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TRAJECTORY OPTIONS FOR LOW-COST MISSIONS TO ASTEROIDS

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Abstract—We consider a wide variety of gravity-assist trajectories using Venus, Earth and Mars to obtain low launch energy trajectories to four large asteroids in the main belt. These trajectories are constructed by analytic and numeric search techniques. We optimize promising trajectories for minimum total ΔV and search for additional (nontargeted) asteroid flybys. Several optimized opportunities with multiple asteroid flybys are reported, followed by a discussion of general characteristics of the various trajectory types. © 1998 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

Asteroids are of keen scientific interest, holding clues to the formation and evolution of our solar system. Current plans for planetary and solar system exploration call for innovative, exciting missions which are low cost. Missions to asteroids with low launch energies and short flight times can fulfill all of these goals. If the trajectory is well-designed, a single mission can incorporate flybys of a variety of asteroids and achieve several scientific objectives.

For this study we concentrate on four large asteroids at a range of distances from the Sun (Tables 1 and 2). In terms of size, Ceres ranks as the largest known asteroid with Vesta third, Hygiea fourth and Sylvia eighth. Vesta has a larger semimajor axis than about 25% of all asteroids. It has the smallest semimajor axis of asteroids with radii greater than 100 km. Sylvia, on the other hand, has a larger semimajor axis than more than 95% of all asteroids. It has the largest semimajor axis of main-belt asteroids with radii greater than 100 km. By finding trajectories to these four asteroids, we can gain insight into general characteristics which govern trajectories to all of the main-belt asteroids.

In this paper we examine many combinations of gravity assists and impulsive maneuvers to reach the asteroids. The bodies considered for gravity assists are Earth, Venus and Mars. We restrict our analysis to flyby missions. The related topic of rendezvous missions is discussed in the literature [1].

Table 1. Asteroid physical characteristics

Number	Name	Type	Radius [km]
1	Ceres	G	470
4	Vesta	V	260
10	Hygiea	C	210
87	Sylvia	P	140

2. SEARCH TECHNIQUES AND PARAMETERS

We follow a systematic method of searching for viable flyby trajectories to the main-belt asteroids. We begin our study of the trajectory options with an analytic approach using simplified orbits in order to achieve some general results. Guided by the insight provided by the analytic theory, we perform a broad search for trajectories with launch dates from 2000 through 2009 using an automated version of STOUR [2] (Satellite Tour Design Program). STOUR is used for the design of patched-conic gravity-assist trajectories with realistic (noncircular, noncoplanar) planetary orbits.

We select a few of the particularly promising trajectories obtained from the broad search and optimize them with MIDAS [3] (Mission Design and Analysis Software) to minimize total ΔV. (Launch ΔV is determined assuming launch from a circular parking orbit of 240 km altitude.) We then search for close flybys of additional asteroids and reoptimize to produce a mission with multiple asteroid flybys. [The additional asteroids will be referred to in the following manner: name (number, radius in km, type if known).]

Direct trajectories from Earth to the asteroids (i.e. with no gravity assists) require launch $V_\infty s$ ranging from 5 km/s to reach Vesta to 7 km/s to reach Sylvia. (The corresponding launch ΔVs range from 4.3 to 5.3 km/s.) A single Mars gravity assist (MGA) can reduce the required launch V_∞ by

Table 2. Asteroid orbit characteristics

Name	Semimajor axis [AU]	Eccentricity	Inclination [degrees]
Vesta	2.36	0.090	7.14
Ceres	2.77	0.077	10.60
Hygiea	3.14	0.121	3.84
Sylvia	3.49	0.082	10.87

more than 1 km/s and the corresponding launch ΔV by over 0.5 km/s [4,5]. However, the MGA trajectories occur less frequently than the direct trajectories. An extra phasing orbit around the Sun either before or after the Mars flyby can be used to increase the number of opportunities, but the flight times are significantly longer in these cases.

Another important characteristic of MGA trajectories is their potential to significantly reduce the V_∞ of the asteroid flyby. A second Mars gravity assist can lower the flyby $V_\infty s$ even more. For a rendezvous mission the arrival V_∞ at the asteroid must be eliminated with a propulsive maneuver, so having a small V_∞ magnitude is highly desirable for these missions. Yen [1] indicates that an M^2GA trajectory is the best type to use to rendezvous with a majority of the main-belt asteroids.

