ABSTRACT

The network disturbance effect can be considered as either a perturbation or as a pure time delay for the exchanged signals. The network-induced time delay is one of the main challenges when a network is inserted in the feedback loops of a control system. In this paper, our objective is to improve the behavior of a Networked Control System (NCS) by analyzing the time-delay given that the decentralized control design method is adopted. First, we review an observer-based control method for decentralized control systems. Second, we establish a map between the decentralized non-networked system, and the typical NCS state-space representation. The main idea the mapping is to put the Decentralized Networked Control System (DNCS) in a general form and then map the resulting system to the typical NCS form. Third, we derive the global dynamics of the DNCS. Fourth, an upper bound for the time-delay is derived that guarantees the stability of LTI DNCSs. Finally, we present a numerical example that illustrates the applicability of the derived bound.

1 INTRODUCTION

Recent advances in integrated circuits and computer networks technologies in addition to the integration of physical systems with communication networks have led to the emergence of inexpensive large-scale Cyber Physical Systems (CPS). Various CPSs are monitored and controlled by Networked Control Systems (NCS) [1]. Wireless distributed robotic systems, power networks, and automotive vehicles are some of the most complex large-scale NCSs that are inherently decentralized and distributed. These systems integrate complex distributed control systems and modern communication networks. Hence, they are viewed as Decentralized Networked Control Systems (DNCS), which is an important sub-class of NCSs. In addition to the stakeholders’ decision making aspect of these systems, the overall performance and resilience of CPSs and DNCSs are significantly influenced by the network communication and control aspect for such systems.

NCSs technologies have great impact in industrial systems where all levels of system components are being integrated.
through different types of communication networks. The increase in the usage of NCS-based technologies have highlighted the need to do more research in this area, specifically to tackle the challenges that are caused by the existence of the communication network in the feedback loops of control systems. Some of these challenges include but are not limited to: time-induced delay, network perturbation and disturbance, cyber-attacks, overall system stability, and packets loss. Furthermore, NCSs are found in a variety of CPSs and applications such as: passenger cars, trucks and buses, aircraft and aerospace electronics, factory automation, medical equipment, mobile sensor networks and many more [3].

The growing interest in NCSs is motivated by many benefits they offer such as the ease of maintenance and installation, systems’ weight reduction, increase in reliability, the large flexibility and the low cost [2]. Sampling, encoding, decoding, medium access scheduling, data packet dropouts and the finite bandwidth of the network, are all factors that challenge the insertion of communication networks in the feedback loops of any control system. All these factors cause a network-induced delay which in turn causes a deterioration of the system performance or can even destabilize it.

In this paper, we address the network-induced delay challenge. In particular, we derive an upper bound on the time-delay induced by the communication network. In addition, the determination of an upper bound on the networked-induced time-delay is crucial in the design of an NCS so that a suitable sampling period is chosen.

1.1 Motivation and Problem Description

As mentioned in the previous section, one of the main challenges in NCS design and analysis is the problem of the network-induced delay. There are two approaches to control the network-induced delay: either through the “control of network” actions, or through “control over network” strategies.

The proposed methods and formulations in this paper are related to the “control over network” design strategy. Generally, there are several types of control and various methods to design controllers; not all of them are applicable for networked systems. For example, control systems are classified as either centralized or decentralized. In complex dynamical systems, centralized control can be optimal, but in many cases it is neither robust nor scalable. This is due to the high computational complexity of employing such centralized controllers. In addition, the centralized control for a distributed system over a large geographical region needs a large amount of information to be exchanged through a communication network. Accordingly, this causes long delays and loss of data that degrades the quality of the data transmitted and received. Furthermore, adapting centralized control makes it harder to apply physical changes to systems. Moreover, large-scale systems are generally composed of smaller subsystems. When the communication between the smaller subsystems is poorly modeled, the centralized control method becomes ineffective. On the other hand, the decentralized control is more suitable for NCSs because it reduces the traffic to be exchanged, which in turn reduces the network traffic and the expected time-delay for each network user.

