Exploring the geometry and material space in gear design

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Exploring the geometry and material space in gear design

Nagesh Kulkarnia, B.P. Gauthama, Pramod Zagadea, Jitesh Panchalb, Janet K. Allen∗c and Farrokh Mistreec

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Gear design is a complex process. When new materials and manufacturing processes are introduced, traditional empirical knowledge is unavailable and considerable effort is required to find starting design concepts. This forces gear designers to go beyond the traditional standards-based design methods. The method presented here, the concept exploration method, is based on the compromise decision support problem and is demonstrated for gear design which requires the simultaneous exploration of geometry, material and manufacturing spaces to exploit synergies. The results obtained are in agreement with existing knowledge. To further develop design guidelines, ternary contour plots of goal achievement are created to show the interdependence among various design goals. These ternary charts help gear designers to visualize and understand the complexity and compromises involved and help them to make trade-offs among individual goals.

Keywords: gear design; concept exploration method; design and material space exploration

Nomenclature

- $b$ face width (mm)
- $C$ cost (Indian rupees, INR)
- $d$ centre distance (mm)
- $d_1^+, d_2^+, d_3^+$ deviation variables to account for overachievement
- $d_1^-, d_2^-, d_3^-$ deviation variables to account for underachievement
- $G$ gear ratio
- $m$ module of gear (mm)
- $N$ number of teeth
- $R$ reliability (%)
- $R_c$ contact ratio
- $\rho$ density (kg/m$^3$)
- $S_U$ tensile strength
- $S_{U\text{MIN}}$ minimum tensile strength
- $S_{U\text{MAX}}$ maximum tensile strength
- $\sigma_{\text{bending}}$ allowable bending stresses

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Traditionally, gears have been designed using design codes developed by professional bodies such as the American Gear Manufacturers Association (AGMA) (ANSI/AGMA 1993, 2010) and International Organization for Standardization (ISO) (ISO 2006). These design procedures are similar and contain empirical design factors obtained from experiments and experience. These methods are based on simple assumptions such as those on the geometry and they also offer correction factors for requirements such as precision and quality. Some of these factors account for uncertainties in operating conditions, material and manufacturing processes. Gears designed using these design codes are usually safe and reliable but suboptimal as the conservative nature of design codes results in overdesign and large margins of safety. For example, for a multispeed gear box, the standards-based design priorities are usually given to strength, followed by compactness, efficiency and wear (MacAldener 2001). Apart from these demands, new demands such as higher quality, low noise and high safety have emerged owing to increased competition, stringent legal requirements, etc. Further, the manufacturers face additional demands for designing gears rapidly and inexpensively. Currently, this is a US $45 billion a year industry. The gear industry vision report (Gear Industry Vision 2004) highlights the vision, challenges, and strategic goals for the gear industry in 2025. In these documents various goals are listed, as well as key technological challenges and innovations required to achieve them, and it is clear that newer paradigms must be explored for gear design.

With the many possible requirements, a designer should have the capability to explore the design space and understand the appropriate compromises. The concept exploration method (CEM) is proposed, which has the following characteristics:

- the capability to concurrently explore coupled geometry and material spaces
- the capability to explore a complex multidimensional space in presence of constraints and conflicting objectives
- the capability to provide insights into the final solution as well as the progression towards solution with understanding of the behaviour of design variables, constraints and design objectives.

A vast amount of literature is available on gear design. The key articles are reviewed here and the major findings are summarized. Buckingham (1949) studies the kinematics of gear mesh and, based on this, offers a design procedure for gears. Dudley (1962) and Shigley (1977) provide a method of designing gears based on achieving desired factors of safety. Tucker (1980) and Estrin (1980) study the effects of gear mesh parameters such as addendum ratio and pressure angle, and present a procedure for modifying standard gear geometry to obtain more suitable gears. Wang, Luo, and Wu (2010) study the effect of the sliding coefficient on gear design and outline a procedure to design a gear tooth based on the desired sliding coefficients. Yeh, Yang, and Tong (2001) and Tsai and Tsai (1998) develop design procedures for high load carrying capacity gears. Handschuh and Zakrajsek (2011) study the effect of high values of pressure angle on the performance of gears to determine the feasibility of designing gears for aeronautical and space applications. Lin
et al. (1998) deal with the effects of dynamic analysis in compact spur gear design; they compare their in-house method of calculation of dynamic factor to that of the AGMA (2010) recommended method. They observe that the size of an optimal gear set is significantly influenced by dynamic factors and that the peak dynamic factor estimated at system natural frequencies dominates the design of gear sets operating over a wide range of speeds. They create design charts that can be used either to design a gear for single speed or over a range of speeds. Hewitt (1992) discusses the selection of gear materials and the estimation of reliability. He considers a statistical distribution for material strength obtained under different processing as well as precision conditions.

