Bilevel Formulation of a Policy Design Problem Considering Multiple Objectives and Incomplete Preferences

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A bilevel optimization formulation of policy design problems considering multiple objectives and incomplete preferences of the stakeholders is presented. The formulation is presented for Feed-in-Tariff (FIT) policy design for decentralized energy infrastructure. The upper-level problem is the policy designer’s problem and the lower-level problem is a Nash equilibrium problem resulting from market interactions. The policy designer has two objectives: maximizing the quantity of energy generated and minimizing policy cost. The stakeholders decide on quantities while maximizing net present value and minimizing capital investment. The Nash equilibrium problem in the presence of incomplete preferences is formulated as a stochastic linear complementarity problem and solved using expected value formulation, expected residual minimization formulation, and Monte-Carlo technique. The primary contributions in this paper are mathematical formulation of the FIT policy, the extension of computational policy design problems to multiple objectives, and the consideration of incomplete preferences of stakeholders for policy design problems.

Keywords: Multi-objective optimization; bilevel optimization; distributed generation; energy policy; Nash equilibrium

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Number of stakeholders</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Payment per unit energy generation over the market price</td>
</tr>
<tr>
<td>( T )</td>
<td>Duration of the policy</td>
</tr>
<tr>
<td>( q_i )</td>
<td>Quantity produced by the ( i^{th} ) stakeholder</td>
</tr>
<tr>
<td>( Q )</td>
<td>Total market quantity</td>
</tr>
<tr>
<td>( Q_{\text{norm}} )</td>
<td>Normalization factor for overall quantity</td>
</tr>
<tr>
<td>( C_{\text{norm}} )</td>
<td>Normalization factor for policy cost</td>
</tr>
<tr>
<td>( \alpha, \beta )</td>
<td>Constants in the market price model</td>
</tr>
<tr>
<td>( V_i )</td>
<td>Net present value of ( i^{th} ) stakeholder</td>
</tr>
<tr>
<td>( I_i )</td>
<td>Capital investment of ( i^{th} ) stakeholder</td>
</tr>
<tr>
<td>( I )</td>
<td>Capital investment per unit quantity generated</td>
</tr>
<tr>
<td>( V_{\text{norm}} )</td>
<td>Normalization factor for net present value</td>
</tr>
<tr>
<td>( I_{\text{norm}} )</td>
<td>Normalization factor for capital investment</td>
</tr>
<tr>
<td>( C_m )</td>
<td>Operation and maintenance cost</td>
</tr>
<tr>
<td>( C )</td>
<td>Policy cost</td>
</tr>
<tr>
<td>( p_m )</td>
<td>Market price</td>
</tr>
<tr>
<td>( E_q )</td>
<td>Set of quantities satisfying the equilibrium constraints</td>
</tr>
<tr>
<td>( j )</td>
<td>Discount rate</td>
</tr>
<tr>
<td>( w_Q )</td>
<td>Policy designer’s weight for quantity</td>
</tr>
<tr>
<td>( w_{v,i} )</td>
<td>Weight for ( i^{th} ) stakeholder’s net present value</td>
</tr>
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1. Introduction

1.1. Policy Design for Sustainable Energy Systems

With the increasing use of small-scale energy generation from renewable sources and increasing deregulation of the energy sector, an alternate paradigm of energy generation and distribution is emerging and leading towards the smart grid architecture. A smart electric grid is a large-scale complex system consisting of decision makers ranging from consumers to utilities, and micro-grid operators to participants of the distribution infrastructure. For such large-scale complex systems, the stakeholders independently make technical decisions within rules and regulations, to achieve their objectives of system performance, reliability, security and load demand while maximizing their profits. The decisions of the stakeholders can be influenced by designing appropriate policies, provision of incentives and development of standards, thereby affecting the design of the entire system. For example, the consumers can play an active role as energy producers for actively managing their demand. They make decisions about a) which technologies to invest in, b) how much energy to generate, c) how much energy to buy and from whom, d) how much energy to sell and e) how much to participate in actively managing their energy demand (Chicco and Mancarella 2009). Other stakeholders include power producers (e.g., utility companies), grid operators, transmission companies (TRANS CO), distribution companies (DISTCO) and regulators (e.g., government and other regulating authorities). The goals of different stakeholders, as shown in Figure 1, are often conflicting in nature.

Examples of policy decisions include establishing renewable portfolio standards, carbon taxes, and incentives for adoption of renewable energy technology. These policy decisions
are driven by various technical, environmental, economic and social objectives (Cou-
ture et al. 2010). The technical objectives include replacing fossil fuel generating plants
with renewables, minimization of system losses, maintaining required stability/ security/
reliability, ensuring balance between demand and supply, meeting power quality require-
ments, peak shaving, targeting high efficiency systems, innovation and early adoption
of technologies and meeting the energy needs. The environmental objectives are min-
imization of greenhouse gas emissions and hazardous materials. Economic objectives
include minimization of policy costs and ratepayer impact. Social objectives include job
creation, economic development, meeting long-term energy requirements, policy trans-
parency, fairness and quality of life. Hence, these policy decisions have an impact on the
overall sustainability of large-scale complex systems.

Policy design problems share a number of similarities with multi-disciplinary engineer-
ing design problems including the presence of multiple stakeholders, multiple objectives,
and the underlying decision-making nature. However, there are two fundamental dif-
f erences between traditional engineering design problems and policy design problems:
a) the nature of the stakeholders’ objectives, and b) coordination mechanisms between
stakeholders. In engineering design problems, the objectives of sub-system design prob-
lems are coordinated to achieve system-level objectives, while the stakeholders in the
policy design problems have their own objectives and make decisions in a self-interested
manner. The individual stakeholders’ objectives may not be aligned with the policy de-
signers’ objectives. Additionally, coordination between stakeholders in a policy design
problem is generally through market-based mechanisms. Hence, there is a need to model
the market-based interactions among stakeholders.

Prior work on computational policy design is based on modeling the problem as a mul-
tilevel decision-making problem with policy decisions representing higher-level problems
and stakeholders’ technical decisions as lower-level problems. The multilevel problems are
converted into mathematical programming problems with equilibrium constraints repre-
senting the outcomes of interactions between stakeholders (see literature review in Sec-
tion 1.2). Existing work on computational policy design is limited because it is assumed
that: a) stakeholders have a single-objective, and b) policy-makers have complete knowl-
gedge of the stakeholders’ preferences. Both these assumptions result in single-objective
bilevel optimization problems. However, as discussed above, both the policy maker and
the stakeholders have multiple objectives, and the policy maker may not have complete
information about the preferences of the stakeholders. In this paper, we present a bilevel
formulation and evaluate alternate solution approaches for addressing these two limi-
tations of existing approaches. The formulation is based on Nash equilibria of games
with incomplete preferences, and mathematical programs with equilibrium constraints
(MPECs). We present the approach using an illustrative example from the design of
feed-in-tariff (FIT) policy. The main contributions of this paper include a mathema-
tical formulation of the FIT policy, extension of computational policy design problems to
multiple objectives, and the consideration of incomplete preferences of stakeholders. In
the following section, a detailed review of the literature is presented.