In control theory, there is rich literature on decentralized control covering various methods of design and analysis of controllers. Most of the decentralized control theory had been introduced before the emergence of networked systems that close the feedback loops through communication networks. As a result, the area of DNCS has recently emerged. In this paper, we introduce a general framework that maps the framework of decentralized control of non-networked systems to the framework of NCS. This mapping process is important to conduct a research that applies the theory of decentralized control of non-networked systems into modern networked control systems.

1.2 Literature Review

In modern control theory, one of the fundamental problems that has been a subject of significant research, is the development of methods for control of dynamical systems that are composed of complex interconnected subsystems [4]– [11]. Generally, the controller design problem has been addressed for large-scale dynamical systems by considering either centralized or decentralized control. The robust design of the decentralized control strategies has been introduced in [12]– [14]. In [15], the authors proposed an observer-based control algorithm for linear systems where the design uses low-order linear functional observers. The individual subsystem states are estimated in [16] and [17] by using a dynamic observer. Observer-based control design for non-linear systems is introduced in [18]– [21]. The key feature of the design proposed in [18] is that the separation principle for linear systems holds in their design of non-linear systems. Another approach for controlling large-scale systems is the quasi-decentralized control which compromises between the complex centralized control and decentralized control that has performance limitations [19]– [21]. Quasi-decentralized control can be considered as a distributed control strategy in the sense that most control signals are collected and processed locally. In the mean time, some signals are transferred between the local units and controllers to properly account for the interactions and reduce the propagation of process failures.

In general, there is a rich literature that deals with the control of large-scale systems, whether these systems adapt the centralized, decentralized or mixed as in the quasi-decentralized
control methods. Nonetheless, most of the aforementioned approaches are introduced for non-networked systems. There are relatively few efforts [22]–[24] that tackle the decentralized control problem for large-scale systems that is concerned with networked dynamical systems. In [27], we analyzed the network effect as perturbation to the signals exchanged through the network. In this paper, we provide a general framework that maps the general framework of decentralized control of non-networked systems to the framework of NCS. This framework is then used to design and analyze an observer-based control for decentralized networked control systems, assuming that the network effect is modeled as a pure time-delay.

This paper is organized as follows. In Section 2 we review the mathematical formulation of the observer-based control design. In Section 3 we present the DNCS configuration and map the DNCS setup to the typical NCS setup. The stability analysis of the proposed design is introduced in Section 4. Numerical examples and simulation results are discussed in Section 5. Section 6 is dedicated for closing remarks and conclusions.

2 OBSERVER BASED CONTROL DESIGN FORMULATION REVIEW

In this section we review the observer-based design of the controller of the non-networked system, which is followed by the DNCS configuration in Section 3. In this paper, we are considering the observer based control design from [15]. Consider a large-scale system where the plant dynamics are described as follows:}

\[
\begin{align*}
\dot{x}_p &= A_p x_p + \sum_{i=1}^{N} B_i u_i, \\
y_i &= C_i x_p, \quad i = 1, 2, \ldots, N
\end{align*}
\]

where \( x \in \mathbb{R}^n \) is the state vector of the plant of the system, \( u_i \in \mathbb{R}^m \) is the input vector of the \( i \)th subsystem and \( y_i \in \mathbb{R}^p \) is the output vector of the \( i \)th subsystem. \( A_p \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times m_i}, \) and \( C_i \in \mathbb{R}^{p_i \times n} \) are real constant matrices. Without loss of generality, \( A_p \) could be a block-diagonal concatenation of \( N \) different sub-systems or plants, assuming that there is no interaction between the states of each individual sub-system. Furthermore, if there are interdependencies between the subsystems, \( A_p \) won’t be a block-diagonal matrix. Let

\[
\begin{align*}
&u = [u_1^\top \ldots u_N^\top]^\top, \quad y = [y_1^\top \ldots y_N^\top]^\top \\
&B_p = [B_1 \ldots B_N], \quad C_p = [C_1^\top \ldots C_N^\top]^\top.
\end{align*}
\]

Then the plant dynamics can be written in the following compact form:

\[
\begin{align*}
\dot{x}_p &= A_p x_p + B_p u \\
y &= C_p x_p.
\end{align*}
\]

