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Abstract: Observer-based actuator fault detection and sensor fault reconstruction for a class of discrete-time nonlinear systems with actuator and sensor faults are investigated in this paper. A descriptor Takagi-Sugeno (T-S) fuzzy model is employed to construct observer-based systems for the purpose of fault detection and sensor fault reconstruction. Two methods for observer design are proposed. In the first method, the observer gains are computed off-line. In the second method, the observer gains are computed on-line at each iteration. The observer designs are formulated using linear matrix inequalities. Sufficient conditions for the existence of the observer-based fault detection and sensor fault reconstruction systems are provided. Comparative simulation study to illustrate the validity of the proposed methods is performed.

Keywords: Discrete-time systems, fault diagnosis, singular systems, state estimation.

1. INTRODUCTION

The increasing demands of reliability and safety for large-scale complex control systems resulted in many research centers investigating fault detection and isolation (FDI) methods, see, for example, [1–6] and references therein. Different approaches to fault estimation or fault reconstruction have been proposed. In particular, observer-based fault detection and isolation methods are reported in [7–14]. Overview papers, [7] and [8], summarized observer-based fault diagnosis techniques that include H_∞ theory, nonlinear unknown input observer theory, as well as adaptive observer theory. In [9] a state observer has been proposed that is used to cancel the system process dynamics so that a residual vector signal sensitive only to faults, and disturbances can be constructed. The design algorithm is formulated in terms of linear matrix inequalities using state-space techniques. An observer-based integrated robust fault estimation and accommodation for a class of discrete-time uncertain nonlinear systems was presented in [13]. Both full-order and reduced-order fault estimation observers in frequency domain for discrete-time systems are proposed in [14]. A robust sliding mode descriptor observer design method to estimate state and disturbance simultaneously using singular system theory was proposed in [10]. An integrated design scheme for affine nonlinear systems using L_2-stability theory based on the proof that an L_∞ and an L_2 observer-based fault detection systems exist for output re-constructability and weak output re-constructability, respectively, were presented in [12]. Although there are many significant results that have been proposed, most of them are given for linear systems [1,9,14] and some for special classes of nonlinear systems [3,4,10,12].

For more general nonlinear systems, the most common technique is based on the Takagi-Sugeno (T-S) fuzzy models [15]. The T-S fuzzy model, first proposed in [16], has become an attractive approach to the control and observation of dynamic systems and found a number of applications in real-life systems [17–19]. In [19] a T-S model is employed to represent a truck-trailer system. T-S fuzzy models are also used to describe networked control systems with different network-induced delays, the pretreatment of wastewater, and space vehicle attitude dynamics, see, for example, [17, 18, 20]. Methods for approximating a nonlinear system using T-S fuzzy models

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can be found in [16] and in [21, 22]. T-S fuzzy models, have been analyzed from many different angles, such as stability analysis [23, 24], observer design [15, 25–27], fault detection and isolation [28–32], fault-tolerant control [33,34]. Only a handful of papers address the issues of fault estimation for T-S fuzzy models, and most of the proposed methods consider sensor faults and actuator faults separately. Proportional integral (PI) observers have been used to deal with systems with both actuator and sensor faults [35–37], but most of PI observers have their own conservatism, for they are usually good at coping with slow varying parameters or time-invariant unknown inputs or faults [35, 36]. Chadli et al. [29] made a significant contribution to the simultaneous state and fault estimation using multiple Lyapunov functions with slack variables to reduce conservatism in the observer design. Most of the proposed design methods are based on off-line computing [15, 25, 27, 29], little work of on-line observer gain computation can be found in the literature.

In this paper, we discuss the problem of simultaneous actuator fault detection and sensor fault reconstruction for nonlinear systems with bounded disturbances using a descriptor T-S framework. The main contributions of this paper are: (i) new sufficiency conditions are given for actuator fault detection and sensor fault reconstruction in terms of linear matrix inequalities (LMIs); (ii) the proposed schemes are shown to be robust to additive disturbances; (iii) novel on-line approach to simultaneous sensor and actuator fault detection is proposed.

