6	16.7	18.7	20.8	22.8
	6	0.39	1.85	3.55	5.62	7.77	9.93	12.1	14.2	16.3	18.4	20.4	22.5
	7	0.35	1.64	3.18	5.17	7.27	9.43	11.6	13.7	15.9	18.0	20.1	22.1
	8	0.31	1.47	2.87	4.75	6.79	8.92	11.1	13.3	15.4	17.5	19.6	21.7
	9	0.28	1.34	2.61	4.39	6.34	8.43	10.6	12.7	14.9	17.1	19.2	21.3
	10	0.26	1.22	2.39	4.06	5.92	7.96	10.1	12.2	14.4	16.6	18.7	20.9
	12	0.22	1.04	2.04	3.52	5.20	7.10	9.12	11.2	13.4	15.5	17.7	19.9
	14	0.19	0.90	1.77	3.09	4.61	6.36	8.27	10.3	12.4	14.5	16.7	18.9
	16	0.17	0.80	1.57	2.75	4.12	5.74	7.52	9.44	11.5	13.5	15.7	17.8
	18	0.15	0.71	1.41	2.48	3.72	5.21	6.87	8.68	10.6	12.6	14.7	16.8
	20	0.14	0.64	1.28	2.25	3.38	4.77	6.31	8.02	9.85	11.8	13.8	15.9
	24	0.12	0.54	1.07	1.90	2.86	4.06	5.40	6.91	8.55	10.3	12.2	14.1
	28	0.10	0.46	0.93	1.64	2.47	3.52	4.70	6.05	7.52	9.12	10.8	12.6
	32	0.09	0.41	0.81	1.44	2.18	3.11	4.16	5.37	6.69	8.15	9.71	11.4
	36	0.08	0.36	0.73	1.29	1.94	2.78	3.72	4.81	6.02	7.34	8.78	10.3
	C', in.	2.94	13.2	26.5	47.0	71.4	103	138	180	226	279	337	400

s, in.	e_x, in.	Number of Bolts in One Vertical Row, n								
		1	2	3	4	5	6	7	8	9
	2	0.84	2.54	4.48	6.59	8.72	10.8	12.9	15.0	17.0
	3	0.65	2.03	3.68	5.67	7.77	9.91	12.1	14.2	16.3
	4	0.54	1.67	3.06	4.86	6.84	8.93	11.1	13.2	15.4
	5	0.45	1.42	2.59	4.21	6.01	8.00	10.1	12.2	14.4
	6	0.39	1.22	2.25	3.69	5.32	7.17	9.16	11.2	13.4
	7	0.35	1.08	1.99	3.27	4.74	6.46	8.33	10.3	12.4
	8	0.31	0.96	1.78	2.93	4.27	5.86	7.60	9.50	11.5
	9	0.28	0.86	1.60	2.65	3.87	5.34	6.97	8.75	10.7
	10	0.26	0.78	1.46	2.42	3.53	4.90	6.42	8.10	9.91
3	12	0.22	0.66	1.24	2.06	3.01	4.19	5.51	7.01	8.63

Interpolate for e_x = 7.5 in → C=3.1

Interpolation within tables OK; interpolation *between tables not OK* (use next lower angle of loading)



7/8” A325N bolts

$$\phi R_n = C\phi r_n$$

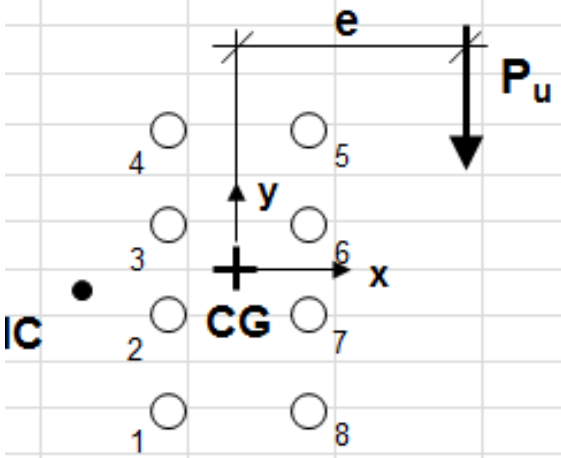
$$\phi R_n = 3.1x24.3 = 75.3kips$$

Bolt properties						Eccentricity (in)		Spacing (in)	
F_{ub}	120	*ksi*	**Dia.**	0.875	*in*	**e**	7.5	**x**	3
N or X	1	*N=1, X=0*						**y**	3

	x	**y**
IC	-1.69	0

Δ_{max}	0.34
d_{max}	5.52

	wrt CG		*wrt IC*							
Bolt	**x**	**y**	x_i	y_i	d_i	Δ_i	R_i	$R_i x_i/d_i$	$R_i d_i$	$R_i y_i/d_i$
1	-1.5	-4.5	0.19	-4.5	4.50	0.28	31.37	1.32	141.30	-31.34
2	-1.5	-1.5	0.19	-1.5	1.51	0.09	24.68	3.10	37.31	-24.48
3	-1.5	1.5	0.19	1.5	1.51	0.09	24.68	3.10	37.31	24.48
4	-1.5	4.5	0.19	4.5	4.50	0.28	31.37	1.32	141.30	31.34
5	1.5	4.5	3.19	4.5	5.52	0.34	31.90	18.45	175.95	26.02
6	1.5	1.5	3.19	1.5	3.53	0.22	30.41	27.52	107.20	12.94
7	1.5	-1.5	3.19	-1.5	3.53	0.22	30.41	27.52	107.20	-12.94
8	1.5	-4.5	3.19	-4.5	5.52	0.34	31.90	18.45	175.95	-26.02
							SUM	100.8	923.5	0.00



	ΣFy	ΣM	
P_u =	75.59		
P_u =		75.37	
	diff	0.29%	
	ΣFx		0