We assume that the assumptions on the system dynamics from [15] are also valid in this paper (system controllability, observability, . . . ). As in [15], we also assume that a global state feedback control exists such that \( u = -F x \), where \( F \in \mathbb{R}^{m \times n} \). The global state feedback control gain \( F \) can be obtained by using any standard state feedback control method. Partitioning the global controller \( u \), we get:

\[
\begin{bmatrix}
  u_1 \\
u_2 \\
\vdots \\
u_N
\end{bmatrix} =
\begin{bmatrix}
  F_1 \\
F_2 \\
\vdots \\
F_N
\end{bmatrix} x.
\]

The authors in [15] proposed the following decentralized controller:

\[
u_i = -F_i x \approx -(K_i L_i + W_i C_i)x \approx -K_i z_i - W_i y_i,
\]

where \( z_i \in \mathbb{R}^{n_i} \) is an estimate of the weighted plant state (\( z_i \) tracks \( L_i x \)) that has the following dynamics:

\[
z_i = E_i z_i + L_i B_i u_i + G_i y_i, \quad (2)
\]

where

\[
E_i \in \mathbb{R}^{n_i \times n_i}, L_i \in \mathbb{R}^{n_i \times m_i}, K_i \in \mathbb{R}^{m_i \times n_i}, W_i \in \mathbb{R}^{m_i \times p_i}, \quad \text{and } G_i \in \mathbb{R}^{n_i \times p_i},
\]

are all real matrices that represent the controller design parameters [15], and

\[
F_i \approx K_i L_i + W_i C_i. \quad (3)
\]

The observation error vector is defined as:

\[
e_{o_i} = z_i - L_i x, \quad i = 1, 2, \ldots, N.
\]

Therefore, the observation error dynamics are:

\[
e_{o_i} = z_i - L_i x.
\]
After some simple manipulations, we obtain the following equation:

\[
\dot{e}_i = E_i e_i + (G_i C_i - L_i A + E_i L_i) x - L_i B_i u_i. \tag{4}
\]

\(B_i\) is a partition of \(B, B = [B, B_1]\), where \(B_1 \in \mathbb{R}^{n \times (m - m_i)}\) is the input matrix for \(u_i(t)\) which contains \((N - 1)\) input vectors of the remaining \((N - 1)\) subsystems. With this particular partition of the input matrix \(B\), the dynamics of the plant states are:

\[
x = Ax + B_i u_i + B_j u_j, \quad i = 1, 2, \ldots, N.
\]

Choosing \(E_i\) to be asymptotically stable, (2) can be viewed as a decentralized linear observer if \(L_i\) and \(G_i\) fulfill the following set of constraints:

\[
L_i B_i = 0 \tag{5}
\]

\[
K_i L_i + W_i C_i = F_i \tag{6}
\]

\[
G_i C_i - L_i A + E_i L_i = 0, \tag{7}
\]

Ha and Trinh in [15] and Elmahdi et al in [27] proposed two different methods to solve the above system of equations. After solving for the system design unknowns \((G_i, L_i, K_i, W_i)\), we now have all the design parameters for the observer-based controller. In the next section, we will map the observer-based control dynamics' parameters to the typical NCS formation.

3 PROBLEM FORMULATION AND DNCS SETUP

To analyze the DNCS design, we map the DNCS formation to the equivalent NCS setup. In other words, we start with a controller design method for the non-networked system of decentralized control, then we map the closed-loop non-networked system to its equivalent configuration in networked dynamical system in order to apply the stability analysis tools. The general setup of a Large Scale System (LSS) DNCS is shown in Figure 1.

3.1 Mapping the DNCS to the NCS Setup

We now convert the DNCS setup to the general setup of the NCS. For simplicity, we consider the case of a lumped delay between the sensor and the controller as shown in Figure 2. The controller's output \((u(t))\) and input \((\hat{y}(t))\) are defined as:

\[
u(t) = C_p x_p(t) + D_p C_p x_p(t - \tau)
\]

\[
\hat{y}(t) = y(t - \tau) = C_p x_p(t - \tau).
\]

In our discussion we analyze the behavior of the system between transmission times. The plant and controller state dynamics can be written as:

\[
\dot{x}_p(t) = A_p x_p(t) + B_p C_p x_p(t - \tau)
\]

\[
\hat{x}(t) = A_p x_p(t) + B_p C_p x_p(t - \tau).
\]