1.2. Literature Review

1.2.1. Modeling stakeholder decisions using non-cooperative games

Non-cooperative game theory (Nash 1951) is used as the natural framework for ana-
lyzing systems that involve multiple interacting decision makers. Non-cooperative games
have been used in engineering design, primarily as a way to represent decentralized design scenarios (Marston and Mistree 2000, Chanron and Lewis 2006) where designers are modeled as decision-makers. The designers’ decisions are in equilibrium if no designer can unilaterally improve their payoff by changing their own decisions. This equilibrium is referred to as the Nash equilibrium.

Computation of Nash equilibria has received significant attention in various fields (McKelvey and McLennan 1996). One of the widely adopted approaches for finding Nash equilibria is to use the first order necessary conditions of optimality of the individual stakeholders’ decisions, and to formulate the problem as a complementarity problem (Ferris and Pang 1997, Facchinei and Pang 2003). The complementarity problem is a special case of a variational inequality problem (Billups and Murty 2000, Facchinei and Pang 2003, Nabetani et al. 2011). The complementarity models for Nash equilibria have been used in a number of applications related to market modeling (Harker and Pang 1990). For example, Hobbs (2001) presents a model of bilateral markets with imperfect competition between electricity producers using linear complementarity models. Gabriel and co-authors (Gabriel et al. 2005a,b, Zhuang and Gabriel 2008, Gabriel et al. 2009) develop an equilibrium model for natural gas markets with different types of market participants including producers, storage operators, pipeline operators, marketers, and consumers.

Egging et al. (2010) present a multi-period mixed complementarity model of the World Gas Model (WGM). The authors consider different types of stakeholders including producers, traders, pipelines and storage operators. Other examples of multi-period energy market models include (Arroyo and Conejo 2002, Tovar-Ramirez and Gutierrez-Alcaraz 2008). Arroyo and Conejo (2002) discuss a multi-period market clearing model based on the intersection between supply and demand curves for the ISOs in a pool-based deregulated market. The overall objective of the market operator is to maximize the net social welfare. Tovar-Ramirez and Gutierrez-Alcaraz (2008) discuss a multi-period formulation for generation companies (GENCOs). The authors model hourly updates of generation quantities, demand, and price.

1.2.2. Policy-design as a bilevel problem

While market equilibrium modeling is an important problem, the goal from a policy-design standpoint is to design the Nash equilibria by influencing stakeholders’ decisions. This problem of designing equilibria can be viewed as a higher-level problem with design variables (e.g., incentives or penalties) that can be used to modify the Nash equilibria of the lower-level equilibrium problem. These design problems represent a special class of bilevel programs (Colson et al. 2005). Within mathematical programming literature, such bilevel problems are called mathematical programs with equilibrium constraints (MPECs) (Luo et al. 1996). Ye (1999, 2005) presents necessary and sufficient conditions for optimality for bilevel programs and MPECs.

MPECs are challenging because the optimality conditions in the lower level problems lead to combinatorial issues, and the potential lack of convexity and/or closedness of the feasible region (Luo et al. 1996). A number of specialized algorithms have been developed to address these challenges (Luo et al. 1996, Pieper 2001, Fletcher and Leyrer 2004, Benson et al. 2006). Examples of the algorithms include piecewise sequential quadratic programming, penalty interior-point algorithm, implicit function based approaches, and smooth non-linear programs (Pieper 2001). Some of these algorithms are implemented in commercial platforms for optimization such as GAMS and Matlab (Dirkse and Ferris 1999). Applications of MPEC include electricity markets (Hobbs et al. 2000, Gabriel and
Leuthold 2010), highway tax policy design (Labbe et al. 1998), and critical infrastructure planning (Scaparra and Church 2008).

1.2.3. Research Gap

As highlighted in Section 1.1, existing work on designing policies using bilevel programming techniques has two main limitations. The first limitation is that the problems are modeled as single-objective problems. Recently, there have been some efforts within the mathematical programming area on extending the MPEC formulation to multi-objective optimization problems with equilibrium constraints (Ye and Zhu 2003, Mordukhovich 2009). The current work in that direction is focused on deriving the necessary conditions for optimality (Bao et al. 2007, Ye 2011). However, such formulations have not yet been utilized for policy design problems.

The second limitation is that the higher level policy designer is assumed to have complete knowledge of the preferences of the lower-level decision makers. The common assumption is that the stakeholders are profit-maximizing firms and their only objective is to maximize their profits. However, the stakeholders may have multiple objectives. Consider an example of a policy decision at the federal level, which affects the decisions made by local (or state) policy makers. In this case, the federal policy design is the upper-level problem and the local policy design is the lower level problem in MPEC, whose goal is not simply to maximize profit. Even the profit maximizing firms have objectives that cannot be directly quantified in terms of profit. Examples of such objectives include service quality, brand recognition (through reduced green-house gas emissions), and community service. Even in cases where the multi-objective nature is acknowledged, it is implicitly assumed that the policy decision-maker knows the stakeholders’ tradeoffs in advance, allowing the lower-level decisions to be modeled as single-objective optimization problems. Hence, it is assumed that the stakeholders’ objectives can be combined into a single objective function (such as a utility function). This single objective function satisfies the completeness axiom of vonNeumann and Morgenstern’s utility theory (vonNeumann and Morgenstern 1947) and can be used to compare all alternatives. However, this assumption may be invalid in three scenarios which are particularly relevant to real policy design problems (Aumann 1962). First, the policy decision maker may not have complete information about the preferences of the individual decision makers. Second, the lower-level decision makers may represent groups of individuals (e.g., committees), leading to incomplete social preferences. Third, the decision makers may be indecisive, and hence, unable to rank all combinations of alternatives in a multi-objective scenario (Rabin 1998).

The lack of knowledge of the lower-level decision maker’s preferences is one of the reasons for the failure of policies, specifically the FIT policy (Grace et al. 2008). Assumptions can be made about the objectives and how these objectives are weighted but they may not be accurate. This inaccuracy can have detrimental effects creating an expensive non-successful FIT policy. Hence, there is a need to develop approaches to account for uncertainty resulting from the lack of complete information about stakeholder’s payoffs.

vonNeumann and Morgenstern’s utility theory has been extended to utility theory with incomplete preferences (Dubra et al. 2004, Mandler 2005, Nau 2006, Ok 2002) to address the incomplete nature of preferences. While existing work on utility theory with incomplete preferences is focused on modeling the decisions, there is limited understanding of strategic interactions between players with incomplete preferences. Bade (2005) shows that the Nash equilibria for any game with incomplete preferences can be characterized in terms of certain derived games with complete preferences. Additionally, if the players’ preferences are concave, the Nash equilibria can be determined from derived complete
games by a simple linear procedure. Bade (2005) discusses the Nash equilibrium of a game where a) each decision maker has multiple objectives, b) decision makers are able to rank alternatives based on each objective individually, and c) the decision makers are unable to make tradeoffs among different objectives. Such games are also referred to as games with vector payoffs (Somasundaram and Baras 2008) or multi-objective games (Zhao 1991, Borm et al. 2003). The equilibria of games with vector payoff are referred to as Pareto equilibria (Borm et al. 1988, Krieger 2003).