Kapelevich (2007) and Goldfarb, Kapelevich, and Tkachev (2008) propose a method where the tooth profile of gear teeth is optimized for desired performance. In this, unlike the traditional methods, the constraints on the geometry are not severely constrained by the manufacturing processes and the manufacturing has only secondary effects. The method results in non-standard geometry, which requires non-standard tooling and affects gear interchangeability. With the advances in computational techniques, finite element analysis (FEA) has also been widely used in gear design (Brauer 2004; Kawalec, Wiktor, and Ceglarek 2006; Wang and Howard 2006; Kirov 2011). Brauer (2004) provides a general method to create a finite element model of involute gears. Kawalec, Wiktor, and Ceglarek (2006) perform comparative analysis of tooth root strength obtained using design standards such as AGMA (2010) and ISO (2006) and verifying them with FEA. They conduct analysis for spur and helical gears, for various key geometric (gear design), manufacturing (racks and gear tools) and performance (load location) parameters. Wang and Howard (2006) investigate large numbers of two- and three-dimensional gear models using FEA. They study parameters such as torsional stiffness, tooth stresses and stress intensity factors, and suggest that caution should be exercised when two-dimensional assumptions are used for modelling gears. Kirov (2011) studies and compares gear design using AGMA and FEA. He argues that it is difficult to directly compare AGMA and FEA for gear design, and suggests that FEA should be preferred and used whenever AGMA does not provide a design method.

While the literature provides equations and methods for use in design of gears for given performance requirements, gear design involves satisfying multiple conflicting goals such as high reliability, compactness and low cost. High reliability demands gears of larger size and/or high-strength material, which conflicts with the compactness and/or low cost requirements. Smaller gears are not necessarily low-cost gears as they may require expensive materials and manufacturing processes (e.g. advanced heat treatment or shot peening). Dudley (1964) and Tucker (1980) observed that the cost trade-off generally favours small gears with high hardness. Savage, Coy, and Townsend (1982) and Carroll and Johnson (1984) provide literature surveys as well as methods for the optimal design of spur gear sets.


Typically, a range of materials is available to a gear designer for preliminary material selection (Ashby 2005). However, a designer uses these as look-up databases for shortlisting material options for further exploration. The in-service performance of the material depends on the effect of the manufacturing process in addition to the material composition. Integrated computational
materials engineering (ICME) is conceived as a way in which future materials development will be done in close association with end-product design (Pollock et al. 2008; McDowell et al. 2010; Schmitz and Prahl 2011). It is based on the use of modelling and simulation tools to facilitate exploration of the materials and product design space simultaneously and thus reduce time-consuming and expensive experimentation. These tools are expected to map the composition, processing, structure and properties of materials, and ultimately link them to product performance. However, the current state of the art of materials science maps these only partially, with significant gaps (Pollock et al. 2008; McDowell et al. 2010; Schmitz and Prahl 2011; Fullwood et al. 2010). Attempts to develop materials for specific needs of gears can be found in Schmitz and Prahl (2011) and Wright et al. (2010). Schimitz and Prahl (2011) discuss development of microalloyed steels for automotive gear applications and Wright et al. (2010) have designed and developed high-performance gear materials for aerospace applications. However, in these studies the material space and the geometric design space are not explored simultaneously and are thus limited to the development of materials and manufacturing processes for achieving specified material properties. The simultaneous exploration of materials and products could lead to better products.

A vast number of parameters of composition and processing governs the properties of materials. This makes the design space very large. Besides this, industry prefers to use standardized material compositions and simplify the incorporation of material variables in design. In view of this, as a starting point, the material space is restricted to be represented by equivalent material tensile strength and to develop a method for concurrently exploring geometric design and material spaces for gears. This involves large numbers of constraints and conflicting goals and therefore has been a difficult problem to solve. In this work an attempt has been made to fill this gap by offering a method to explore the complex and coupled geometry and material space in the design of gears using an example of the conceptual design of a spur gear. When seeking a conceptual design, designers typically explore a large region of the design space to identify a good design, which then can be used as a starting place for further evaluation and design.