The paper is organized as follows: The problem statement and some preliminary results are given in Section 2. In Sections 3 and 4, two methods for constructing a system for fault detection and sensor fault reconstruction are presented in the form of linear matrix inequalities. Next, we discuss the method for actuator fault detection by constructing an appropriate residual. In Section 5, two simulation examples are presented to demonstrate the validity of the proposed method. Conclusions are drawn in Section 6.

2. PROBLEM STATEMENT

We consider a class of discrete-time, nonlinear systems with actuator fault and sensor fault subject to bounded uncertainties modeled as,

$$x(k+1) = \phi(x(k), u(k), f_a(k), \eta(k)), \tag{1a}$$
$$y(k) = Cx(k) + F f_s(k), \tag{1b}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $y(k) \in \mathbb{R}^p$ is the measured output, $f_a(k) \in \mathbb{R}^q$ models the actuator fault and $f_s(k) \in \mathbb{R}^s$ models the sensor fault. The vector $\eta(k) \in \mathbb{R}^d$ models the uncertainty/disturbance in the system. We make the following assumption.

Assumption 1: The matrix $C \in \mathbb{R}^{p\times n}$ has full row rank and $F \in \mathbb{R}^{p\times s}$ has full column rank.

Remark 1: Assumption 1 is a common condition for observer designing, and similar assumptions can be found in [1, 3, 4, 10, 33]. Note that the assumption that $F \in \mathbb{R}^{p\times s}$ has full column rank implies that $p \geq s$.

We refer to the model (1) as the truth model. For the design purpose, we construct a design model using the T-S fuzzy model,

$$\begin{aligned} x(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \, [A_i x(k) + B_i u(k) \\ + D_i f_a(k) + G_i \eta(k)], \end{aligned} \tag{2a}$$
$$y(k) = Cx(k) + F f_s(k), \tag{2b}$$

where $\xi(k)$ is a vector of measured variables, for example, we can have $\xi(k) = y(k)$. The activation functions $\mu_i(\cdot)$ $(i = 1 \dots r)$ are known functions which satisfy $\mu_i(\xi(k)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(\xi(k)) = 1$, where $r \in \mathbb{N}$ is the number of local models comprising the T-S model. For a method of generating these local models, see, for example, [21] and [22].

A consequence of Assumption 1 is that we can construct a state variable transformation such that the output matrix C of the plant model (1) in the new coordinates will have the following form,

$$C = \begin{bmatrix} I_p & 0 \end{bmatrix}. \tag{3}$$

Therefore, without loss of generality, we consider C to be of the form (3).

To proceed, we represent the above model as a descriptor T-S fuzzy system. We first partition the state as,

$$x_1 = Cx, \text{ and } x_2 = \begin{bmatrix} 0 & I_{n-p} \end{bmatrix} x.$$

To this end, we define an augmented state,

$$\bar{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ f_s(k) \end{bmatrix}.$$

Let,

$$\bar{E} = \begin{bmatrix} I_n & 0 \\ 0 & 0_s \end{bmatrix} = \begin{bmatrix} I_p & 0 & 0 \\ 0 & I_{n-p} & 0 \\ 0 & 0 & 0_s \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0_s \end{bmatrix} = \begin{bmatrix} A_{11,i} & A_{12,i} & 0 \\ A_{21,i} & A_{22,i} & 0 \\ 0 & 0 & 0_s \end{bmatrix},$$

$$\bar{B}_i = \begin{bmatrix} B_i \\ 0_{s\times m} \end{bmatrix} = \begin{bmatrix} B_{1,i} \\ B_{2,i} \\ 0_{s\times m} \end{bmatrix},$$

$$\bar{D}_i = \begin{bmatrix} D_i \\ 0_{s\times q} \end{bmatrix} = \begin{bmatrix} D_{1,i} \\ D_{2,i} \\ 0_{s\times q} \end{bmatrix},$$

$$\bar{G}_i = \begin{bmatrix} G_i \\ 0_{s\times d} \end{bmatrix} = \begin{bmatrix} G_{1,i} \\ G_{2,i} \\ 0_{s\times d} \end{bmatrix},$$

and,

$$\bar{C} = \begin{bmatrix} C & F \end{bmatrix} = \begin{bmatrix} I_p & 0 & F \end{bmatrix}.$$