In this paper, we extend the existing literature on computational policy design by considering the multi-objective nature of the policy design problem and the incomplete preferences of stakeholders. The rest of the paper is organized as follows. An overview of the FIT policy and the scope of the design problem are presented in Section 2. Mathematical formulation of the multi-objective FIT policy with incomplete preferences is presented in Section 3. Results of the illustrative example are presented in Section 4. Discussion of limitations and future research opportunities are presented in Section 5.

2. Framing the FIT Design Problem for Sustainable Energy Systems

Energy policies can be adopted at different levels including the federal, state, local, and utility levels. The policy options include incentives for investment, guidelines for energy conservation, taxation and other public policy techniques (Larsen and Rogers 2000, Smith et al. 2002). Specific examples include emission taxes, incentives to non-polluters and renewable energy, incentive for demand response, emission cap-and-trade systems, emission intensity standards and regulations, and alternative allocations of emission rights to regions and sectors. In several counties, including Germany and Spain, one of the mechanisms which has been particularly successful in addressing environmental, reliability, and security issues associated with decentralized energy is feed-in-tariff (FIT) (Couture et al. 2010), discussed in the following section.

2.1. Overview of feed-in-tariff (FIT) policies

A feed-in-tariff is an energy supply policy that offers a guarantee of payments to renewable energy (RE) developers for the electricity they produce (Couture and Cory 2009). The objectives of these policies are to motivate the deployment of RE technologies and to increase renewable generation while reducing dependencies on fossil-fuels. FIT policies support decentralized infrastructure and motivate individuals along with companies to invest in renewable energy technologies. FIT can be designed by the utilities or the state government. Moreover, FIT can be designed to work in conjunction with other US state policies such as renewable portfolio standards (RPSs) and net-metering, and federal policies such as the Production Tax Credit (PTC) and the Investment Tax Credit (ITC).

The design of FIT programs can be categorized into two classes - market independent design, and market dependent design. In the market independent design, the investors are paid a fixed price per unit electricity produced, independent of the market price. Different variations of the market independent design include fixed price with full/partial inflation adjustment, front-end loaded design, and spot market gap model. In the market dependent FIT policy design (see Figure 2), the payment depends on the market price of electricity. Variations of the market dependent design include premium price model (fixed premium on top of the market price), variable premium model (includes caps and floors), and percentage of retail price. The price can be based on factors such as cost of generation,
Table 1. The objectives and design variables of the policy design problem

FIT Policy Design Objectives (Couture and Cory 2009)

**Economic:** job creation, economic development, economic transformation, stabilization of electricity prices, lower electricity prices, grow economy, revitalize rural areas, attract new investment, develop community ownership, develop future export opportunities

**Environmental:** clean air benefits, greenhouse gas emission reduction, preserve environmentally sensitive areas, manage waste streams, reduce exposure to carbon legislation

**Energy Security:** secure abundant energy supply, reduce long-term price volatility, reduce dependence on natural gas, promote a resilient system

**Renewable Energy (RE) Objectives:** rapid RE deployment, technological innovation, drive cost reductions, meet renewable portfolio standards, reduce fossil fuel consumption, stimulate green energy economy, barriers to renewable development

**Design Variables**

**Alternatives:** 1) Premium price model: price per kWh; 2) Variable premium price model: premium price in access of the market price; 3) Percentage of retail value: % in access of the market price.

**Payment differentiation** based on technology and fuel type, project size, resource quality, resource quantity, and location. Other design variables include tariff degression, and inflation adjustment.

value of the system (to the utility, consumers, etc.), and auction-based price discovery. Additionally, the payment differentiation can be based on technology and fuel type, project size, resource quality, and location. Based on the design of the policy, the investors (individuals or companies) decide to invest in different technologies to maximize their objectives. The resulting increase in generation capacity affects the market equilibrium and the corresponding energy prices. Tamas et al. (2010) recently presented a formal mathematical model for the FIT policy and compared it with tradeable green certificates.

The formulation of policy decision involves the identification of policy objectives, design variables, and constraints. Various objectives can be considered while designing FIT policies (see Table 1). We use the market-dependent design, specifically the premium price model, in this paper. The stakeholders are modeled as decision makers who invest in different technologies based on the incentives determined by the policymakers. An overview of the decisions of the policy designer and each stakeholder is provided in Table 2. The policy designer has two objectives: a) to maximize the total quantity of energy generated by all the stakeholders \( Q \), and b) to minimize the cost of implementing the policy \( C \). The first objective is related to the policy goal of renewable energy
penetration and the second objective is an economic goal associated with most policies. The stakeholders’ decisions are driven by two objectives: a) maximization of the net-present value (NPV) of their investment and b) the minimization of capital investment.

### 2.2. Assumptions

The problem formulation presented in this section is based on the following assumptions:

1. **There is only one policy designer** - In a real policy design problem, there are multiple entities involved in the policy making process. Different entities may have different objectives. An entity (such as environmental protection agency) may have an objective to reduce $CO_2$ emissions, while another may be more interested in the economic impacts of a policy. Here, we consider that one policy maker is responsible for satisfying all the objectives and has complete control of the design variables.

2. **Fixed premium-price FIT model payment option** - We assume a FIT policy where the payment is a fixed premium price ($\Delta$). In other words, the investors are paid a set amount, $\Delta$, in addition to the market price of electricity.

3. **The policy-maker can only control two decision variables** - We set the policy designer’s decision variables as the premium-price of the policy, $\Delta$, and the duration of the policy, $T$. These two variables are the primary design variables in a premium-price model. In a general FIT design scenario, the premium price may be different for different types of technologies, and different sizes of generation facilities installed by the stakeholders.

4. **Stakeholders can only control the quantity of generation** - Each stakeholder $i$ can only control the quantity of electricity ($q_i$) generated. Based on the duration of the policy, the premium-price, and the market demand, each stakeholder decides upon the quantities.

5. **The policy options are not dynamic** - In many FIT policies the policy options (decision variables) can vary with time. For example, the premium price may be high during the initial period to encourage investment but may be reduced over time. Alternatively, the premium price may increase with time to account for
inflation. In this paper, we assume that $\Delta$ is fixed for the duration of the policy.

(6) **Incomplete information about preferences** - It is assumed that the policy maker has complete information of the stakeholder’s individual objectives (maximization of net-present value and minimization of capital investment). However, the policy maker does not have complete information about how each stakeholder makes tradeoffs among these objectives.