For the conceptual design of a spur gear design, a designer is interested in a region of the design space containing satisficing design solutions, rather than a single optimum solution. The decision support problem (DSP) technique proposed by Mistree and co-authors (Bras and Mistree 1991; Kamal et al. 1987; Mistree et al. 1989, 1990), especially the compromise decision support problem (cDSP) (Mistree, Hughes, and Bras 1993), has been developed specifically to assist designers in this context. The cDSP is useful when multiple conflicting objectives are present and a designer seeks a compromise among these objectives. The cDSP has been used in a wide variety of applications for the design of components and complex systems (Jivan and Mistree 1985; Lyon and Mistree 1985; Nguyen and Mistree 1986; Bascaran, Mistree, and Bannerot 1987; Marinopoulos et al. 1987; Shupe and Mistree 1987; Rolander et al. 2006; Kulkarni et al. 2013; Kumar et al. 2013). The cDSP differs from standard optimization formulations in that it is a hybrid formulation based on mathematical programming and goal programming. The cDSP is implemented in the DSIDES (Decision Support In the Design of Engineering Systems) software. The cDSP, along with DSIDES, has been used extensively to solve a variety of multi-objective, nonlinear problems in engineering, such as in the design of turbulent convective systems (Rolander et al. 2006), the preliminary design of ships (Lyon and Mistree 1985), the conceptual design of an aircraft (Marinopoulos et al. 1987), damage-tolerant structural systems (Shupe and Mistree 1987), thermal design (Bascaran, Mistree, and Bannerot 1987), the design of horizontal pressure vessels (Nguyen and Mistree 1986), the design of helical compression springs (Jivan and Mistree 1985), the design space exploration for continuous casting of steel (Kumar et al. 2013) and the design of gears (Kulkarni et al. 2013).

In this work, the cDSP gear design formulation (Kulkarni et al. 2013) is extended to make it possible to explore the combined design space of the material and the gear geometry and offer the designer and the customer compromises or trade-offs among the various goals.
Engineering Optimization

Table 1. Gear design goals and the associated conflicts.

<table>
<thead>
<tr>
<th>Goal</th>
<th>Requirements</th>
<th>Conflicts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum reliability design</td>
<td>Larger gear geometry (gear module, $m$, number of teeth, $N$, and face width, $b$) and high-strength material</td>
<td>Compact design, minimum cost design</td>
</tr>
<tr>
<td>Compact design</td>
<td>Smaller $m$ and $N$, and high-strength material</td>
<td>High-reliability design, possible weak conflict with cost</td>
</tr>
<tr>
<td>Minimum cost design</td>
<td>Smaller gear geometry ($m$, $N$ and $b$) and low-cost material</td>
<td>High-reliability design</td>
</tr>
</tbody>
</table>

2. Geometry and material space exploration

Designing a gear that satisfies user-specified functional and performance requirements and performs satisfactorily in service is a challenging task. Gear designers have different design goals, such as high reliability (which is important for the user’s safety), low cost (which is important for the company’s profitability) and compactness (which is important from the geometry point of view as the gearbox must fit in the given space). These goals are conflicting and couple tightly the three aspects of gear design, namely, geometry design, material selection and manufacturing, and final production cost. Thus, it is important that a designer considers geometry, material and manufacturing processes in an integrated way to explore the synergy among them. An integrated gear design approach will overcome time-consuming design iterations and reduce product development time and costs.

To show the advantages of concurrent exploration of design spaces, the scope of the current study is limited to the concurrent exploration of the combined geometric and material space for the design of a gear and will show the effectiveness of the proposed method. The details of the various goals considered in this study and associated conflicts are presented in Table 1.

The concurrent exploration of geometry design and material space also involves large numbers of constraints and conflicting goals. The cDSP, which is a mathematical construct capable of modelling and addressing such problems, is used in this article (Mistree, Hughes, and Bras 1993; Rolander et al. 2006). The cDSP enables the construction of different practical scenarios in a multi-objective formulation by offering the opportunity for assigning appropriate weights to different goals and exploring the compromises among them. In the cDSP, the difference between the desired (the target, $G_i$) and the achieved ($A_i(\vec{x})$) value of a goal is minimized. The difference between these values is modelled as two deviation variables, $d_i^+$ and $d_i^-$, which represent overachievement and underachievement, respectively, for each goal. A cDSP is constructed such that deviations are always positive and simultaneous overachievement or underachievement is not allowed. The cDSP template formulation is shown in Table 2. Further detail on the formulation and application of the cDSP for the current design problem is given in Section 3.2.