Here, $x_1(k) \in \mathbb{R}^p$, $A_{11,i} \in \mathbb{R}^{p\times p}$, $B_{1,i} \in \mathbb{R}^{p\times m}$, $D_{1,i} \in \mathbb{R}^{p\times q}$, $G_{1,i} \in \mathbb{R}^{p\times d}$. In the following, we omit the arguments in the membership functions so that no confusion arises.

Next, we represent model (2) as a descriptor T-S fuzzy system as follows,

$$\begin{aligned} \bar{E}\bar{x}(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \big[& \bar{A}_i\bar{x}(k) + \bar{B}_i u(k) \\ & + \bar{D}_i f_a(k) + \bar{G}_i \eta(k) \big], \end{aligned} \tag{4a}$$

$$y(k) = \bar{C}\bar{x}(k). \tag{4b}$$

Note that if,

$$M = \begin{bmatrix} I_p & -F & 0 \\ 0 & 0 & I_{n-p} \\ 0 & I_s & 0 \end{bmatrix} \in \mathbb{R}^{(n+s)\times(n+s)},$$

then

$$\bar{C}M = [\; I_p \quad 0 \;].$$

Let $\theta(k) = M^{-1}\bar{x}(k) \in \mathbb{R}^{(n+s)\times(n+s)}$. Then the model (4) can be represented as,

$$\begin{aligned} \tilde{E}\theta(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \big[& \tilde{A}_i\theta(k) + \bar{B}_i u(k) \\ & + \bar{D}_i f_a(k) + \bar{G}_i \eta(k) \big], \end{aligned} \tag{5a}$$

$$y(k) = \tilde{C}\theta(k). \tag{5b}$$

where $\tilde{E} = \bar{E}M$, $\tilde{A}_i = \bar{A}_i M$, and $\tilde{C} = \bar{C}M$.

In the next two sections, we propose LMI conditions for the design of robust observers with constant and time-varying gains. These observers are capable of rejecting disturbances while, at the same time, being sensitive to actuator faults $f_a(k)$ under a different set of sufficient conditions. In addition, these observers can be used to reconstruct the sensor faults $f_s(k)$.

3. OBSERVER WITH CONSTANT GAINS

3.1. Technical result

To proceed, we require the following lemma.

Lemma 1: There is a nonsingular matrix $N \in \mathbb{R}^{(n+s)\times(n+s)}$ such that,

$$N\tilde{E} = \begin{bmatrix} E_1 & 0 \\ E_2 & I_{n+s-p} \end{bmatrix}, \tag{6}$$

where $E_1 \in \mathbb{R}^{p\times p}$ and $E_2 \in \mathbb{R}^{(n+s-p)\times p}$.

Proof: Since $F \in \mathbb{R}^{p\times s}$ is full column rank it follows from Assumption 1 that there exists a nonsingular matrix $S \in \mathbb{R}^{p\times p}$ such that

$$SF = \begin{bmatrix} 0_{(p-s)\times s} \\ I_s \end{bmatrix}.$$

We note that

$$\tilde{E} = \bar{E}M = \begin{bmatrix} I_p & -F & 0 \\ 0 & 0 & I_{n-p} \\ 0 & 0_s & 0 \end{bmatrix}.$$

Hence, the nonsingular matrix,

$$N = \begin{bmatrix} -\begin{bmatrix} I_{p-s} & 0 \\ 0 & 0_s \end{bmatrix} S & 0 & \begin{bmatrix} 0 \\ I_s \end{bmatrix} \\ -\begin{bmatrix} 0 & I_s \end{bmatrix} S & 0 & 0 \\ 0 & I_{n-p} & 0 \end{bmatrix},$$

satisfies (6), which completes the proof. □

3.2. Design of the observer with constant gains

We begin with the following state transformation that relates $z(k)$ to $\theta(k)$,

$$\begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} = \sum_{j=1}^{r} \mu_j(\xi(k-1)) \begin{bmatrix} I_p & 0 \\ -E_2 - L_jE_1 & I_{n+s-p} \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}.$$