(7) **Single period model** - The model is based on a single time period. It is assumed that the equilibrium price remains the same throughout the duration of the policy.

(8) **Electricity market is modeled using Cournot Nash equilibrium** - We assume that the electricity market is modeled as Nash equilibrium of a Cournot competition game.

### 2.3. Simple model of the electricity market

The electricity market modeling literature consists of two types of models for market equilibrium arising from profit maximizing participants: the Cournot equilibrium (Hobbs 2001) and supply function equilibrium (SFE) (Hobbs et al. 2000). Both concepts are based on the Nash equilibrium, but differ in the decision makers’ variables. In Cournot equilibrium model, the participants compete in quantity of energy produced, whereas in the SFE model, the participants compete both in quantity and price.

Consider a simple example of two producers deciding on their production quantities $q_1$ and $q_2$ (Aliprantis and Chakrabarti 1999). Assume that the market price is determined by the overall quantity produced ($Q$) through a linear function: $p(Q) = (A - Q)$ where $A$ is a constant. The profit of each firm is:

$$\pi_i(q_1, q_2) = (A - q_1 - q_2)q_i - c_i q_i$$

where $c_i$ is the cost of production for player $i$. The resulting Nash equilibrium is:

$$q_1^* = \frac{A + c_2 - 2c_1}{3}, \quad q_2^* = \frac{A + c_1 - 2c_2}{3}$$

Cournot equilibrium is more flexible and tractable because it results in a set of algebraic equations for the Nash equilibrium whereas SFE results in a set of differential equations (Ventosa et al. 2005). Hence, the Cournot equilibrium model has attracted significant attention from the electricity market modeling community (Hobbs 2001). For a given price and duration of a policy, the stakeholders decide the quantities ($q_i$) which correspond to the Cournot equilibrium value based on the market and stakeholder preferences. The policy makers can achieve their objectives by designing the price and duration.

### 2.4. Details of the decisions made by stakeholders and the policy designer

The objectives, design variables, and constraints for the policy designer and the stakeholders are shown in Table 3. The first objective of a stakeholder is the net present value ($V_i$), which is the outcome of the time series of cash inflow and outflow. The second objective is minimization of capital investment ($I_i$). As previously stated, the policy designer has control over two variables, the premium-price of the policy ($\Delta$), and the duration of
Table 3. Multi-objective policy design problem with multi-objective decisions of stakeholders

<table>
<thead>
<tr>
<th><strong>Policy Design Problem</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives:</strong></td>
</tr>
<tr>
<td>$\max Q = \sum_{i=1}^{N} q_i$</td>
</tr>
<tr>
<td>$\min C = \sum_{i=1}^{N} q_i(p_m + \Delta)$</td>
</tr>
<tr>
<td><strong>Decision Variables:</strong></td>
</tr>
<tr>
<td>$\Delta$, $T$</td>
</tr>
<tr>
<td><strong>Constraints:</strong></td>
</tr>
<tr>
<td>$0 \leq \Delta \leq 0.20$/KWh</td>
</tr>
<tr>
<td>$T \leq 21$ years</td>
</tr>
<tr>
<td>$q_i \in E_q$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>$i^{th}$ Stakeholder’s Decision</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives:</strong></td>
</tr>
<tr>
<td>$\max V_i$</td>
</tr>
<tr>
<td>$\min I_i$</td>
</tr>
<tr>
<td><strong>Decision Variable:</strong></td>
</tr>
<tr>
<td>$q_i$</td>
</tr>
<tr>
<td><strong>Constraint:</strong></td>
</tr>
<tr>
<td>$q_i \geq 0$</td>
</tr>
</tbody>
</table>

The policy $(T)$. We assume that the market price can be modeled as

$$p_m = (\alpha - \beta Q) \quad \text{where} \quad Q = \sum_{i=1}^{N} q_i$$

Here, $\alpha$ and $\beta$ are two constants based on the energy market. The cash inflow per year for stakeholder $i$ is $(p_m + \Delta)q_i$ and the outflow per year is the operation and maintenance cost $(C_m)$ along with a one-time capital investment $(I_i)$. The capital investment is assumed to be proportional to the quantity generated (i.e., $I_i = q_i I$). Assuming that $j$ is the discount rate, the net present value for the $i^{th}$ stakeholder is:

$$V_i = \sum_{t=1}^{T} q_i \left( \frac{\alpha - \beta \sum_{i=1}^{N} q_i}{(1 + j)^t} + \Delta \right) - \left[ \sum_{t=1}^{T} \frac{C_m q_i}{(1 + j)^t} + q_i I \right] \tag{1}$$

The decision variable for each stakeholder is the quantity, $q_i$, which is constrained to be positive. Ideally, $q_i < 0$ would indicate that instead of generating energy, the stakeholder purchases it from other stakeholders. However, we do not consider that scenario here.

The constraints on the policy level design problem include bounds on the premium price ($0 \leq \Delta \leq 0.20$ per KWh), the policy duration ($T \leq 21$ years), and the market equilibrium constraint ($q_i \in E_q$). According to the market equilibrium constraint, the quantities of energy generated by each stakeholder must belong to the Cournot Nash equilibrium set.

If the preferences of each stakeholder are known precisely, and their tradeoffs among
the objectives can be formulated as utility functions, then stakeholders’ decision problems can be formulated as single-objective utility maximization problems. In that scenario, based on the premium-price ($\Delta$) and the duration of the policy ($T$) chosen by the policy designer, and the preferences from the stakeholders, the stakeholders choose the quantities ($q_i$) according to the Cournot Nash equilibrium. The policy maker can maximize his/her own payoff based on these decisions. Within game theory, such interactions between decision makers are modeled as a Stackelberg game (vonStackelberg 2011) where one of the players (in this case the policy maker) moves first and the rest of the players (the stakeholders) move based on the decision made by the player moving first. The Stackelberg game can be mathematically modeled as a bilevel optimization problem with policy design as the upper optimization problem and the stakeholders’ decisions as the lower-level Nash equilibrium problem (Fudenberg and Tirole 1993).

In general cases, however, the tradeoff functions of the decision makers may be uncertain. As highlighted in Section 1.2.3, this uncertainty may arise due to a) the lack of knowledge of each stakeholder’s preferences, b) diversity among different stakeholders, c) stakeholders’ decisions representing groups of individuals, and d) indecisiveness of the stakeholders themselves. In special cases, it may be feasible to identify what the different objectives are, but not feasible to determine how the tradeoffs between different objectives are made by each stakeholder. We consider such a special case in this paper. It is assumed that the policy designer is aware that the stakeholders will make their decisions using the two objectives: maximizing net present value and minimizing capital investment. However, there is uncertainty in how important these objectives are to different stakeholders. Hence, the policy designer’s goal is to design the FIT policy in the presence of this uncertainty. In the following section, we discuss a mathematical formulation and different solution approaches for the policy decision.