To analyze stability, we need to combine the nominal system with the perturbation in one state. The perturbation represents the error due to the network existence in the feedback loops of the system, and the observation error. From the observer-based control design we have the following representation:

\[
\text{Plant: } \begin{cases}
\dot{x}_p(t) = A_p x_p(t) + B_p u(t) \\
y_p(t) = C_p x_p(t)
\end{cases} \tag{8}
\]

\[
\text{Controller/Observer: } \begin{cases}
z(t) = E_p z(t) + L_p u(t) + G_p \hat{y}(t) \\
u(t) = -K z(t) - W \hat{y}(t)
\end{cases} \tag{9}
\]
where \( E, L, G, K \) and \( W \) are the observer-based control design parameters in compact form.

Recall that the observer-based controller’s goal is to track a linear weighted combination of the plant state, \( \{z(t) \rightarrow Lx_p(t - \tau)\} \), and the observation error can be written as: \( \epsilon_o(t) = z(t) - Lx_p(t) \), while the plant output is \( \hat{y}(t) = y(t - \tau) = C_p x_p(t - \tau) \). The controller’s output \( u(t) \) can be found as follows:

\[
u(t) = -Kz(t) - W\hat{y}(t)
\]

\[
= -K(Lx_p(t - \tau) + e_o(t)) - WC_p x_p(t - \tau)
\]

\[
= -[(KL + WC_p) - (F - (KL + WC_p))] x_p(t - \tau)
\]

\[
-Ke_o(t),
\]

where \( F \approx KL + WC_p \) and \( \Delta F = F - (KL + WC_p) \). We can write \( u(t) \) as:

\[
u(t) = -Fx_p(t - \tau) + \Delta F x_p(t - \tau) - Ke_o(t). \tag{10}
\]

Substituting (10) in the plant dynamics \((\dot{x}_p(t))\) equation, we get:

\[
x_p(t) = A_p x_p(t) + (B_p \Delta F - B_p F) x_p(t - \tau) - B_p Ke_o(t). \tag{11}
\]

The controller dynamics \((\dot{z}(t))\) or \((\dot{x}(t))\) can be written as:

\[
z(t) = E z(t) + LB_p u(t) + G \hat{y}(t)
\]

\[
= E z(t) + LB_p (-Kz(t) - W\hat{y}(t)) + G \hat{y}(t)
\]

\[
= (E - LB_p K)z(t) + (G - LB_p W)\hat{y}(t).
\]

Letting \( z(t) = x(t) \), we can rewrite the controller dynamics as:

\[
\dot{x}(t) = A_p x(t) + B_p \hat{y}(t) = A_p x(t) + B_p C_p x_p(t - \tau). \tag{12}
\]

where \( A_p = E - LB_p K \), \( B_p = G - LB_p W \). and from \( u(t) = -Kz(t) - W\hat{y}(t) \), we get \( C_c = -K \) and \( D_c = -W \).

\[4.3 \text{ Observation Error Dynamics and Closed-Loop States Augmentation} \]

The observation error dynamics can be formulated as:

\[
\dot{e}_o(t) = E z(t) + LB_p u(t) + G \hat{y}(t) - LA_p x_p(t)
\]

\[
- LB_p u(t) + ELx_p(t) - ELx_p(t)
\]

\[
= E (z(t) - Lx_p(t)) + GC_p x_p(t - \tau)
\]

\[
- LA_p x_p(t) + ELx_p(t)
\]

\[
= E e_o(t) + GC_p x_p(t - \tau) - LA_p x_p(t)
\]

\[
+ ELx_p(t) + GC_p x_p(t - \tau) - GC_p x_p(t)
\]

\[
\dot{e}_o(t) = E e_o(t) + (GC_p - LA_p + EL) x_p(t)
\]

\[
+ GC_p x_p(t - \tau) - GC_p x_p(t)
\]

Hence,

\[
\dot{e}_o(t) = E e_o(t) + (\Delta M - GC_p) x_p(t) + GC_p x_p(t - \tau), \tag{13}
\]

where \( \Delta M = GC_p - LA_p + EL \). Using the previous derivations in (12), (11), and (13), we can augment the general state \( x(t) \) with the observation error as follows:

Let \( w = [x_p(t) \hspace{1cm} x(t) \hspace{1cm} e_o(t)] \)

\[
A_0 = \begin{bmatrix} A_p & O & B_p C_p \\ O & A_p & O \\ \Delta M - GC_p & O & E \end{bmatrix}
\]

\[
A_1 = \begin{bmatrix} -B_p F & O & O \\ B_p C_p & O & O \\ GC_p & O & O \end{bmatrix}
\]

Hence, we can write the augmented state dynamics as,

\[
w(t) = A_0 w(t) + A_1 w(t - \tau). \tag{14}
\]
Equation (14) represents the dynamics of the overall system, combining the nominal system dynamics and the perturbation due to the network. We can obtain an expression for \( w(t) \) by using the Taylor series expansion formula as follows,

\[
\begin{align*}
  w(t) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} w^{(n)}(t) \\
  &= w(t) - \tau w(t) + R_2(w, \tau).
\end{align*}
\]

Neglecting the higher order terms, we get the following approximation for \( w(t - \tau) \):

\[
w(t - \tau) \approx w(t) - \tau w(t).
\] (15)

Substituting (15) into (14), we get:

\[
\dot{w}(t) = A_0 w(t) + A_1 w(t) - \tau A_1 \dot{w}(t).
\]

Rearranging,

\[
w(t) = (I + \tau A_1)^{-1} (A_0 + A_1) w(t).
\] (16)

If there is no network in the feedback-loops of the control system, then the system representation will be reduced to the nominal non-networked system which is

\[
w(t) = (A_0 + A_1) w(t) \Rightarrow \dot{w}(t) = A w(t).
\]

In what follows, we provide a bound on the time-delay (\( \tau \)) that guarantees the asymptotic stability of the global DNCS state dynamics (\( w(t) \)).

**Theorem 1.** For the DNCS represented in (8,9), let the globally exponentially stable equilibrium point of the closed-loop non-networked system be \( w = 0 \), and \( V(w) = w^T P w \) be a Lyapunov function that satisfies \( A^T P + P A = -2Q \), where \( P \) is a symmetric positive definite matrix. If the origin is a globally exponentially stable equilibrium point of the DNCS, then the network-induced time delay \( \tau \) satisfies:

\[
\tau^2 \|PA_1^2 A\| - \tau \|PA_1 A\| - \lambda_{\text{max}}(Q) < 0.
\] (17)

**Proof.** We have the Lyapunov function \( V(w) = w^T P w \). Taking the derivative, we get,

\[
\dot{V}(w) = 2w^T P \dot{w} = w^T P \dot{w} + \dot{w}^T P \dot{w}.
\] (18)

Substituting (16) into (18), we get:

\[
\dot{V}(w) = w^T A^T (I + \tau A_1)^{-1} P w + w^T (I + \tau A_1)^{-1} A w.
\]

Let \( H = (I + \tau A_1)^{-1} \Rightarrow \dot{V}(w) = w^T A^T H^T P w + w^T PHAw \). We can write \( \dot{V}(w) \) as:

\[
\begin{align*}
  \dot{V}(w) &= w^T A^T P P^{-1} H^T P w + w^T PPH^{-1} PA w \\
  &= w^T A^T (PP^{-1}H^T P w + w^T PPH^{-1} PA w \\
  &= w^T A^T (PP^{-1}H^T P w + w^T PPH^{-1} PA w \\
  &= w^T P (H - I) Aw - 2w^T Q w \\
  &= 2w^T (PHP^{-1} - I) PA w - 2w^T Q w \\
  &= 2w^T P (H - I) Aw - 2w^T Q w.
\end{align*}
\] (19)

Using the Neumann series formula for the inverse of the sum of two matrices,

\[
H = (I + \tau A_1)^{-1} = I - \tau A_1 + \tau^2 A_1^2 - \tau^3 A_1^3 + \ldots
\] (20)

Substituting (20) into (19),

\[
\dot{V}(w) = 2w^T P(I - \tau A_1 + \tau^2 A_1^2 - \tau^3 A_1^3) Aw - 2w^T Q w \\
\]

In addition, for any symmetric matrix \( Q \) we have:

\[
\lambda_{\text{min}}(Q) \|x\|^2 \leq x^T Q x \leq \lambda_{\text{max}}(Q) \|x\|^2.
\]