Therefore,

$$z_1(k) = \theta_1(k), \tag{7a}$$

$$\begin{aligned} z_2(k) &= \sum_{j=1}^{r} \mu_j(\xi(k-1))\,(L_jE_1 + E_2)\,\theta_1(k) + \theta_2(k) \\ &= \sum_{j=1}^{r} \mu_j(\xi(k-1)) \begin{bmatrix} L_j & I \end{bmatrix} N\tilde{E}\theta(k), \end{aligned} \tag{7b}$$

where, $\theta_1(k), z_1(k) \in \mathbb{R}^p$, $L_j \in \mathbb{R}^{(n+s-p)\times p}$ and the selection of gains L_j will be discussed later in this subsection.

Remark 2: The above state transformation is used in the observer design to eliminate the effects of uncertainties/disturbances on the plant state estimate.

Note that to transform the system model into the z-coordinates we need an expression for $N\tilde{E}\theta(k+1)$. Since N is nonsingular, we can pre-multiply by N both sides of (5a) to obtain

$$\begin{aligned} N\tilde{E}\theta(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \big[& N\tilde{A}_i\theta(k) + N\bar{B}_i u(k) \\ & + N\bar{D}_i f_a(k) + N\bar{G}_i \eta(k) \big], \end{aligned} \tag{8a}$$

$$y(k) = \tilde{C}\theta(k) = \begin{bmatrix} I_p & 0 \end{bmatrix} \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} = \theta_1(k), \tag{8b}$$

where $\theta_1(k) \in \mathbb{R}^p$.

Next, we partition the matrices in (8a) as follows:

$$\mathcal{A}_i = N\tilde{A}_i = \begin{bmatrix} \mathcal{A}_{1,i} & \mathcal{A}_{2,i} \\ \mathcal{A}_{3,i} & \mathcal{A}_{4,i} \end{bmatrix},\ \mathcal{B}_i = N\bar{B}_i = \begin{bmatrix} \mathcal{B}_{1,i} \\ \mathcal{B}_{2,i} \end{bmatrix},$$
$$\mathcal{D}_i = N\bar{D}_i = \begin{bmatrix} \mathcal{D}_{1,i} \\ \mathcal{D}_{2,i} \end{bmatrix},\ \mathcal{G}_i = N\bar{G}_i = \begin{bmatrix} \mathcal{G}_{1,i} \\ \mathcal{G}_{2,i} \end{bmatrix},$$

where all the partitioned matrices are of appropriate dimensions. We are now ready to use the transformation given by (7) to obtain

$$z_1(k) = y(k), \tag{9a}$$
$$\begin{aligned} z_2(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} &\mu_i(\xi(k))\mu_j(\xi(k)) \\ &\times [(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})z_2(k) \\ &+ \sum_{l=1}^{r}\mu_l(\xi(k-1))((L_j\mathcal{A}_{1,i} + \mathcal{A}_{3,i}) \\ &-(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})(L_lE_1 + E_2))y(k) \\ &+(L_j\mathcal{D}_{1,i} + \mathcal{D}_{2,i})f_a(k) \\ &+(L_j\mathcal{G}_{1,i} + \mathcal{G}_{2,i})\eta(k) \\ &+(L_j\mathcal{B}_{1,i} + \mathcal{B}_{2,i})u(k)]. \end{aligned} \tag{9b}$$

Using (9), we propose the observer with constant gains of the form,

$$\hat{z}_1(k) = y(k), \tag{10a}$$
$$\begin{aligned} \hat{z}_2(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} &\mu_i(\xi(k))\mu_j(\xi(k)) \\ &\times [(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})\hat{z}_2(k) \\ &+ \sum_{l=1}^{r}\mu_l(\xi(k-1))((L_j\mathcal{A}_{1,i} + \mathcal{A}_{3,i}) \\ &-(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})(L_lE_1 + E_2))y(k) \\ &+(L_j\mathcal{B}_{1,i} + \mathcal{B}_{2,i})u(k)]. \end{aligned} \tag{10b}$$