3. Compromise decision support problem-based geometry and material space exploration for spur gear design

3.1. Problem statement

The current analysis deals with a problem motivated by industry, the design of a pinion for the first gear reduction for a compact sized automobile. The problem statement is as follows:

Design a compact, highly reliable, compact and low-cost pinion of a commercial spur gear system of AGMA precision number 2 8 to carry a torque of 113 Nm at 4500 rpm with a gear ratio
of 3.5. The design must allow for moderate shock in the driving engine and moderate shock in the driven machinery. The target for reliability should be 99.99% and should not less than 95%. The gear teeth must have a pressure angle of 20° and they will be cut using a rack cutter. The expected fatigue life is 10^9 cycles. The steel for the gear is available in the range of tensile strengths of 800–1600 MPa. The maximum allowable centre distance is 300 mm. Standard gear geometry as per AGMA standard is desired.

3.2. Formulation

Following the example in Rolander et al. (2006) and the template described in Table 2, the cDSP for the design of a gear is given below.

3.2.1. Given

A spur gear pinion is required to be designed based on AGMA (1993, 2010) standards based on the 20° standard pressure angle system. The gear is required to carry 113 Nm torque. The required gear ratio, G, is 3.5.

The standard geometry of a gear is defined by module, m, number of teeth, N, and face width, b (ANSI/AGMA 1993, 2010). Descriptions of gear geometry in terms of module, number of teeth and face width, material strength and reliability can be found in ANSI/AGMA (1993, 2010) and Shigley (1977). The material space is determined by the ultimate tensile strength of the steel, S_U, and is used as a design variable for materials design. Reliability R also has a range of minimum permissible to maximum targeted values and there is scope for exploration in the design. Thus, there are five design variables for the design problem.

The physical limitations on design are represented as bounds and system constraints. The bounds on these design variables are selected based on the knowledge available to the designer. Based on
the problem statement and the AGMA requirements (ANSI/AGMA 1993, 2010), seven constraints are selected. Two constraints are specified on the factor of safety in bending and contact. The space constraint is incorporated using the maximum allowable centre distance. Considering uniform load application on the entire face of the gear, AGMA (1993, 2010) recommends restricting the minimum and maximum allowable face width as a function of module and these become two additional constraints. The contact ratio is an important factor for smooth running of the gear drive and two additional constraints are specified on the minimum and maximum allowable values for contact ratio based on AGMA (1993, 2010) recommendations.

Three design goals are selected for investigation. These are maximization of reliability, maximization of compactness (i.e., minimization of centre distance between gear and pinion) and minimization of cost. These lead to six deviation variables, one each corresponding to the overachievement and underachievement of three design goals. The cDSP allows designers to incorporate given objectives as constraints as well as goals with desirable target values. In this study, compactness limit is modelled as constraint with a desirable goal.

**Assumptions:** Helical gears are preferred for the application described above. However, this study is restricted to spur gears for the purposes of illustration of the method. The material space is represented by considering only the ultimate tensile strength ($S_U$) of the material as many of the material properties, such as hardness and fatigue endurance strength, can be related to the ultimate material strength. In practice, modules for the gear have discrete values due to tooling limitations. However, for simplicity, the module is assumed to be a continuous variable.

### 3.2.2. **Find**

The standard gear geometry is expressed using AGMA standards (ANSI/AGMA 2010) in terms of:

- **module** ($m$) in mm: the module is the ratio of pitch circle diameter to the number of teeth; it has units in millimetres and it is the index of tooth size in SI
- **number of teeth** ($N$)
- **face width** ($b$) in mm: the face width of a gear refers to the thickness of the gear teeth.

These geometry variables are used along with

- **material strength** ($S_U$) in MPa
- **reliability** ($R$).

Find design variables $m, N, b, S_U$ and $R$ and the deviation variables, $d_i^+, d_i^-$.  

### 3.2.3. **Satisfy**

**Design constraints**

\[
\begin{align*}
C_1 & : \text{Minimum face-width}^* : b \geq 3m \\
C_2 & : \text{Maximum face-width}^* : b \leq 5m \\
C_3 & : \text{Maximum limit on centre distance}^# : d = m(1 + G)N/2 \leq 300 \text{ mm} \\
C_4 & : \text{Bending stress induced}^{**} : \sigma_{\text{allowable}}^{\text{bending}} - \sigma_{\text{induced}}^{\text{bending}} \geq 0
\end{align*}
\]
C5 : Contact stress induced∗# : \( \sigma_{\text{contact}}^{\text{allowable}} - \sigma_{\text{contact}}^{\text{induced}} \geq 0 \)  
(5)

C6: Minimum contact ratio∗ : \( R_c \geq 1.4 \)  
(6)

C7 : Maximum contact ratio∗ : \( R_c \leq 1.8 \)  
(7)

*as per AGMA guidelines (ANSI/AGMA 1993, 2010); # as per problem statement.

