EPACS Workshop: Notes on Turbojet Engine Modeling

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- Taylor linearization of the non-linear model

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- This is accomplished by means of a *controller* whose task is to produce the required system's inputs that in turn result in the desired system's outputs
- Constructing a controller is a part of the *control problem*

Essential Elements of the Control Problem

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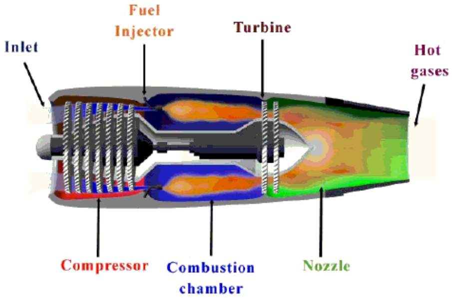
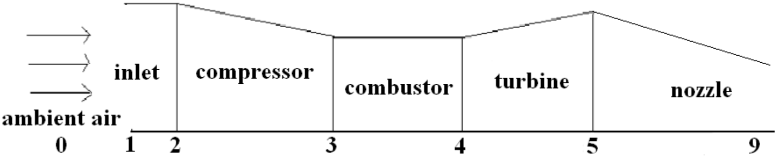
4. a means of measuring the performance of any given control strategy to evaluate its effectiveness

Single spool turbojet engine

Figure comes from Jayachandran Kamaraj, *Modeling and Simulation of Single Spool Jet Engine*, MS Thesis in Aerospace Engineering and Engineering Mechanics, The University of Cincinnati, April, 2003, p. 8



Turbojet engine station numbering



Surrounding Atmosphere Model

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- Mean sea level conditions, International Standard Atmosphere (ISA):

T_{std}	Ambient static temp	288.15 °K
P_{std}	Ambient static pressure	101325 Pa
a_{std}	Speed of sound	340.294 m/s
ρ_{std}	Density	1.225 kg/m^3

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- ISA 11,000 m to 25,000 m altitude
- $$T_{\text{amb}} = 216.69 \quad \text{and} \quad P_{\text{amb}} = 22632 * e^{(1.733 - 0.000157 * \text{alt})}$$

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- Stagnation values of the temperature and pressure at the diffuser exit

$$T_{t2} = T_{t0} \quad \text{and} \quad P_{t2} = \eta_I P_{t0},$$

where the inlet efficiency $\eta_I = 1.0$ if $M \leq 1$ and for $M > 1$,

$$\eta_I = 1.0 - 0.075(M-1)^{1.35}$$

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- Temperature of the incoming air also increases with pressure in the compressor
- Compressor performance maps are obtained from the actual rig test of the engines
- Corrected parameter values used to eliminate the dependence of the performance characteristics on the values of the inlet temperature and pressure

Corrected parameters at the i-th engine station

- Dimensionless ratios of pressure, temperature, and density

$$\delta = \frac{P}{P_{\text{std}}}, \quad \theta = \frac{T}{T_{\text{std}}}, \quad \sigma = \frac{\rho}{\rho_{\text{std}}},$$

where $P_{\text{std}} = 101.3\,\text{kPa}$, $T_{\text{std}} = 288.15^\circ\,\text{K}$, and $\rho_{\text{std}} = 1.225\,\text{kg/m}^3$

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$$\dot{m}_{ci} = \dot{m}_i \frac{\sqrt{\theta_i}}{\delta_i}$$

- Corrected engine speed

$$N_{ci} = \frac{N}{\sqrt{\theta_i}}$$

Mass Flow Parameter (MFP)

- One-dimensional mass flow equation,

$$\dot{m} = \rho A V,$$

where $\dot{m}$ is the mass flow rate per unit area, ρ is density, A is the duct area, and V is the velocity

L. C. Jaw with J. D. Mattingly, *Aircraft Engine Controls: Design, System Analysis, and Health Monitoring*, American Institute of Aeronautics and Astronautics, Inc. Reston, VA, 2009, p. 313

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$$P = \rho R T,$$

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- Combining the above equations gives

$$\frac{\dot{m}}{A} = \rho V = \frac{PV}{RT} = \frac{V}{\sqrt{\gamma g_c R T}} \frac{P\sqrt{\gamma g_c}}{\sqrt{RT}}$$

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- Manipulate more to obtain

$$\frac{\dot{m}}{A} = \frac{V}{\sqrt{\gamma g_c RT}} \frac{P\sqrt{\gamma g_c}}{\sqrt{RT}} = M\sqrt{\frac{\gamma g_c}{R}} \frac{P}{P_t} \sqrt{\frac{T_t}{T}} \frac{P_t}{\sqrt{T_t}}$$