Theorem 1: Let

$$Z_{ij} = \frac{1}{2}(P\mathcal{A}_{4,i} + Q_j\mathcal{A}_{2,i} + P\mathcal{A}_{4,j} + Q_i\mathcal{A}_{2,j}),$$

and let $f_a(k) = 0$ in (9). If there exist matrices $P = P^\top \in \mathbb{R}^{(n+s-p)\times(n+s-p)}$ and $Q_i \in \mathbb{R}^{(n+s-p)\times p}$ such that the following linear matrix inequalities

$$\min \delta, \tag{11a}$$
$$\begin{bmatrix} -\delta I & \begin{bmatrix} Q_j & P \end{bmatrix}\mathcal{G}_i \\ (\begin{bmatrix} Q_j & P \end{bmatrix}\mathcal{G}_i)^\top & -\delta I \end{bmatrix} \prec 0, \tag{11b}$$
$$\begin{bmatrix} -P & Z_{ij} \\ Z_{ij}^\top & -P \end{bmatrix} \prec 0, \tag{11c}$$
$$P \succ 0, \tag{11d}$$
$$i < j. \tag{11e}$$

are feasible, then the system (10) characterized by gains $L_j = P^{-1}Q_j$ produces error dynamics that are asymptotically stable.

Proof: Note that (11a) and (11b) implies that

$$\begin{bmatrix} Q_j & P \end{bmatrix}\begin{bmatrix} \mathcal{G}_{1,i} \\ \mathcal{G}_{2,i} \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} L_j & I \end{bmatrix}\begin{bmatrix} \mathcal{G}_{1,i} \\ \mathcal{G}_{2,i} \end{bmatrix} = 0$$
$$\Leftrightarrow (L_j\mathcal{G}_{1,i} + \mathcal{G}_{2,i}) = 0.$$

Let $\tilde{z}_2(k) = z_2(k) - \hat{z}_2(k)$, and subtract (10b) from (9b), we obtain the error dynamic system,

$$\begin{aligned} &\tilde{z}_2(k+1) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k))\mu_j(\xi(k))[(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})\tilde{z}_2(k) \\ &\quad + (L_j\mathcal{D}_{1,i} + \mathcal{D}_{2,i})f_a(k)]. \end{aligned} \tag{12}$$

For $f_a(k) = 0$, the above takes the form,

$$\begin{aligned} &\tilde{z}_2(k+1) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k))\mu_j(\xi(k))(L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i})\tilde{z}_2(k) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(k))\mu_j(\xi(k))S_{ij}\tilde{z}_2(k), \end{aligned} \tag{13}$$

where $S_{ij} = L_j\mathcal{A}_{2,i} + \mathcal{A}_{4,i}$.

Now, consider the Lyapunov function candidate $V(k) = \tilde{z}_2^\top(k)P\tilde{z}_2(k)$. We evaluate the first forward time difference of $V(k)$ on the trajectories of the error dynamic system (13) to obtain (14).

$$\begin{aligned} \Delta V &= \tilde{z}_2^\top(k+1)P\tilde{z}_2(k+1) - \tilde{z}_2^\top(k)P\tilde{z}_2(k) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{t=1}^{r}\mu_i\mu_j\mu_l\mu_t\tilde{z}_2^\top(k)\left[S_{ij}^\top P S_{ij} - P\right]\tilde{z}_2(k) \\ &= \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{t=1}^{r}\mu_i\mu_j\mu_l\mu_t\tilde{z}_2^\top(k) \\ &\quad \times \left[(S_{ij} + S_{ji})^\top P(S_{kl} + S_{lk}) - 4P\right]\tilde{z}_2(k) \\ &\le \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\tilde{z}_2^\top(k) \\ &\quad \times \left[(S_{ij} + S_{ji})^\top P(S_{ij} + S_{ji}) - 4P\right]\tilde{z}_2(k) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\tilde{z}_2^\top(k) \\ &\quad \times \left[\left(\frac{S_{ij} + S_{ji}}{2}\right)^\top P\left(\frac{S_{ij} + S_{ji}}{2}\right) - P\right]\tilde{z}_2(k). \end{aligned} \tag{14}$$