These constraints are normalized based on the method given by Mistree, Hughes, and Bras (1993).

Goals

Based on the problem statement, maximization of reliability, minimization of centre distance and minimization of cost are selected as the three design goals. The target value for the three design goals is chosen (based on the guidelines in Mistree, Hughes, and Bras 1993) such that

- it is greater than or equal to the maximum expected value for maximization goals
- it is less than or equal to the minimum expected value for minimization goals.

The goals are defined as given below:

- **G1**: Achieve the maximum desired reliability \( R \).
  
  The goal for maximum reliability is taken as 99.99% (which is the maximum achievable reliability given in AGMA). To model the greatest achievement of the goal (Mistree, Hughes, and Bras 1993), the reliability goal is expressed as:

\[
\frac{R}{0.9999} + d^{-1} - d^{+1} = 1
\]

(8)

In AGMA-based design, the influence of reliability is brought in by a correction factor \( Y_z \) in the allowable stresses during gear design through constraints C4 and C5. The details are available in Shigley (1977).

- **G2**: Minimize centre distance \( d \) (makes the design compact).

  The centre distance \( (d) \) is defined as

\[
d = \frac{m(1 + G)N}{2}
\]

(9)

The target centre distance is taken a value corresponding to the minimum \( m \) and minimum \( N \), which is calculated as \( d = 162 \) mm. The centre distance goal, with a target of 162 mm, is expressed as

\[
\frac{162}{d} + d^{-2} - d^{+2} = 1
\]

(10)

- **G3**: Minimize cost \( C \).

  The total cost consists of the material cost and the manufacturing cost and is calculated using an empirical relationship as assumed below:

\[
C = W \left( a_0 + b_0 \left( 1 + \left( \frac{S_U - (S_U)_{\text{min}}}{(S_U)_{\max} - (S_U)_{\min}} \right)^{1.5} \right) \right)
\]

(11)

where \( W \) is the weight of a component, \( a_0 \) is the manufacturing cost [taken as Indian rupees (INR) 5/kg], and \( b_0 \) is the material cost (taken as INR 40/kg for 800 MPa \( S_U \) steel). The increase in material cost with increase in tensile strength is given in Equation (11). The above equation for
cost is valid between \((S_U)_{\min}\) and \((S_U)_{\max}\), which are the minimum and maximum tensile strengths of available materials. The target value of cost corresponds to a gear with a minimum volume and lowest material strength, which is calculated to be \(C = \text{INR 57.00}\). The cost goal is expressed as

\[
\frac{57}{C} + d_3^- - d_3^+ = 1
\]  

(12)

Bounds on design and deviation variables are given by:

- B1: \(4 \leq m \leq 8\) (mm)
- B2: \(18 \leq N \leq 40\)
- B3: \(40 \leq b \leq 80\) (mm)
- B4: \(800 \leq S_U \leq 1600\) (MPa)
- B5: \(0.95 \leq R \leq 0.9999\)
- B6–B11: restriction on deviation variables, \(d_i^- \cdot d_i^+ = 0\) for \(i = 1 \ldots 3\) and \(d_i^+, d_i^- \geq 0\) for \(i = 1 \ldots 3\)

3.2.4. Minimize

The goals are normalized as suggested by Mistree, Hughes, and Bras (1993). The Archimedean approach is used to construct the effective deviation function and it is solved using DSIDES software.

\[
Z(d^-, d^+) = W_1(d_1^- - d_1^+) + W_2(d_2^- - d_2^+) + W_3(d_3^- - d_3^+)
\]

where \(\sum_{i=1}^{3} W_i = 1\) and \(W_i \geq 0\) \(\forall i\)

(13)

3.3. Gear design scenarios

The cDSP allows a designer to explore compromises among different goals by generating various design scenarios by giving different weights to each of the goals. A schematic of the different design scenarios considered in this study is shown in Figure 1.

At each vertex, an individual design goal has been given the full weight of 1.0, e.g. Scenario S1 corresponds to a highly reliable design. Hence, the reliability goal is given a weight of 1.0 and the other two goals have weights of 0.0 each. Other scenarios are generated by considering appropriate contributions from each of these goals, e.g. Scenario S4 corresponds to a reliable and compact design, with weights of 0.5 each for reliability and compactness, and cost is given a weight of 0.0. Scenario S7 corresponds to a reliable, low-cost and compact design, with equal weights of 0.33 for each of the goals.