In the next three sections, we examine trajectory types that have the potential to reach the asteroids with lower total ΔV than direct and MGA trajectories. The new trajectories provide many opportunities for flyby missions to asteroids with low total ΔV. Using Earth and/or Venus as the primary gravity-assist bodies, we search for trajectories that have a launch V_∞ less than 5 km/s and a total ΔV of about 4.5 km/s or less. Short flight times are preferred over long flight times; trajectories with total flight times longer than 7 years are not considered.

3. EARTH GRAVITY ASSIST

Hollenbeck [6] introduced ΔV-EGA (delta-velocity Earth gravity-assist) trajectories in which the spacecraft completes approximately one orbit following launch before returning to Earth for a gravity assist. Sims *et al.* [7] analyze multiple-revolution ΔV-EGAs and introduce the following designation for all ΔV-EGA trajectories:

$$K : L(M)^{\pm},$$

where K is the number of Earth orbit revolutions, L is the number of spacecraft orbit revolutions, M is the spacecraft orbit revolution on which the maneuver is performed and $+(-)$ indicates Earth encounter just after (before) the spacecraft passes through perihelion. The first number, K, gives the approximate time in years between launch from Earth and the Earth gravity assist. The number in parentheses may be omitted when not important to the discussion or if the number of spacecraft revolutions is one.

ΔV-EGAs can significantly reduce the total ΔV required by direct trajectories to reach a given aphelion radius. This reduction in total ΔV comes with the cost of a longer flight time and larger post-launch ΔV.

A 2 year orbit with perihelion at 1 AU has an aphelion radius of 2.17 AU, which is slightly larger

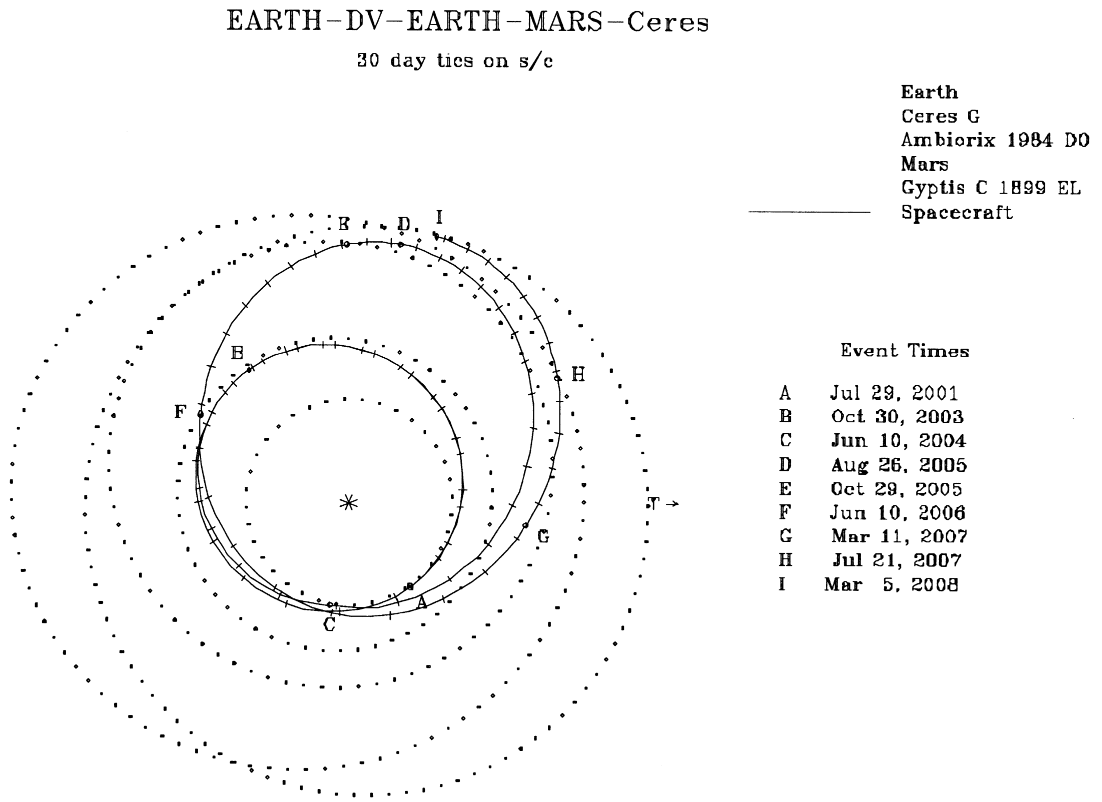

Fig. 1. ΔV-EMGA trajectory to Ceres.