Therefore, $\Delta V < 0$ if

$$\left(\frac{S_{ij} + S_{ji}}{2}\right)^\top P\left(\frac{S_{ij} + S_{ji}}{2}\right) - P \prec 0.$$

Then (11c) is obtained using the Schur complement and choosing $Q_j = PL_j$. This completes the proof. □

Following [38], we further evaluate (14) to obtain

$$\begin{aligned}
&\Delta V\\
&\leq \sum_{i=1}^{r}\mu_i^2(\xi(k))\tilde{z}_2^\top(k)\left[S_{ii}^\top PS_{ii}-P\right]\tilde{z}_2(k)\\
&\quad+2\sum_{i<j}^{r}\mu_i(\xi(k))\mu_j(\xi(k))\tilde{z}_2^\top Q\tilde{z}_2(k)\\
&\leq \sum_{i=1}^{r}\mu_i^2(\xi(k))\tilde{z}_2^\top(k)\left[S_{ii}^\top PS_{ii}-P\right]\tilde{z}_2(k)\\
&\quad+(\gamma-1)\sum_{i=1}^{r}\mu_i^2(\xi(k))\tilde{z}_2^\top Q\tilde{z}_2(k)\\
&=\sum_{i=1}^{r}\mu_i^2(\xi(k))\tilde{z}_2^\top(k)\left[S_{ii}^\top PS_{ii}-P+(\gamma-1)Q\right]\tilde{z}_2(k),
\end{aligned} \tag{15}$$

where $0<\gamma<r$, $i<j$, $j=1,2,...,r$. Using (15), we obtain the following relaxed sufficient conditions,

$$\min\delta, \tag{16a}$$

$$\begin{bmatrix} -\delta I & \begin{bmatrix} Q_j & P \end{bmatrix}\mathcal{G}_i \\ (\begin{bmatrix} Q_j & P \end{bmatrix}\mathcal{G}_i)^\top & -\delta I \end{bmatrix} \prec 0, \tag{16b}$$

$$\begin{bmatrix} -P & P\mathcal{A}_{4,i}+Q_i\mathcal{A}_{2,i} \\ \mathcal{A}_{4,i}^\top P+\mathcal{A}_{2,i}^\top Q_i & -P+(\gamma-1)Q \end{bmatrix} \prec 0, \tag{16c}$$

$$\begin{bmatrix} -P & Z_{ij} \\ Z_{ij}^\top & -P \end{bmatrix} \prec 0, \tag{16d}$$

$$P \succ 0, \tag{16e}$$

$$Q \succ 0. \tag{16f}$$

The estimated states in the original coordinates are obtained by applying the inverse transformation to obtain,

$$\hat{x}(k)=M\sum_{j=1}^{r}\mu_j(\xi(k-1))\begin{bmatrix} I & 0 \\ -E_2-L_jE_1 & I \end{bmatrix}\hat{z}(k).$$

Remark 3: Although the above LMIs form relaxed conditions for constructing constant gains for the proposed observer, the infeasibility may still be an issue, as demonstrated in Example 2. To remedy this problem, we develop a new methodology for the observer design that is presented in Section 4.

3.3. Actuator fault detection

We note that we stated and proved Theorem 1 under the assumption that no actuator fault occurs. When the actuator fault occurs, that is, $f_a(k)\neq 0$, the observer error dynamic is governed by (12). For the actuator fault detection purpose, the following result is crucial.