4. Results and discussion

DSIDES provides many control options, namely, opportunities to control exploration of the design space, specification of the maximum number of iterations and criteria for convergence of design/deviation variables. In this work, the maximum number of iterations is set to 100 and the stationarity (which is part of the stopping criteria and defines the convergence tolerance) of deviation and design variables to 2%.

The results from DSIDES of the seven different scenarios illustrated in Figure 1 are analysed in detail. While analysing the results of different scenarios, the following observations are made.
Figure 1. Schematic representation of different design scenarios. Studying these scenarios offers an understanding and exploration of the design space. At each scenario location, the goals are weighted to assign their importance. In S1, S2 and S3 (reliability, compactness and cost goals, respectively) are weighted 1 and the other goals are assigned zero weight. In scenarios S4, S5 and S6, the goals indicated are each assigned a value of 0.5 and the remaining goal is assigned a value of 0. In S7, all goals are assigned a value of 0.33.

In scenarios such as compact design (S2) or high-reliability design (S1), oscillations in the convergence behaviour of design variables and deviation variables are observed. The convergence behaviour of design variables for Scenario S2 is shown in Figure 2(a). The variables defining compact designs (i.e. minimum \( m \) and minimum \( N \)) converge within the first few iterations. The other three design variables oscillate among three possible states. In all of these states the contact stress constraint is active and violated within acceptable limits. This convergence behaviour is expected as consideration of the compact design goal alone leads to an underconstrained system with no constraints on face width, reliability or tensile strength, and these three parameters can take theoretically infinite feasible combinations. Similarly, in other scenarios such as S1, the same reliability can be achieved by different combinations of geometry (\( m, N \) and \( b \)) and material (\( S_U \)). For Scenario S4, which is a combination of scenarios S1 and S2, oscillation is observed in \( S_U \) and \( b \) in a few initial iterations and finally convergence is obtained.

The cost goal involves all of the design variables and hence it is well constrained. This is observed from the convergence behaviour of the design variable (Figure 2b) for the design scenarios S3, S5, S6 and S7. The output design variables and achieved goal values for all these scenarios are listed in Table 3.

A comparison of normalized goals achieved for different scenarios is shown in Figure 3. These results are in line with the known properties of gear design; for example, when a compact design is required, the material strength needs to be high, and high reliability requires an increase in the size of gear as well as the strength of the material.

Observations from the results show that the minimum cost design is simultaneously a compact design and a minimum face-width design. However, a compact design is not necessarily a minimum cost design as it is a design with larger face width compared to the minimum cost design. The requirement for high reliability conflicts with compactness or cost, as can be expected. Scenarios S4, S6 and S7 are the best gear design scenarios, which have high reliability and exploit compromises among various goals.

To observe the achievability of goals visually, additional scenarios are considered with different weights of goals, formulated with the corresponding cDSP and solved using DSIDES. All possible
Figure 2. Convergence behaviour of design variables for (a) compact design, S2: this convergence behaviour suggests that there are multiple acceptable designs; and (b) minimum cost design, S3: this convergence behaviour suggests that there is only one acceptable design.

Table 3. Results: design variables and goals for different scenarios.

<table>
<thead>
<tr>
<th>Scenario #</th>
<th>Module, ( m ) (mm)</th>
<th>Face width, ( b ) (mm)</th>
<th>Number of teeth, ( N )</th>
<th>Tensile strength, ( S_{UTS} ) (MPa)</th>
<th>Reliability, ( R ) (%)</th>
<th>Centre distance, ( d ) (mm)</th>
<th>Cost, ( C ) (INR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>6.59</td>
<td>67.29</td>
<td>18</td>
<td>1221</td>
<td>99.99</td>
<td>266.94</td>
<td>349.48</td>
</tr>
<tr>
<td>S2</td>
<td>4.00</td>
<td>57.32</td>
<td>18</td>
<td>1444</td>
<td>99.30</td>
<td>162.00</td>
<td>134.55</td>
</tr>
<tr>
<td>S3</td>
<td>4.00</td>
<td>40.26</td>
<td>18</td>
<td>1464</td>
<td>95.40</td>
<td>162.00</td>
<td>96.14</td>
</tr>
<tr>
<td>S4</td>
<td>4.99</td>
<td>76.72</td>
<td>18</td>
<td>1550</td>
<td>99.99</td>
<td>201.95</td>
<td>307.71</td>
</tr>
<tr>
<td>S5</td>
<td>4.00</td>
<td>40.26</td>
<td>18</td>
<td>1464</td>
<td>95.40</td>
<td>162.00</td>
<td>96.14</td>
</tr>
<tr>
<td>S6</td>
<td>5.69</td>
<td>57.59</td>
<td>18</td>
<td>1550</td>
<td>99.99</td>
<td>230.53</td>
<td>300.96</td>
</tr>
<tr>
<td>S7</td>
<td>4.99</td>
<td>76.70</td>
<td>18</td>
<td>1550</td>
<td>99.99</td>
<td>210.96</td>
<td>307.67</td>
</tr>
</tbody>
</table>