than the perihelion radius of Vesta. So a 2:1 ΔV-EGA does not offer any advantage in reaching Vesta at its minimum radius. For asteroid orbits beyond that of Vesta, a 2:1 ΔV-EGA can reduce the total ΔV required by a direct trajectory. To reach the semimajor axis of Sylvia, a direct trajectory requires a total ΔV of 5.45 km/s, whereas a 2:1 ΔV-EGA requires only 4.64 km/s.

The 3:2 ΔV-EGA can be used to reduce the total ΔV even more than the 2:1 ΔV-EGA for aphelion radii below 4.3 AU, which encompasses the orbits of nearly all of the main-belt asteroids. The 4:3 ΔV-EGA outperforms the 3:2 ΔV-EGA below about 2.4 AU, but adds an additional year to the flight time. Several other characteristics of multiple-revolution ΔV-EGAs are described by Sims [5] and Sims *et al.* [7].

We can add ΔV-EGA trajectories to the beginning of direct and MGA trajectories in order to reduce the total ΔV. MGA trajectories with low total ΔV and up to one phasing orbit occur very infrequently. A ΔV-EGA can make some of the MGAs with higher launch energies more accessible, thereby increasing the number of opportunities for trajectories with low total ΔV. For example, a family of MGA trajectories with short flight times to Ceres has launch dates around June 2004; however, these trajectories have launch $V_\infty s$ of more than 6.0 km/s (and total ΔV greater than 4.80 km/s). A $3{:}2(2)^-$ ΔV-EGA trajectory reduces the total ΔV to 4.18 km/s. Flybys of Gyptis (♯444, 85, C) and Ambiorix (♯3519, 4) can be added for additional ΔVs of 0.07 and 0.08 km/s, respectively. The resulting trajectory is shown in Fig. 1.

In general, the aphelion of the nominal orbit of the 3:2 ΔV-EGA is 1.6 AU, slightly larger than the semimajor axis of Mars. So a Mars flyby would occur near aphelion and could be used to offset or entirely replace the aphelion ΔV, thereby making these Mars–Earth gravity-assist (MEGA) trajectories very efficient in terms of propellant usage. Some MEGA trajectories are analogous to a 3:2 ΔV-EGA while others resemble a 4:3 ΔV-EGA.

We discovered a MEGA trajectory [similar to a $4{:}3(2)^+$ ΔV-EGA] to Vesta with a launch V_∞ of 2.97 km/s (with a total ΔV of 3.61 km/s) and noticed that it passed within 12.5 million km of Ceres. Normally we would not pursue a nontargeted flyby at such a large distance, but since it was Ceres, the largest asteroid, we reoptimized the trajectory to include flybys of both Vesta and Ceres. Adding Ceres required a post-launch maneuver of 0.63 km/s. A flyby of Gyptis can be added with 0.11 km/s more ΔV. For a total ΔV of 4.35 km/s the trajectory will fly by Gyptis at 7.7 km/s, Ceres at 5.4 km/s and Vesta at 5.4 km/s, in less than 5.88 years. This trajectory is shown in Fig. 2. Removing the EME portion of the trajectory, we can launch