3. Mathematical Formulation of FIT Policy Design Problem and Solution Approach

Bilevel problems with higher-level (single-objective) optimization problem and lower-level equilibrium problem can be mathematically formulated and executed as Mathematical Programs with Equilibrium Constraints (MPEC). MPEC is discussed in Section 3.1. The formulation of FIT design problem with complete information about stakeholder preferences as an MPEC is discussed in Section 3.2. Finally, the solution of the policy design problem under uncertainty is discussed in Section 3.3.

3.1. Mathematical programming with equilibrium constraints (MPEC)

MPEC is a type of constrained nonlinear programming problem where some of the constraints are defined as parametric variational inequality or complementarity system (Luo et al. 1996). These constraints arise from some equilibrium condition within the system, and hence, are called equilibrium constraints (Dempe 2002). MPEC is applicable to a variety of problems in engineering such as optimal design of mechanical structures, network design, motion-planning of robots, facility location, and equilibrium problems in economics. Examples of problems related to economic equilibrium where MPEC has been used include maximizing revenue from tolls on a traffic system (Patriksson and Rockafellar 2002), optimal taxation (Light 1999), and demand adjustment problems (Chen and
Bilevel Formulation of a Policy Design Problem

Florian 1996). Mathematically, a MPEC problem can be represented using two sets of variables, \( x \) and \( y \). Here, \( x \) belongs to the upper-level problem and \( y \) solves the lower-level equilibrium problem. The solution of \( y \) depends on the value of \( x \) chosen for the upper-level problem. The overall objective function \( f(x, y) \) is minimized.

\[
\min_{(x,y)} f(x, y)
\]

subject to:

\[
(x, y) \in \Omega \text{ and } y \in S(x)
\]

where \( \Omega \) is the joint feasible region of \( x \) and \( y \); and \( S(x) \) is a set of variational inequalities that represent the equilibrium problem. The function \( f(x, y) \) represents a system-level function that quantifies the goodness of the solution. For the lower-level Nash equilibrium problem, the set \( S(x) \) corresponds to the feasible Nash space. The Nash equilibrium point can be formulated as a variational inequality using the first order necessary conditions for optimality such as Karush Kuhn Tucker (KKT) conditions (Bertsekas 1999).

Solving the MPEC problems is challenging due to the non-linearities in the problem, non-convex feasible space, combinatorial nature of constraints, disjointed feasible space, and multi-valued nature of the lower equilibrium problem (Luo et al. 1996). Significant research efforts have been devoted to developing efficient algorithms for solving MPEC problems. These include piecewise sequential quadratic programming (PSQP) (Luo et al. 1998), penalty interior-point algorithm (PIPA) (Luo et al. 1996), implicit function-based approaches (Outrata et al. 1998), and smooth sequential quadratic programming (Pieper 2001). Recently, few efforts have been carried out by Ye (Ye 2011), Mordukhovich (Mordukhovich 2009), and Bao et al. (Bao et al. 2007) on deriving the necessary conditions for optimality of multi-objective problems with equilibrium constraints.

3.2. Formulation of the FIT design problem with complete information as MPEC

Games within which players can have several possibly conflicting objectives are called “games with vector payoffs” or “multi-objective games” (Born et al. 1988, 2003). Due to the multi-objective nature of the decisions, the concepts of rational reactions and Nash equilibria need to be generalized. In the games with single objectives, a rational reaction (best reply) of a player to other players’ decisions is a point that maximizes his/her payoff. In the case of games with vector payoffs, a player’s rational reaction to the decisions of other players is a set of Pareto optimal solutions, also referred to as Pareto Best Replies. In single-objective games, the points of intersection of the rational reaction sets are the Nash equilibria. In games with vector payoff, the concept of Nash equilibrium is replaced by the concept of Pareto equilibria defined as the pairs of strategies which are Pareto best replies to one another (Zhao 1991, Krieger 2003). Under certain conditions, games with vector payoffs can be reduced to games with single objectives by combining each player’s objectives using the Archimedean weighting scheme. The reduced single objective game is called a tradeoff game (Born et al. 1988) or a derived game with complete information. Hence, one approach to solve the policy design problem with uncertain preferences is to solve multiple derived games with complete information. We consider the MPEC with the equilibrium for the derived game in this section.

The derived game is formulated by using a weighted combination of the stakeholders’
objectives to define the payoffs, $\pi_i$, of the stakeholders in the games with complete information. The payoff function for stakeholder $i$ is listed in Equation (2). $V_{\text{norm}}$ and $I_{\text{norm}}$ are used to normalize the net present value and capital investment. A set of games with complete information are obtained by choosing different values of the weights ($w_{v,i}$) for stakeholder $i$.

$$
\pi_i(V_i, I_i, w_{v,i}) = w_{v,i} \frac{V_i}{V_{\text{norm}}} + (1 - w_{v,i}) \left[ 1 - \frac{I_i}{I_{\text{norm}}} \right]
$$

(2)

This payoff function is used to determine the KKT conditions for each stakeholder.

$$
\nabla \pi_i + \sum \lambda_i \nabla g_i = 0
$$

$$
\lambda_i \geq 0, \quad \lambda_i g_i = 0, \quad g_i \leq 0
$$

where $\pi_i$ is the objective function and $g_i \leq 0$ are the inequality constraints in the generic formulation. The necessary optimality conditions for stakeholder $i$ can be written as:

$$
w_{v,i} \frac{V_{\text{norm}}}{V_i} \left[ (\alpha + \Delta - C_m) f - I - 2\beta f q_i - \sum_{k=1,k\neq i}^N \beta f_{qk} \right] - (1 - w_{v,i}) \left( \frac{I_i}{I_{\text{norm}}} \right) - \lambda_i = 0
$$

$$
\lambda_i \geq 0, \quad \lambda_i g_i = 0, \quad g_i \leq 0
$$

where,

$$
f = \sum_{t=1}^T \frac{1}{(1+j)^t}
$$

For fixed values of $w_{v,i}$, these optimality conditions for $N$ players constitute a linear complementarity problem (LCP) of the form:

$$
q_i F(q_i) = 0, \quad F(q_i) \geq 0, \quad q_i \geq 0 \quad \forall i
$$

(3)

$$
F(q_i) = w_{v,i} \frac{V_{\text{norm}}}{V_i} \left[ (\alpha + \Delta - C_m) f - I - 2\beta f q_i - \sum_{k=1,k\neq i}^N \beta f_{qk} \right] - (1 - w_{v,i}) \left( \frac{I_i}{I_{\text{norm}}} \right)
$$

LCPs have been extensively studied in the mathematical programming literature (Murty and Yu 1992, Cottle et al. 2009). An LCP is a special case of nonlinear complementarity problems (NCP). There are various algorithms to solve the deterministic LCPs. The algorithms can be broadly classified into pivoting methods and iterative methods. The pivoting methods require recursive solution of systems of linear equations whereas the iterative methods converge in the limit. Iterative methods are more suitable...
for large $N$. We use the iterative methods with NCP functions ($\phi : \mathbb{R}^2 \to \mathbb{R}$) to solve the complementarity problem listed in Equation (3). The NCP functions have the following property:

$$\phi(q_i, F(q_i)) = 0 \iff q_i \geq 0, \quad F(q_i) \geq 0, \quad q_i F(q_i) = 0$$