Note: In S1, S2 and S3 respectively, the reliability, compactness and cost goals are weighted 1 and the other goals are assigned zero weight. In scenarios S4, S5 and S6, the goals indicated are assigned values of 0.5 and the remaining goal is assigned a value of 0. In S7, all goals are assigned a value of 0.33.
feasible designs obtained during the analysis are collated. Three ternary charts are created (as shown in Figure 4), considering the averaged value of the normalized achievement values of the three goals to study the design scenarios, for each of the goals.

In Figure 4 (a) the visual representation of the design space where the feasible designs are collated is shown, and a ternary plot is created for the reliability goal achievement. The ternary charts shown in Figure 4(b) and (c) are created in similar way for achieved cost and achieved compactness goals, respectively. Maximum and minimum values of the achieved goals are shown in Table 4.

These achieved goal values are normalized between 0 and 1 such that 1 (blue) represents maximum achievement of the given goal and 0 (red) represents minimum achievement. To illustrate, the achievement of maximum reliability (99.99%) is represented by blue and that of minimum reliability (95.4%) is represented by red (Figure 4a); in the case of cost the objective is to minimize it, so the achieved minimum cost (INR 96) is represented by blue and the achieved maximum cost (INR 350) is represented by red (Figure 4b). For compactness, the objective is maximization, which corresponds to minimization of the centre distance, so the achieved minimum centre distance (162 mm) is represented by blue and the maximum achieved centre distance (267 mm) is represented by red.

These ternary charts can be used to compare and observe the implications of a designer’s decision on the selection of goals for gear design. To illustrate, if the designer is interested in high reliability design (the blue region marked by the green dashed line in Figure 4a), its implication (i.e. maximum achievable values for compactness and cost goals) on the design space of other goals (shown by the green dashed line in the corresponding region in Figure 4b and c) can be seen. Similarly, the implications of the designer’s emphasis on low cost (the region shown by the black dashed line in Figure 4b) and compactness (the region shown by the purple dashed line in Figure 4c) on the other goals can be seen. The reliability goal is in direct conflict with the other two (i.e. low cost and compactness) goals. The low-cost design is simultaneously a compact design, but a compact design is not necessarily a low-cost design. This can be explained as follows.

- A low-cost design corresponds to a design having lower weight and lower material cost. A design with lower weight corresponds to a design having minimum centre distance (compact
Figure 4. Visualization of the design space for different goals. The colour bars indicate the degree to which the goal is maximized in the solution; blue indicates the maximum achievement of the given goal while red indicates the minimum achievement of the goal. (a) Solution space of scenario where the goal is maximization of reliability; (b) solution space of scenario where the goal is minimization of cost; (c) solution space of scenario where the goal is maximization of compactness. In all the scenarios (a, b and c), the desirable values are represented in blue. The green dashed line shows the boundary of the region (blue region) in (a) where the normalized reliability achievement is more than 80%, while this same line in (b) and (c) shows the corresponding region where reliability achievement is more than 80% in (a). Similarly, the black dashed line is the boundary of the region where minimum cost achievement is more than 80% (in b and the corresponding region in a and c) and the purple dashed line is the boundary of the region where maximum compactness achievement is more than 80% (in c and the corresponding region in a and b).
Table 4. The maximum and minimum values achieved for each goal obtained in any of the scenarios.

<table>
<thead>
<tr>
<th>Goal</th>
<th>Achieved value</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability (%)</td>
<td></td>
<td>95.4</td>
<td>99.99</td>
</tr>
<tr>
<td>Cost (INR)</td>
<td></td>
<td>96</td>
<td>350</td>
</tr>
<tr>
<td>Centre distance (mm)</td>
<td></td>
<td>162</td>
<td>267</td>
</tr>
</tbody>
</table>

Table 5. Validation of results obtained using DSIDES with AGMA design.