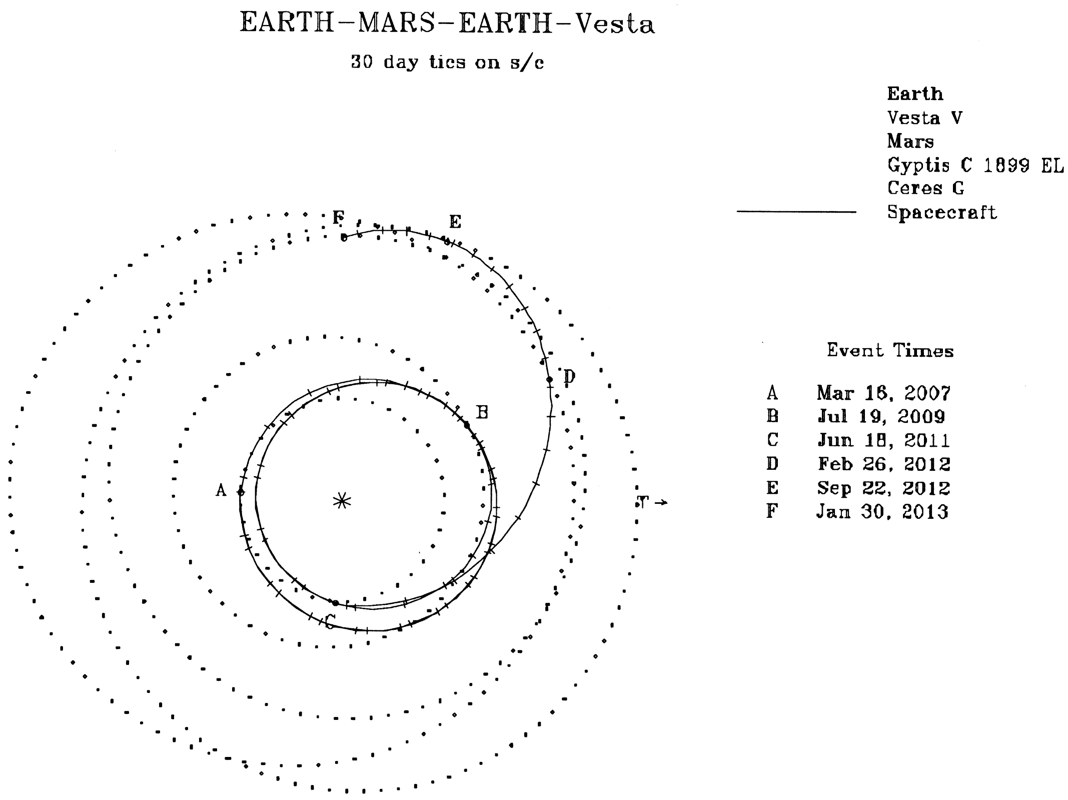

Fig. 2. MEGA trajectory to Ceres and Vesta.