We can now upper bound the derivative of the Lyapunov candidate function \( \dot{V}(w) \) as follows:

\[
\begin{align*}
  \dot{V}(w) &\leq 2\tau^2 \|PA_1^2 A\| \|w\|^2 - 2\tau \|PA_1 A\| \|w\|^2 \\
  &\leq -2\lambda_{\text{max}}(Q) \|w\|^2.
\end{align*}
\]

From the above, when the origin is a globally exponentially stable equilibrium point of the DNCS, then

\[
\tau^2 \|PA_1^2 A\| - \tau \|PA_1 A\| - \lambda_{\text{max}}(Q) < 0.
\]
5 NUMERICAL SIMULATIONS

In this section, and to further assess the applicability of the derived bound on the maximum time delay ($\tau$), we introduce a numerical example to analyze the effect of the pure time-delay considering the observer-based decentralized control.

Consider a fourth order system with the following plant state space representation:

$$\begin{cases}
\dot{x}_p(t) = A_p x_p(t) + B_p u(t) \\
y_p(t) = C_p x_p(t),
\end{cases}$$

(21)

where,

$$A_p = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 5 & 6 & 7 & -8 \\ 9 & 10 & 11 & -12 \\ 13 & 14 & 15 & -16 \end{bmatrix} \in \mathbb{R}^{4 \times 4}, B_p = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 11 & -12 \\ 21 & 43 \\ 31 & 25 \end{bmatrix} \in \mathbb{R}^{4 \times 4},$$

and $C_p = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{6 \times 4}$.

The eigenvalues of $A_p$ are:

$$\text{eig}(A_p) = \{1.00 + 5.56i, 1.00 - 5.56i, 0.0\}.$$

Hence, the non-networked non-controlled plant is initially unstable. In addition, the plant dynamics satisfies the assumptions in [15]. We now start with the design of the observer based decentralized controller by solving the system of equations (5–7) as in [15]. The algorithm used to design the non-networked controller is highlighted in [15, p. 725]. We assume that the first two columns of $B_p$ are equivalent to $B_1$, while the rest are $B_{i \neq 1}$. Also, the first three rows of $C_p$ constitute $C_1$. In this example, we use a full-order observer of size four (i.e., $\hat{x}_o(t) = z(t) \in \mathbb{R}^4$) and that the observation error ($\varepsilon_o(t)$) is neglected. Choosing $E$ to be Hurwitz, we can find the observer-based controller parameters ($F, L, K, W, G$), and write the controller’s dynamics as

$$C: \begin{cases}
z(t) = E z(t) + L B_p u(t) + G y(t) \\
u(t) = -K z(t) - W y(t),
\end{cases}$$

where,

$$E = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, L = \begin{bmatrix} 0.81 & 0.19 & 0.23 & -0.38 \\ -0.44 & 0.65 & 0.35 & -0.38 \\ 0.81 & 0.19 & 0.23 & -0.38 \\ -0.44 & 0.65 & 0.35 & -0.38 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.53 & -0.09 & -1.57 & 0 & 0 \\ 1.04 & 0.15 & -1.63 & 0 & 0 \\ 0 & 0 & -0.17 & 0.69 & -1.57 \\ 0 & 0 & 0 & 1.52 & 1.55 & -1.75 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.03 & -0.45 & 0 & 0 \\ 0.27 & 1.45 & 0 & 0 \\ 0 & 0 & 0.92 & 0.52 \\ 0 & 0 & -0.07 & -0.65 \end{bmatrix},$$

and $W = \begin{bmatrix} 0.21 & -0.01 & 0.26 & 0 & 0 & 0 \\ 0.14 & 0.30 & -0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.21 & 0.94 & -0.66 \\ 0 & 0 & 0 & 0.06 & -0.44 & 0.87 \end{bmatrix}$.