(4)

Hence, NCP functions can be used to replace the complementarity constraints. Examples of NCP functions commonly used in the iterative algorithms include the “min” function:

$$\phi(q_i, F(q_i)) = \min(q_i, F(q_i))$$

and the Fischer-Burmeister (FB) function (Fischer 1992):

$$\phi(q_i, F(q_i)) = q_i + F(q_i) - \sqrt{q_i^2 + F(q_i)^2}$$

For $N$ stakeholders, formulating the complementarity system using the NCP functions converts the equilibrium constraints into $N$ nonlinear equations. Since the FB function is non-smooth at $(q_i = 0, F(q_i) = 0)$, the derivatives do not exist. To facilitate the solution of these equations, Kanzow (Kanzow 1996) proposed a smooth approximation of the Fischer-Burmeister function, which can be written for the above complementarity constraint as follows:

$$\phi_\mu(q_i, F(q_i)) = q_i + F(q_i) - \sqrt{q_i^2 + F(q_i)^2} + 2\mu, \quad \mu > 0$$

Using the smooth function, an approximation of the complementarity problem is obtained, whose solution approaches the solution of the complementarity problem as $\mu \to 0$.

The smooth formulation can be solved by using Newton-based methods. Using the smooth approximation of the Nash equilibrium problem, an approximation of the overall policy design problem (shown in Table 3) is obtained. The policy designer’s objectives are formulated as a weighted combination of the two objectives: maximize $Q$ and minimize $C$.

$$\Pi(q_i, T, \Delta, w_Q) = w_Q \frac{\sum_{k=1}^N q_k}{Q_{\text{norm}}} + (1 - w_Q) \left[ 1 - \frac{\sum_{i=1}^N q_i \left( \alpha - \beta \sum_{k=1}^N q_k + \Delta \right)}{C_{\text{norm}}} \right]$$

$Q_{\text{norm}}$ and $C_{\text{norm}}$ are used to normalize the values of total quantity and policy cost. Different values of weights ($w_Q$) result in different MPEC problems. For a given preference of the policy designer, the resulting approximation of the MPEC is:

$$\min_{q_i, T, \Delta} \Pi(q_i, T, \Delta, w_Q)$$

subject to: $\phi_\mu(q_i, F(q_i)) = 0 \quad \forall i$
3.3. **Approach for policy decision making under incomplete preferences of stakeholders**

For the scenario where the stakeholders’ preferences (i.e., $w_{v,i}$) are not completely known, the Nash equilibrium problem in Equation (3) can be formulated as the following stochastic LCP (Luo and Lin 2009):

$$q_i F(q_i, W_{v,i}) = 0, \quad F(q_i, W_{v,i}) \geq 0, \quad q_i \geq 0, \quad W_{v,i} \in \Omega \quad \forall i$$

where $W_{v,i}$ is assumed to be a random number indicating the uncertainty in the preference of the $i^{th}$ stakeholder. The function $F(q_i, W_{v,i})$ can be written as:

$$F(q_i, W_{v,i}) = \frac{W_{v,i}}{\text{V}_{\text{norm}}} \left[ (\alpha + \Delta - C_m) f - I - 2\beta f q_i - \sum_{k=1, k\neq i}^{N} \beta f q_k \right] - (1 - W_{v,i}) \left( \frac{I}{\text{I}_{\text{norm}}} \right)$$

The equilibrium problem in terms of $F(q_i, W_{v,i})$, with random $W_{v,i}$, is a stochastic LCP problem. A survey of techniques for solving stochastic LCP problems is presented by Lin and Fukushima (2010). The state of the art approaches to solving the stochastic LCP involve developing appropriate deterministic formulations. Examples of such deterministic formulations are a) expected value (EV) formulation, and b) expected residual minimization (ERM) formulation. The EV formulation (Gurkan et al. 1999) is aimed at finding $q_i$ that satisfy the complementarity condition for the expected value of $F(q_i, W_{v,i})$ with respect to $W_{v,i}$:

$$q_i E(F(q_i, W_{v,i})) = 0, \quad E(F(q_i, W_{v,i})) \geq 0, \quad q_i \geq 0, \quad W_{v,i} \in \Omega \quad \forall i$$

For the LCP where the stakeholders’ preferences are independent of each other, this is equivalent to:

$$q_i F(q_i, E(W_{v,i})) = 0, \quad F(q_i, E(W_{v,i})) \geq 0, \quad q_i \geq 0, \quad W_{v,i} \in \Omega \quad \forall i$$

The expected residual minimization (ERM) method is based on a different formulation of the complementarity problem. This method is based on finding the vector of quantities $q_i$ that minimizes the expected residual for the complementarity problem listed in Equation (4):

$$\min_{q \geq 0} E \left[ \| \Phi(q, F(q, W)) \|^2 \right]$$

where

$$\Phi(q, F(q, W)) = \left( \begin{array}{c} \phi(q_1, F(q_1, W_{v,1})) \\ \vdots \\ \phi(q_N, F(q_N, W_{v,N})) \end{array} \right)$$
and

\[ q = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}, \quad W = \begin{pmatrix} W_{v,1} \\ \vdots \\ W_{v,N} \end{pmatrix} \]

The ERM approach can be used with different NCP functions. The expected value of the residual is calculated by generating samples of \( W \) and evaluating the values of the residual at those sample points. As expected, the accuracy of this approach depends on the number of samples used.

In addition to using these deterministic reformulations of the stochastic LCP, we also use the Monte-Carlo technique to determine the distribution of the Nash equilibria for known distributions of \( W \). While the Monte-Carlo technique provides detailed information about the distribution of Nash equilibria, the approach is only suitable for small \( N \). We utilize the EV formulation, ERM formulation and the Monte-Carlo technique to determine the Nash equilibria for the FIT policy design problem with uncertain preferences. The results are presented in the following section.