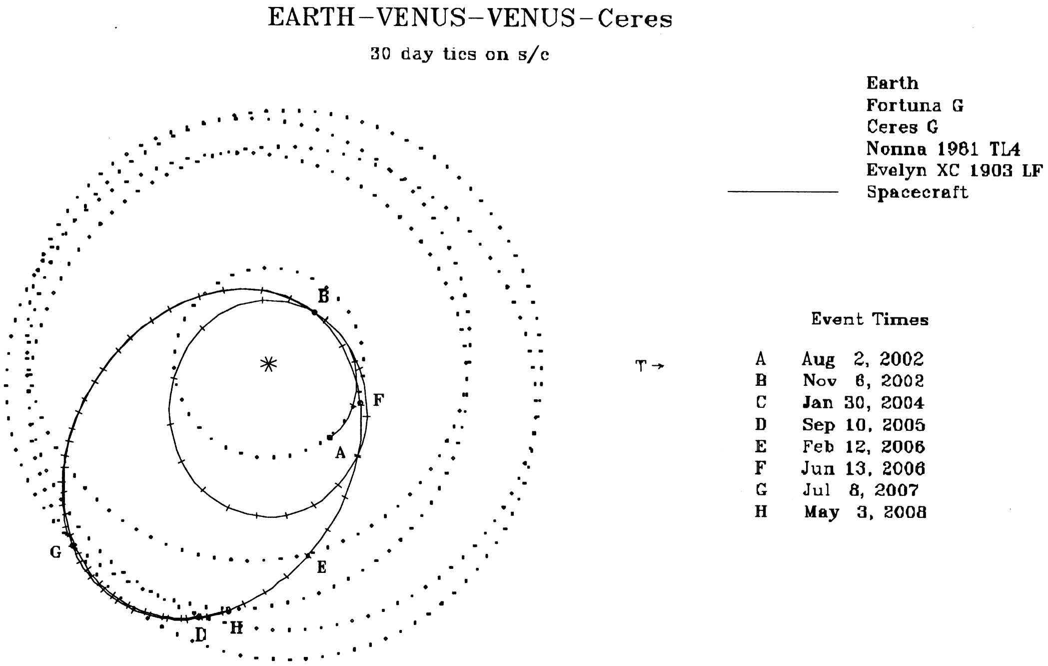

Fig. 3. V^2GA trajectory to Ceres.

from Earth, fly by Ceres, and encounter Vesta in 1.65 years with a total ΔV of 5.87 km/s.

The ΔV-EGA and MEGA trajectories are special cases of V_∞ leveraging [5,7]. The trajectories described in this section use the V_∞ leveraging primarily to increase the energy (and aphelion) of the orbit. V_∞ leveraging can also be used to increase the inclination of an orbit [8]. The NEAR mission [9] is using a $2{:}1^-$ ΔV-EGA to increase the inclination by about 10°.

4. VENUS GRAVITY ASSIST

A single Venus gravity assist with no post-launch maneuvers can be used to reach Mars and a small fraction of the main-belt asteroids, but not Vesta or the other asteroids in our study. Two Venus gravity assists (VVGA or, more succinctly, V^2GA) can be used to reach slightly beyond Sylvia. The minimum launch V_∞ (using a step size of 0.5 km/s) found by STOUR for V^2GA trajectories to the asteroids are as follows: Vesta, 4.0 km/s; Ceres, 4.5 km/s; Hygiea, 5.0 km/s; and none for Sylvia less than or equal to 5.0 km/s. (STOUR did find some V^3GAs to Sylvia with launch $V_\infty s$ of 5.0 km/s.) An example of an optimized V^2GA to Ceres has an August 2002 launch date with a launch V_∞ of 4.39 km/s (corresponding to a total ΔV of 4.06 km/s) and a flight time of 3.14 years. This trajectory can be altered to fly by Fortuna (♯19, 100, G) after a total of 5.7 years for an additional cost of 0.17 km/s in total ΔV. We can also add flybys of Nonna (♯4022, 5) and Evelyn (♯503, 40, XC) for additional total ΔVs of 0.06 and 0.08 km/s, respectively. This V^2GA trajectory to Ceres, Nonna, Evelyn and Fortuna, which is depicted in Fig. 3, has a total ΔV of 4.37 km/s and a flight time of 5.75 years.

Trajectories with multiple Venus flybys can use V_∞ leveraging with Venus by adding a maneuver near aphelion of a Venus–Venus leg resulting in a ΔV-VGA trajectory analogous to a ΔV-EGA trajectory [5]. The aphelia of 2:1 ΔV-VGAs are in the vicinity of the orbit of Mars. So, as with the MEGA trajectories, a Mars flyby would occur near aphelion and could be used to offset or entirely replace the aphelion ΔV. These VMVGA trajectories occur infrequently without additional orbit revolutions used for phasing.

The arrival $V_\infty s$ at the asteroids tend to be higher for trajectories with Venus as the final gravity-assist body than those with Earth as the final gravity-assist body. A Mars gravity assist after the final Venus gravity assist can reduce the V_∞ at Venus required to reach a particular asteroid, and it can also reduce the encounter V_∞ at the asteroid.

5. VENUS AND EARTH GRAVITY ASSIST

Hollenbeck [6] gives a preliminary assessment of the potential of VEGA (Venus–Earth gravity–assist) trajectories and discusses the required phasing for various combinations of Earth–Venus and Venus–Earth legs. (He also gives examples of VEGA trajectories to Jupiter and Saturn.) Bender and Friedlander [10] give characteristics of Earth–Venus–Earth trajectories over about a 1 year time span of Earth returns and demonstrate the 8-year repeatability of these trajectories. They also present two VEGA trajectories which fly by five asteroids.