<table>
<thead>
<tr>
<th>Face width b (mm)</th>
<th>Design #1</th>
<th>Design #2</th>
<th>Design #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using AGMA</td>
<td>77.81</td>
<td>39.23</td>
<td>58.40</td>
</tr>
<tr>
<td>Using DSIDES</td>
<td>78.73</td>
<td>40.00</td>
<td>61.34</td>
</tr>
<tr>
<td>% Error</td>
<td>1.18</td>
<td>1.96</td>
<td>5.03</td>
</tr>
</tbody>
</table>

design) as well as having lower face width. Lower material cost is obtained by using a material of lower strength.

- On the contrary, in the compact design no restriction is placed on material strength (which affects the material cost) or on the face width. Therefore, a compact design corresponds only to a design having minimum centre distance.

Overall, the central region shows the region of compromise. Such design space visualization charts (as shown in Figure 4) are useful for a gear designer to make informed decisions on the selection of design goals and the weights for these goals.

5. Verification

Various designs for different scenarios are obtained using DSIDES, which linearizes constraints, and three of these gear designs are selected randomly for validation. To validate the designs, the design values obtained for \( m, N, S_U \) and \( R \) are substituted in the AGMA gear design formulae and the face width for the selected cases is computed using appropriate factors of safety. The face widths computed using AGMA and those given by DSIDES are given in Table 5.

From Table 5, it can be observed that the results obtained using DSIDES agree well with those obtained using AGMA. The difference in the value of face width for design #3 is due to the presence of active constraint C5 in the formulation.

6. Closing comments

Designing a gear to meet specified functional, non-functional and performance requirements is a challenging task. However, having the option of exploring different combinations of materials and geometry allows a designer to understand trade-offs and explore different options. It is advantageous to explore geometry and material space simultaneously as this can reduce costly design iterations and save design time as well as reduce the final product cost. In this article, a novel method for the design of gears using the concurrent exploration of the geometry and material space, using the CEM, is demonstrated. This method will have significant utility for industry, where the total cost of the product must be optimized with trade-offs among the cost of
material, weight of the transmission system, and its durability and reliability. All of these must be accomplished within the constraints of available space.

Conflicting gear design goals such as compactness, high reliability and low cost are considered. Inspection of the results from simulations provides insights into the behaviour of design variables, deviation variables and constraints. The geometry and material strength requirements for different scenarios that have been obtained are in agreement with existing knowledge of gear design and the method used provides the means for a systematic exploration of the design space.

In this case, a conceptual design is sought for a gear rather than a final design. This work is intended to be used to rapidly explore the design space and provide a starting place for a designer, who then will introduce various considerations of manufacturing, materials availability and purity, etc. Therefore, there are some approximations and limitations in the accuracy of the model. For instance, only the ultimate tensile strength ($S_U$) of the material is used to represent the material space, as many of the material properties, such as hardness and fatigue endurance strength, can be related to the ultimate material strength. However, a more complete and complex representation of the material space could be included in the formulation. This does not detract from the method presented here for conceptual design.

In the longer term, this work is intended to form a module in a demonstration information technology platform for ICME (Pollock et al. 2008) in which it will be possible to simultaneously design both the material and the product, instead of relying on the current procedure of selecting materials to build products. Current work (Bhat et al. 2013; Gautham et al. 2013) at Tata Consultancy Services on a Platform for the Realization of Engineered Materials and Products (PREMAP) (Bhat et al. 2013; Gautham et al. 2013) is enriched with various tools for modelling and simulation, supported by tools for enabling knowledge engineering, collaboration, robust design, decision support and data-mining, along with appropriate databases and knowledge bases that will facilitate the simultaneous exploration of the material and product design spaces. Apart from the proposed method for gear design, this work is useful because of the ternary design charts, which are helpful in the conceptual design phase for gear designers to visualize the complex and coupled design space, and to make informed decisions on weights to be given to individual design goals. This will aid the understanding of a variety of engineers and managers who wish to use information about gear design but do not have the time or resources to calculate this information. Further, the CEM is generalizable to other problems that require trade-offs.

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Notes

1. ‘Satisficing is a decision-making strategy or cognitive heuristic that entails searching through the available alternatives until an acceptability threshold is met.’ http://en.wikipedia.org/wiki/Satisficing
2. AGMA 2000-A88. AGMA has defined a set of quality numbers which define the tolerances for gears of various sizes manufactured to a specified accuracy. Quality numbers 3–7 include commercial-quality gears. Quality numbers 8–12 are precision quality.
3. AGMA (1993, 2010) recommends the overload factor, which is intended to make allowance for all externally applied loads in excess of the nominal tangential load, to be selected appropriately based on operating conditions.

References


ISO. 2006. Calculation of Load Capacity of Spur and Helical Gears Parts 1 to 3. ISO 6336-1 to 3.