After designing the observer-based controller for the system, we can write the networked dynamics of the overall system as in (16) where $A_0$ and $A_1$ are computed based on the non-networked system dynamics parameters. The dynamics of the non-networked ($\tau = 0$) controlled system can be written as

$$\dot{w}(t) = (A_0 + A_1)w(t) = Aw(t).$$

The eigenvalues of the non-networked closed loop system are:

$$\text{eig}(A) = \{-1.76 \pm 5.52i, -1.53, -1.20, -0.40, -0.11, -0.50, -0.10\}.$$

Hence, the designed controller stabilizes the overall closed-loop system, as $A$ is Hurwitz. Figure 3 shows the stable behavior of the plant for random inputs.

To analyze the effect of the time-delay, we apply Theorem 1 by finding a Lyapunov function $V(w) = w^T P w$ and a symmetric positive definite matrix $P$ that satisfies $A^T P + PA = -2Q$, where $Q = Q^T > 0$. As mentioned in the previous section, the objective of this numerical simulation is to test the applicability of the derived time-delay bound derived in Theorem 1, such that the networked system with the observer-based controller is globally stable.
Recall that for the origin to be a globally exponentially stable equilibrium point of the DNCS, then

\[
\tau^2 \|PA_0^2A\| - \|PA_1A\| - \lambda_{\max}(Q) < 0.
\]

After finding \( P \) and calculating \( A_0, A_1 \) and \( \bar{A} = A_0 + A_1 \), the maximum value of the time-delay that would keep the networked system stable is \( \tau_{\text{max}} = 0.2309 \text{sec} \) (computed by evaluating the quadratic bound on \( \tau \) from Theorem 1). We now set \( \tau = \tau_{\text{max}} - \varepsilon \) and simulate the networked dynamics of the system (a reference input is added), where \( \varepsilon \) is a small positive constant since \( \tau < \tau_{\text{max}} \). For \( \tau = \tau_{\text{max}} - \varepsilon = 0.202 \text{sec} \). The eigenvalues of

\[
\bar{A} = (I + \tau A_1)^{-1}(A_0 + A_1)
\]

are: \( \{-13.4 \pm 16.6i, -2.1, -1.2, -0.4, -0.1, -0.5, -0.1\} \).

Checking the stability of (16) for \( \tau = \tau_{\text{max}} \), we find that the system becomes unstable with the following eigenvalues of \( \bar{A} \):

\[\{249.5, -19.6, -2.2, -1.2, -0.4, -0.5, -0.1, -0.1\}\].

Therefore, the derived \( \tau \) bound is very accurate. Starting from random initial conditions and random inputs, Figures 4 shows the stable behavior of the plant for \( \tau = \tau_{\text{max}} - \varepsilon = 0.202 \text{sec} \) assuming random initial conditions and random inputs. The networked-controlled plant states are stable and converging within few seconds, as shown in Figure 4.

#### 6 CONCLUSIONS

The network-induced disturbances is one of the main demerits of installing communication networks in modern control systems. The networked-induced time-delay is an example of networked-induced disturbances. In this paper, we analyze the effect of modeling the network as a pure time-delay on the linearized time-invariant dynamics of any plant. In particular, we study how to improve the behavior of Decentralized Networked Control Systems by establishing realistic bound on the time-delay induced by the communication network. This bound would lead, if satisfied, to the overall closed-loop stability.

First, we review an observer-based decentralized control method for decentralized control systems. Second, we establish a map between the decentralized non-networked system, and the typical NCS state-space representation. The main idea of our design is to put the DNCS in the general form and then map the resulted system to the general form of the NCS. This would facilitate deriving the maximum time-delay bound for the observer-based control of the NCS. Third, treating the network effect as pure time-delay, we derive the global dynamics of the DNCS. Fourth, an upper bound for the time-delay is derived, that guarantees the global exponential stability of any linear time-invariant DNCS. Finally, we present a numerical example that illustrates the applicability of the proposed upper bound of the time-delay. Results show that the overall closed-loop system is stable, assuming that the networked induced delay is within the derived bound.

As mentioned in Section 1, the determination of an upper
bound on the networked-induced time-delay is very crucial in the design of an NCS so that a suitable sampling period is chosen. When the time-delay is larger than the sampling period in an NCS, then the global stability of the overall NCS cannot be guaranteed. Therefore, having an accurate bound on the time-delay for DNCS applications is crucial for both, the design and stability of such systems. In future work, the bound derived can be used to analyze the effect of this time-delay scheduling protocols in NCS applications.

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