We discovered a VEGA trajectory to Ceres with a launch date in May 2007 that returns to Earth after 6.37 years using a Mars gravity assist. This optimized trajectory has a total ΔV of 3.92 km/s, including a launch V_∞ of 3.49 km/s and a post-launch ΔV of 0.17 km/s. (The VEGA trajectory without returning to Earth a second time encounters Ceres in 2.71 years with a launch V_∞ of 3.47 km/s and no post-launch ΔV.) For the trajectory shown in Fig. 4, we have added three additional flybys of asteroids. The trajectory has the following sequence of encounters after launch: Venus, Earth, Vale (♯131, 22, SU), Ceres, Mars, Ulyanov (♯2112, 5), Hedwig (♯476, 61, P) and Earth. This 'grand tour' of the inner solar system encounters the three largest terrestrial planets, the largest asteroid and three other asteroids of various types before returning to Earth in less than 6.37 years after launch. The total ΔV of this trajectory is 4.13 km/s, with a launch V_∞ of 3.49 km/s and a post-launch ΔV of 0.37 km/s. The minimum distance from the Sun is 0.72 AU. Following the first asteroid flyby 2.18 years into the flight, the encounters are well-spaced, ranging from 240 to 400 days between encounters with an average of about 300 days. The return to Earth can be used to bring back samples and/or as a gravity assist to target additional asteroid flybys.

Bender and Friedlander [10] note that, because of the encounter geometry, the Earth flyby cannot turn the $\mathbf{V}_\infty$ vector in the same direction as the Earth's velocity. Hence, generally speaking, the minimum relative velocities of the asteroid encounters will be larger than those resulting from direct trajectories. There are two ways in which we can use a Mars gravity assist to reduce the encounter velocity and to potentially reduce the total ΔV. One way is to fly by Mars after the Venus gravity assist but prior to the Earth gravity assist. The Mars gravity assist can raise the perihelion of the orbit, resulting in a more favorable encounter geometry at Earth. We can also use a Mars gravity assist on the way to an asteroid following the Earth gravity assist. The effect in this case is analogous to that described in Section 2. The Mars gravity assist can reduce both the V_∞ at encounter with the asteroid and the required launch V_∞.

6. DISCUSSION

6.1. *Total* ΔV

Table 3 presents the minimum Earth launch V_∞ and corresponding launch ΔV required to escape the Earth, fly directly to Venus, fly directly to Mars, or fly to Mars with a Venus gravity assist, assuming no deterministic post-launch maneuvers. (In all these cases the total ΔV is the launch ΔV.) Here we assume circular, coplanar orbits for Earth,