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$$\text{MFP} = \sqrt{\frac{\gamma g_c}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Useful Formula Involving Mass Flow Parameter (MFP)

- We have

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- Comparing, we obtain

Formula for computing the mass flow rate, $\dot{m}$, using the MFP function

$$\dot{m} = \frac{P_t A}{\sqrt{T_t}} \text{MFP}(M)$$

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Yarlagadda's Compressor Model

- Mass flow rate and the shaft speed for the compressor are represented in corrected parameters in the compressor performance map

Santosh Yarlagadda, *Performance Analysis of J85 Turbojet Engine Matching Thrust with Reduced Inlet Pressure to the Compressor*, MS Thesis in ME, The University of Toledo, May 2010

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- Notation: $\dot{m}_3$ (kg/s)—mass flow rate at the exit of the compressor, $\dot{m}_{3'}$—mass flow rate in the compressor, $\dot{m}_{3'\text{corr}}$—corrected mass flow rate in the compressor;

$$\dot{m}_{3'\text{corr}} = \frac{\dot{m}_{3'}\sqrt{\frac{T_2}{T_{2\text{ref}}}}}{\frac{P_2}{P_{2\text{ref}}}}$$

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- Notation: $N_{\text{corr comp}}$—corrected speed for the compressor, N_{des}—shaft speed at the design point;

$$N_{\text{corr comp}} = \frac{N}{N_{\text{des}}}\sqrt{\frac{T_{2\text{des}}}{T_2}}$$

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Obtaining $\dot{m}_{3'_{\text{corr}}}$ and $\eta_{\text{Is comp}}$ to compute $\dot{m}_{3'}$

- Corrected mass flow rate of the compressor,

$$\dot{m}_{3'_{\text{corr}}} = f_1\left(\frac{P_3}{P_2}, N_{\text{corr comp}}\right)$$

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- Isentropic compression efficiency,

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- Compute the actual mass flow rate,

$$\dot{m}_{3'} = \frac{\dot{m}_{3'_{\text{corr}}}\frac{P_2}{P_{\text{ref}}}}{\sqrt{\frac{T_2}{T_{\text{ref}}}}}$$

Compressor—computing T_{t3}, $\dot{m}_{c2}$, and $\dot{W}_c$

- Notation—$\eta_{\text{Is Comp}}$ is the isentropic compression efficiency in the compressor

L. C. Jaw with J. D. Mattingly, *Aircraft Engine Controls: Design, System Analysis, and Health Monitoring*, American Institute of Aeronautics and Astronautics, Inc. Reston, VA, 2009, p. 320

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- Temperature $T_{3'}$ of the compressor by Yarlagadda, p. 30,

$$T_{3'} = T_2 \left(1 + \frac{1}{\eta_{\text{Is Comp}}} \left(\left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \right)$$

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- The compressor corrected mass flow rate of Jaw and Mattingly,

$$\dot{m}_{c2} = \sqrt{\frac{T_{t2}}{T_{\text{std}}}} \frac{P_{\text{std}}}{P_{t2}} \frac{P_{t4}}{\sqrt{T_{t4}}} \frac{A_4}{1+f} \text{MFP}(M_4)$$

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- The compressor input power, $\dot{W}_c = \dot{m}_2 c_{pc} (T_{t3} - T_{t2})$

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Compressor Dynamics

- Notation: *Bld*—compressor bleed flow rate (kg/s), W_3—mass of the air in the compressor (kg), V_3–compressor volume in m^3;

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- Let $\dot{m}_{3'}$ be the inflow rate of change of air and $(\dot{m}_3 + Bld)$ be the outflow rate of change of air,

$$\frac{dW_3}{dt} = \dot{m}_{3'} - \dot{m}_3 - Bld$$

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- $$\frac{dT_3}{dt} = \frac{1}{W_3 c_v}\left(\dot{m}_{3'}\left(c_p T_{3'} - c_v T_3\right) - \left(\dot{m}_3 + Bld\right) R T_3\right)$$

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- $$P_3 = \frac{W_3 R T_3}{V_3}$$

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Combustor (Burner) Dynamics

- Pressure drop across the combustor calculated from the formula,

$$\dot{m}_b = \dot{m}_3 = \sqrt{\frac{P_3 - P_4}{R_b}},$$

where $\dot{m}_b$ is the mass flow rate in the cumbustor (burner) in kg/s

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- R_b is the calculated combustor pressure loss coefficient computed from the steady state values of the engine at various speeds
- Because of this pressure drop, the gases can flow in the correct direction and mass flow rate through the combustion chamber is calculated