4. Results of FIT Policy Design Problem

In this section, we present the results of the FIT design problem. The values of the parameters used in the formulation are shown in Table 4. Different scenarios with increasing complexity are considered. First, a 2-player scenario with complete preferences of stakeholders is discussed in Section 4.1. In Section 4.2, scenarios with incomplete preferences are presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.69 $</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 5 \times 10^{-5} ) $/KWh</td>
</tr>
<tr>
<td>( j )</td>
<td>0.06</td>
</tr>
<tr>
<td>( I )</td>
<td>0.0228 $/KWh</td>
</tr>
<tr>
<td>( C_m )</td>
<td>0.0057 $/kWh</td>
</tr>
<tr>
<td>( V_{\text{norm}} )</td>
<td>1</td>
</tr>
<tr>
<td>( I_{\text{norm}} )</td>
<td>1</td>
</tr>
<tr>
<td>( Q_{\text{norm}} )</td>
<td>( 2.5 \times 10^5 ) KWh</td>
</tr>
<tr>
<td>( E_{\text{norm}} )</td>
<td>( 8.0 \times 10^4 ) $</td>
</tr>
</tbody>
</table>

4.1. Sample results for 2-Player scenario with complete preferences

Rational reaction sets for different preferences of the stakeholders (i.e., different values of \( w_v \)) are presented in Figure 3(left). The rational reaction sets are shown for the scenario where the preferences of the stakeholders are identical and the policy decisions are fixed (\( \Delta = 0.10, T = 10 \)). The intersection of the rational reaction sets is the Nash equilibrium. Due to the symmetry in preferences, the resulting quantities \( (q_i) \) are equal. Two different scenarios of policy maker’s decisions with the corresponding rational reaction sets of the stakeholders are shown in Figure 3(right). The two policy designer’s decisions
are: i) $\Delta = 0.10$, $T = 2$, and ii) $\Delta = 0.20$, $T = 21$. As expected, the quantities increase as the payment and the policy duration increase.

Figure 3. Rational reaction sets for the two stakeholders under different preferences of stakeholders (left) and policy maker (right)

Figure 4 displays the set of policy designs for different tradeoffs between policy design objectives and the corresponding values of quantities ($q_i$) associated with Nash equilibria, and the resulting policy objectives. It is assumed that there is no uncertainty in stakeholders' preferences, and $w_{v,i} = 0.5$ for both stakeholders. The overall quantities and the cost are evaluated for the duration of the policy ($T$). If the policy designer is only concerned with the cost of the policy ($w_Q = 0.0$) the design variables will be selected to minimize this cost resulting in a low quantity of generation, which results in low cost and low generation. This point is labeled as “1” in Figure 4. If these preferences are equal ($w_Q = 0.5$, shown as point “2”), the optimum values of the decision variables are $\Delta = 0.0$ $$/ \text{KWh}$, $T = 21$ years resulting in moderate quantity of generation yet a low cost of the policy. The policy will not pay any premium-price yet will maximize the duration of the policy. In this case, the stakeholders will receive payments based on the market value only. No payment will be received in addition to the market value. If the policy maker is only concerned with the quantity of energy being produced ($w_Q = 1.0$, shown as point “3”), the production quantities will be chosen to maximize this value. In this case, the optimum values of the variables are $\Delta = 0.2$ $$/ \text{KWh}$, $T = 21$ years, which results in a high level of generation at a high policy cost.

4.2. Results for scenarios with incomplete preferences

For the scenario where the policy maker has incomplete information about the stakeholders’ preferences, the three approaches discussed in Section 3.3 are utilized. Table 5 shows the scenario where the parameters $W_{v,i}$ are assumed to be uniformly distributed within the range of $[0.0, 1.0]$. This corresponds to the scenario where the policy maker has a complete lack of information about the stakeholders’ preferences. It is observed that despite the uncertainty in the stakeholders’ preferences, the optimum policy decisions are the same under all scenarios. The corresponding outcomes of the policy are uncertain. The expected values of outcomes and the associated standard deviations obtained through the Monte-Carlo technique are also listed in the table.
Bilevel Formulation of a Policy Design Problem

Figure 4. Results from scenario considering complete knowledge of stakeholders’ preferences: Pareto optimal decisions of the policy maker (left), corresponding Nash equilibria (center), and c) the policy outcomes (right)

Table 5. Policy decisions for different preferences of the policy maker for $W_{i,j} = U[0,0.1,0.5]$

<table>
<thead>
<tr>
<th>Policy maker’s preference</th>
<th>Approach</th>
<th>$\Delta$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_Q = 0.0$</td>
<td>MC</td>
<td>0.0</td>
<td>1</td>
<td>$E(Q) = 7272.16$</td>
<td>$E(C) = 2055.97$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 2520.33$</td>
<td>$\sigma_C = 646.72$</td>
</tr>
<tr>
<td></td>
<td>EV</td>
<td>0.0</td>
<td>1</td>
<td>$E(Q) = 8480.16$</td>
<td>$E(C) = 2255.65$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 2852.81$</td>
<td>$\sigma_C = 1066.88$</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.0</td>
<td>1</td>
<td>$E(Q) = 187459.87$</td>
<td>$E(C) = 45125.67$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 15504.53$</td>
<td>$\sigma_C = 2742.38$</td>
</tr>
<tr>
<td>$w_Q = 0.5$</td>
<td>MC</td>
<td>0.0</td>
<td>21</td>
<td>$E(Q) = 190519.72$</td>
<td>$E(C) = 45035.35$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 16962.63$</td>
<td>$\sigma_C = 3796.32$</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.0</td>
<td>21</td>
<td>$E(Q) = 190772.97$</td>
<td>$E(C) = 44980.18$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 16974.97$</td>
<td>$\sigma_C = 3796.32$</td>
</tr>
<tr>
<td>$w_Q = 1.0$</td>
<td>MC</td>
<td>0.2</td>
<td>21</td>
<td>$E(Q) = 246519.72$</td>
<td>$E(C) = 74707.37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_Q = 16962.63$</td>
<td>$\sigma_C = 3796.32$</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.2</td>
<td>21</td>
<td>$E(Q) = 246785.57$</td>
<td>$E(C) = 74631.73$</td>
</tr>
</tbody>
</table>

We also explore a scenario where the policy maker may have some information about the preferences, e.g., when $W_{i,j}$ is within a smaller interval of $[0.1,0.5]$. The results for this scenario are shown in Figure 5 and Table 6. Figure 5 shows the location of the Nash equilibria for different samples of $W_{i,j}$. The distributions of the outcomes (overall quantity and policy cost) for $W_{i,j} = U[0.1,0.5]$ and $w_Q = 1.0$ are shown in Figure 6.

While the impact of incomplete preferences on the location of the Nash equilibrium is important, the important question from the policy maker’s standpoint is - what is the best decision for the policy maker to make? Although the preferences may not be completely known at the stakeholder level, the policy maker still has control over the decision variables. Although the uncertainty in the outcomes ($Q,C$) is significant, the corresponding policy design variables have either no uncertainty or significantly smaller uncertainty. For example, in Table 6, for $w_Q = 0.0$, although $\sigma_Q$ is large, the policy maker’s best decision remains the same.

The results indicate that although there may be high uncertainty in the quantities generated at market equilibria, the policy maker may only have small uncertainty in the decision variables. This is primarily due to the decrease in uncertainty when the equilibria are mapped into the design space. However, we envision that in some policy design problems, low uncertainty in the equilibria may result in high uncertainties in the design space. The results are sensitive to the market parameters ($\alpha, \beta$). By changing
these parameters slightly we observed significant changes in the equilibrium quantities.
Therefore, in a real FIT policy design, the market parameters need to be calibrated for the specific market under consideration. The results are more sensitive to the premium price and less sensitive to the duration of the policy. It was found that the premium-price of the policy motivated stakeholders to generate higher amounts of electricity. Although changing the duration of the policy had an effect, it was found that in most cases the duration of the policy was maximized.