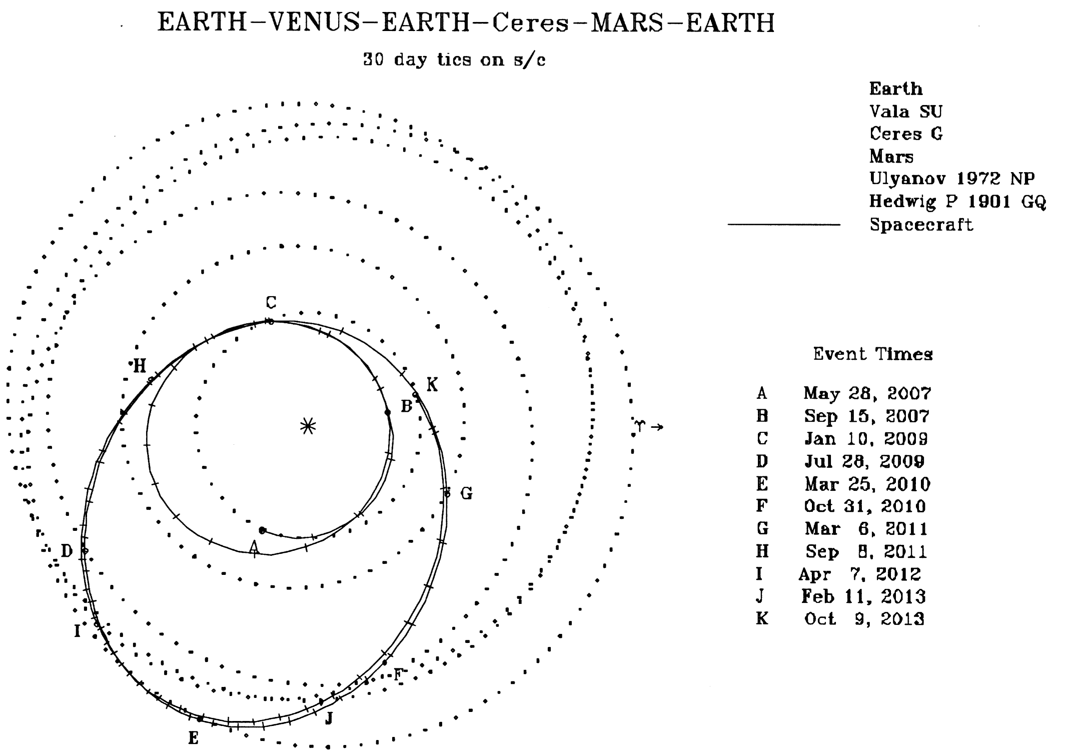

Fig. 4. Inner solar system grand tour.

Table 3. Minimum launch V_∞ and ΔV

Trajectory type	Launch V_∞ [km/s]	ΔV [km/s]
Escape from Earth	0	3.21
Direct to Venus	2.50	3.49
Direct to Mars	2.94	3.60
VGA to Mars	3.10	3.64

Venus and Mars and launch from a circular parking orbit of 240 km altitude.

With these values of total ΔV in mind, we analyze the performance of the trajectories examined in this study. We found MEGAs to Vesta, Ceres and Hygiea with total ΔVs of 3.61 km/s (i.e. launch $V_\infty s$ less than 3.0 km/s). The lowest total ΔV for MEGAs to Sylvia is 3.79 km/s if we constrain the flight time to less than 7 years. VEGAs to Vesta and Hygiea have total ΔVs as low as 3.60 km/s, while the lowest total ΔVs found for VEGAs to Ceres and Sylvia are about 3.75 km/s. This demonstrates the potential of MEGAs and VEGAs to reach a majority of the asteroids in the main belt (at least out to 2.75 AU) with total ΔVs near 3.60 km/s. MEGAs and VEGAs can reach almost all of the asteroids in the main belt with a total ΔV less than 3.80 km/s; it's just a matter of the proper alignment of the three bodies involved.

The minimum total ΔVs we found for VMVGAs to Vesta, Ceres and Hygiea are 3.67, 3.69, and 3.73 km/s, respectively. VMVGAs seem to depend more on the distance to the asteroids than do MEGAs and VEGAs. We did find VMVGAs to Sylvia with total ΔVs less than 4.1 km/s, but the flight times are over 7 years. We also found VMVMGAs to Sylvia with total ΔVs under 4.0 km/s, but again the flight times are long (greater than 7 years).

Both MEGAs and VMVGAs are V_∞ leveraging trajectories with Mars replacing the aphelion ΔV. Using a propulsive maneuver instead of the Mars gravity assist, ΔV-EGAs can be used to reach any of the asteroids by hooking into any direct or MGA trajectory. The ΔV-EGAs are more sensitive to the distance to the asteroids than are the MEGAs. A 3:2 ΔV-EGA can reach Sylvia with a total ΔV less than 4.5 km/s and can reach Vesta with less than 4.0 km/s of total ΔV. A 4:3 ΔV-EGA has lower total ΔV to Vesta than a 3:2 ΔV-EGA, but higher total ΔV to Ceres and beyond. A 4:3 ΔV-EMGA to Vesta has a total ΔV as low as 3.72 km/s, while a 3:2 ΔV-EMGA to Sylvia has a total ΔV as low as 4.23 km/s. The ΔV-VGA cannot be applied as readily (due to the planetary alignment requirement) as the ΔV-EGA, but there are more opportunities for ΔV-VGAs than for VMVGAs. The minimum total ΔVs for ΔV-VGAs are 0.2–0.3 km/s larger than the minimum for VMVGAs to Vesta, Ceres and Hygiea.