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- R_b is the calculated combustor pressure loss coefficient computed from the steady state values of the engine at various speeds
- Because of this pressure drop, the gases can flow in the correct direction and mass flow rate through the combustion chamber is calculated
- W_4—mass of the air in the combustor (kg)

$$\frac{dW_4}{dt} = \dot{m}_3 + \dot{m}_f - \dot{m}_4,$$

  where $\dot{m}_f$ is the mass flow rate of the fuel in kg/s

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$$\frac{dT_4}{dt} = \frac{1}{W_4 c_{vb}} \left(\dot{m}_3 \left(c_{pb} T_b - c_{vb} T_4 \right) + \dot{m}_f \left(\mathrm{HVF} \eta_b - c_{vb} T_4 \right) - \dot{m}_4 R T_4 \right)$$

where η_b is the combustor (burner) efficiency, HVF is the lower heating value of fuel, $\mathrm{HVF} = 43120\,\mathrm{kJ/kg}$, and T_b is the combustor interpolation constant calculated from

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Engine Fuel Flow Analysis

- An expression for the engine fuel flow from application of energy conservation to the burner

$$\dot{m}_3 h_{t3} + \dot{m}_f \eta_b h_{PR} = \dot{m}_4 h_{t4},$$

where $\dot{m}_3$ and $\dot{m}_4$ are the mass flow rates of gas entering and leaving the burner, h_{t3} and h_{t4} are the total enthalpies, η_b is the combustion efficiency, and h_{PR} is the heating value of the fuel (maximum energy released when fuel is burned at 100% efficiency)

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The Rate of Change of the Pressure in the Combustor Volume

- The rate of change of air (gas) masses within the combustor "control surface"

$$\dot{m} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \dot{m}_3 + \dot{m}_f - \dot{m}_4$$

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$$dP = (\rho R)_o \, dT + (RT)_o \, d\rho,$$

where the subscript "o" denotes a nominal operating condition

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- Taking into account that the first term is much smaller than the second term gives

$$\begin{aligned} \frac{dP_4}{dt} &\approx \left(\frac{RT_4}{V_4}\right)_o \dot{m} \\ &= \left(\frac{RT_4}{V_4}\right)_o (\dot{m}_3 + \dot{m}_f - \dot{m}_4) \end{aligned}$$

Turbine Modeling—Corrected Parameters

- Corrected mass flow rate, $\dot{m}_{5'\text{corr}} = \dfrac{\dot{m}_{5'}\sqrt{\frac{T_4}{T_{4\text{ref}}}}}{\frac{P_4}{P_{4\text{ref}}}}$

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Computing actual mass flow rate and temperature

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- Power

$$Pw_t = \dot{m}_{5'} c_{pb} (T_5 - T_4)$$

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Shaft Dynamics

- Newton's second law applied to a shaft of a two-disk system,

$$I\dot{\omega} = \Delta Q,$$

where I is the moment of inertia, ΔQ is the differential torque exerted on the disks, and ω is the angular velocity in rad/s

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- $$\Delta Q = \eta_m Q_t - Q_c - Q_l,$$

where η_m is the turbine efficiency, Q_t—turbine's torque, Q_c—compressor's torque, and Q_l is the loading torque

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- Multiply both sides of $I\dot{\omega} = \Delta Q$ by ω to obtain

$$\begin{aligned} I\dot{\omega}\omega &= (\eta_m Q_t - Q_c - Q_l)\,\omega \\ &= \eta_m \dot{W}_t - \dot{W}_c - \dot{W}_l, \end{aligned}$$

where $\dot{W}_t$ is the turbine output power, $\dot{W}_c$ is the compressor input power, and $\dot{W}_l$ is the loading power

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- We obtain

$$\begin{aligned} I\frac{\pi^2}{900}N\frac{dN}{dt} &= \eta_m \dot{W}_t - \dot{W}_c - W_l \\ &= \eta_m \dot{m}_4 c_{pt}(T_{t4} - T_{t5}) - \dot{m}_2 c_{pc}(T_{t3} - T_{t2}) - W_l \\ &= \eta_m \dot{m}_2(1+f)c_{pt}(T_{t4} - T_{t5}) - \dot{m}_2 c_{pc}(T_{t3} - T_{t2}) - W_l \end{aligned}$$

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- The mass flow rate through the nozzle depends on
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Engine Thrust

- Assumption: the pressure leaving the exhaust nozzle is equal to that of the ambient air, that is, $P_9 = P_0$

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- The exit velocity V_9 for a perfect gas with constant specific heats,

$$V_9 = \sqrt{2 g_c c_{pt} T_{t9}\left[1 - \left(\frac{P_9}{P_{t9}}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]}$$

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- NASA video lectures:
  `http://nescacademy.nasa.gov/video_catalog.php?catid=3&subcatid=1`