In the scenarios presented so far, the outcomes from the three approaches are close to each other. However, if the equilibrium quantities hit the constraint \( q_i \geq 0 \), then the predictions from these methods are substantially different. For example, if \( W_{v,i} = U[0.0, 0.2] \), the equilibrium quantities can go to 0, as shown in Figure 7. The resulting distributions of policy outcomes are shown in Figure 8. The expected value of the overall quantity and policy cost using MC approach are 3471.75 kWh and 1331.99 $ respectively. On the other hand, the overall quantity predicted by the expected value approach is 5901.60 kWh and the overall quantity predicted by the ERM approach is 6753.43 kWh. Hence, these deterministic formulations work well if the equilibrium point is away from the constraint, but may overestimate the production quantities.

![Figure 7. Equilibrium quantities from the scenario considering incomplete knowledge of stakeholders' preferences \( W_{v,i} = U[0.0, 0.2] \) and the overall policy outcomes for \( \Delta = 0 \) and \( T = 1 \) year.](image)

Table 7 presents the results for the scenario where the number of players is 30. As the number of players increases, it is observed that the quantity generated by each individual decreases. However, the overall quantity generated by all the stakeholders increases as \( N \) increases.

5. Closing Remarks

In this paper, we present a computational approach for formulating and executing policy design problems with multiple objectives and incomplete knowledge of preferences of the stakeholders. A specific class of incomplete preferences is addressed. It is assumed that the policy maker has knowledge about the different objectives of the stakeholders but has incomplete knowledge about how the stakeholders tradeoff different objectives.

The approach presented in this paper is based on Nash equilibria of games with incomplete preferences, mathematical programs with equilibrium constraints (MPECs),
Figure 8. Distributions of the outcomes \((Q\) and \(C\)) for \(W_{v,i} = U[0.0, 0.2], \Delta = 0, T = 1\) year

Table 7. Policy decisions for different preferences of the policy maker assuming \(N = 30, W_{v,i} = U[0.3, 0.5]\)

<table>
<thead>
<tr>
<th>Policy maker's preference</th>
<th>Approach</th>
<th>(\Delta)</th>
<th>(T)</th>
<th>(Q)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_Q = 0.0)</td>
<td>MC</td>
<td>0.0</td>
<td>1</td>
<td>(E(Q) = 12033.41)</td>
<td>(E(C) = 1062.66)</td>
</tr>
<tr>
<td></td>
<td>EV</td>
<td>0.0</td>
<td>1</td>
<td>(\sigma_Q = 69.33)</td>
<td>(\sigma_C = 34.87)</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.0</td>
<td>1</td>
<td>(12075.09)</td>
<td>1041.42</td>
</tr>
<tr>
<td>(w_Q = 0.5)</td>
<td>MC</td>
<td>0.0</td>
<td>21</td>
<td>(E(Q) = 276123.85)</td>
<td>(E(C) = 8991.20)</td>
</tr>
<tr>
<td></td>
<td>EV</td>
<td>0.0</td>
<td>21</td>
<td>(276165.47)</td>
<td>8965.20</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.0</td>
<td>21</td>
<td>(276208.04)</td>
<td>8938.59</td>
</tr>
<tr>
<td>(w_Q = 1.0)</td>
<td>MC</td>
<td>0.2</td>
<td>21</td>
<td>(E(Q) = 357413.37)</td>
<td>(E(C) = 13944.75)</td>
</tr>
<tr>
<td></td>
<td>EV</td>
<td>0.2</td>
<td>21</td>
<td>(357455.79)</td>
<td>13916.30</td>
</tr>
<tr>
<td></td>
<td>ERM</td>
<td>0.2</td>
<td>21</td>
<td>(357497.85)</td>
<td>13876.14</td>
</tr>
</tbody>
</table>

and stochastic complementarity problems. The primary contributions in this paper are mathematical formulation of the FIT policy, the extension of computational policy design problems to multiple objectives, and the consideration of incomplete preferences of stakeholders for policy design problems. The consideration of incomplete preferences is important for research on design under uncertainty because the existing design literature is primarily focused on uncertainty about physical phenomena but incomplete knowledge of preferences have received relatively little attention.

While the motivation in this paper is that the policy designer has incomplete knowledge about the stakeholders’ payoffs, the approach can be used in two other situations also: a) the stakeholders may themselves not know what their preferences for tradeoffs are, b) the preferences of the stakeholders may represent group preferences. The second situation is common in many policy decisions because the problem is not only bilevel in nature, it is indeed multilevel in nature, as shown in the Figure 1. The proposed approach has applications to other bilevel problems within engineering design research such as design for market systems (Shiau and Michalek 2009a,b), fuel efficiency and emission policy (Michalek et al. 2004, Shiau et al. 2009a), and plug-in hybrid charging patterns (Shiau et al. 2009b). All these problems are generally multi-objective in nature and require the knowledge of preferences of the stakeholders whose interactions result in market equilibria.
The proposed approach has limitations due to the assumptions listed in Section 2.2. First, it is assumed that the market behavior can be defined in terms of the Nash equilibrium. This is a common assumption made in the energy market modeling literature. However, in reality, the market is a dynamic system. Additionally, the decisions are not generally made by all stakeholders at the same time. The decisions may be made sequentially. Hence, there is a need to model the interactions between stakeholders as dynamic games. Second, the approach presented in this paper is based on the assumption that the lower-level decisions can be converted into equilibrium constraints in the closed form. However, as the decisions of the stakeholders become more complex, deriving the equilibrium constraints in closed form may not be feasible. Finally, we do not consider stability of equilibria. The market equilibria for the problem presented in this paper happen to be stable for the ranges of decision variables considered. In a general case, the stability of the equilibria may change by changing the policy design variables. Price stability is an important aspect for the engineering design of distributed energy systems within smart electric grid. One of the goals of the policy design problem is to ensure the stability of the equilibrium. Integrating the stability considerations in the policy design problem is a challenge especially due to the different possible stability problems such as price stability and voltage stability.

The illustrative example presented in this paper is also simplified. In the example, we only consider energy producers whose quantity of generation is determined by the equilibrium. However, in practice, these energy producers are also required to meet local energy demands. The demand fluctuates with time, and the local producers can also purchase energy from central generation stations. The example presented in this paper is not based on specific RE technologies. One of the characteristics of these RE technologies is that their output is uncertain. In a holistic policy design framework, it is important to account for this uncertainty. The example is also based on the assumption that all stakeholders enter the market and make a decision at the same time. However, in practice, different stakeholders may enter the market at different times. Hence, the decisions are made at different time-steps with different amount of available information. These limitations clearly indicate the potential for further research in this direction.

6. Acknowledgments

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