6.2. Flight time

The direct trajectories can have flight times less than 1 year with a launch V_∞ of 6.0 km/s to Vesta, 7.5 km/s to Ceres and Hygiea, and over 8.0 km/s to Sylvia. Direct trajectories with minimum launch V_∞ have flight times less than 2 years. The MGA trajectories that we found with the minimum launch $V_\infty s$ to each of the asteroids all use a phasing orbit which increases the flight time to over 3 years. However, we did find MGAs to Vesta and Hygiea with launch $V_\infty s$ slightly above the corresponding minimum values that do not require a phasing orbit and have flight times less than 2 years.

The 3:2 ΔV-EGAs have flight times of about 3 years between launch and Earth return, while the 4:3 ΔV-EGAs require about 4 years before the Earth gravity assist. The " – " type ΔV-EGAs are shorter than the " + " type by a few months. Including the time of flight from the Earth gravity assist to the asteroid encounter, the minimum total flight times are a little over 3 years with $3{:}2^-$ ΔV-EGAs and a little over 4 years with $4{:}3^-$ ΔV-EGAs. The MEGAs with minimum launch $V_\infty s$ are analogous to 4:3 ΔV-EGAs and have flight times of close to 5 years or more. There are MEGAs analogous to 3:2 ΔV-EGAs that have flight times of about 4 years or less to each of the asteroids.

The VEGAs with the lowest launch $V_\infty s$ have flight times less than 3 years to Vesta and Ceres and less than 5 years to Hygiea and Sylvia. With slightly larger launch $V_\infty s$ the flight time can drop to less than 3 years to Hygiea and to less than 4 years to Sylvia.

Trajectories with two Venus gravity assists and no Earth gravity assists have flight times as low as about 3 years to Vesta and Ceres and less than 4 years to Hygiea. The VMVGAs with the lowest total ΔVs, however, have flight times over 4 years to Vesta and Ceres and about 5 years to Hygiea. We have not found any trajectories to Sylvia with only two Venus gravity assists that have a total ΔV less than 4.5 km/s and a flight time less than 7 years, but we did find a V^3GA trajectory to Sylvia with a flight time less than 5 years (and a total ΔV less than 4.5 km/s).

6.3. Frequency of launch opportunities

The synodic period between Earth and the asteroids ranges from 1.4 years with Vesta to 1.2 years with Sylvia. Direct trajectories with reasonable launch V_∞ and no phasing orbits occur every synodic period; however, the characteristics of these trajectories vary between synodic periods due to the eccentricity and inclination of the asteroid orbits. A ΔV-EGA trajectory can be added to the beginning of any direct trajectory, reducing not only the total ΔV, but also the variation in total ΔV between synodic periods. The $3{:}2^\pm$ and $4{:}3^\pm$ ΔV-EGAs provide a total of four launch opportunities corresponding to the direct trajectories every synodic period.

The synodic period between Earth and Mars is over 2 years and between Mars and an asteroid ranges from over 2.5 years with Sylvia to nearly 4 years with Vesta. Since MGA trajectories to asteroids require the proper relative alignment of Earth, Mars, and an asteroid, the low launch V_∞ trajectories occur infrequently without the use of phasing orbits.

The MEGA trajectories occur almost as frequently as direct trajectories. The Mars gravity assist can occur on one of three orbit revolutions between Earth launch and Earth return for the MEGAs that are analogous to the 4:3 ΔV-EGAs. The Earth gravity assist can occur either before or after perihelion of the return orbit. The flexibility inherent in these two facts provides many opportunities for MEGA trajectories. The VEGA trajectories provide even more opportunities than the MEGA trajectories. This abundance of opportunities is due primarily to the availability of many combinations of trajectory types between launch and the Venus encounter, and between Venus and the return to Earth [10].

The V^2GA trajectories occur infrequently to Ceres and Hygiea, but more frequently to Vesta. When V_∞ leveraging is included between Venus flybys, the number of opportunities increases dramatically.

7. CONCLUSION

Ample opportunities exist for low launch energy trajectories to the main-belt asteroids. We hope that the trajectories we have identified here may provide useful benchmarks to guide the selection of trajectory types for future low-cost flyby missions. Each trajectory type has its advantages and disadvantages and the final choice is left to mission designers, who must make their decisions based on their intimate knowledge of spacecraft capabilities and mission requirements.

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