Assumption 2: Recall that,

$$\begin{bmatrix} D_i & G_i \end{bmatrix} = \begin{bmatrix} D_{1,i} & G_{1,i} \\ D_{2,i} & G_{2,i} \end{bmatrix},$$

where $D_i\in\mathbb{R}^{n\times q}$, $D_{1,i}\in\mathbb{R}^{p\times q}$, $G_i\in\mathbb{R}^{n\times d}$, and $G_{1,i}\in\mathbb{R}^{p\times d}$. We assume that,

$$\operatorname{rank}\begin{bmatrix} D_i & G_i \end{bmatrix} = q+d,$$

and,

$$\operatorname{rank}\begin{bmatrix} D_{1,i} & G_{1,i} \end{bmatrix} < q+d.$$

Lemma 2: Suppose Assumption 2 holds. If L_j satisfy $L_j\mathcal{G}_{1,i}+\mathcal{G}_{2,i}=0$, then L_j satisfy $L_j\mathcal{D}_{1,i}+\mathcal{D}_{2,i}\neq 0$.

Proof: We prove the lemma by contradiction. Suppose that $L_j\mathcal{G}_{1,i}+\mathcal{G}_{2,i}=0$ and $L_j\mathcal{D}_{1,i}+\mathcal{D}_{2,i}=0$ hold simultaneously, that is, we have $\mathcal{G}_{2,i}=-L_j\mathcal{G}_{1,i}$, and $\mathcal{D}_{2,i}=-L_j\mathcal{D}_{1,i}$, then

$$\begin{aligned}
\operatorname{rank}\begin{bmatrix} \mathcal{D}_i & \mathcal{G}_i \end{bmatrix} &= \operatorname{rank}\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \\ \mathcal{D}_{2,i} & \mathcal{G}_{2,i} \end{bmatrix}\\
&= \operatorname{rank}\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \\ -L_j\mathcal{D}_{1,i} & -L_j\mathcal{G}_{1,i} \end{bmatrix}\\
&= \operatorname{rank}\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \end{bmatrix}.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\operatorname{rank}\begin{bmatrix} \mathcal{D}_i & \mathcal{G}_i \end{bmatrix} &= \operatorname{rank}\begin{bmatrix} N\bar{D}_i & N\bar{G}_i \end{bmatrix}\\
&= \operatorname{rank}\begin{bmatrix} \bar{D}_i & \bar{G}_i \end{bmatrix}\\
&= \operatorname{rank}\begin{bmatrix} D_i & G_i \end{bmatrix}.
\end{aligned}$$

Hence, by Assumption 2,

$$\operatorname{rank}\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} D_i & G_i \end{bmatrix} = q+d.$$

By the definition of $\begin{bmatrix} \mathcal{D}_i & \mathcal{G}_i \end{bmatrix}$, we have

$$\begin{aligned}
\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \\ \mathcal{D}_{2,i} & \mathcal{G}_{2,i} \end{bmatrix} &= \begin{bmatrix} \mathcal{D}_i & \mathcal{G}_i \end{bmatrix}\\
&= N\begin{bmatrix} \bar{D}_i & \bar{G}_i \end{bmatrix}\\
&= N\begin{bmatrix} D_{1,i} & G_{1,i} \\ D_{2,i} & G_{2,i} \\ 0 & 0 \end{bmatrix},
\end{aligned}$$

that is,

$$\begin{aligned}
&\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \\ \mathcal{D}_{2,i} & \mathcal{G}_{2,i} \end{bmatrix}\\
&= \begin{bmatrix} -\begin{bmatrix} I_{p-s} & 0 \\ 0 & 0_s \end{bmatrix}S & 0 & \begin{bmatrix} 0 \\ I_s \end{bmatrix} \\ -\begin{bmatrix} 0 & I_s \end{bmatrix}S & 0 & 0 \\ 0 & I_{n-p} & 0 \end{bmatrix}\begin{bmatrix} D_{1,i} & G_{1,i} \\ D_{2,i} & G_{2,i} \\ 0 & 0 \end{bmatrix},
\end{aligned}$$

and therefore,

$$\begin{aligned}
&\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \end{bmatrix}\\
&= \begin{bmatrix} -\begin{bmatrix} I_{p-s} & 0 \\ 0 & 0_s \end{bmatrix}SD_{1,i} & -\begin{bmatrix} I_{p-s} & 0 \\ 0 & 0_s \end{bmatrix}SG_{1,i} \\ -\begin{bmatrix} 0 & I_s \end{bmatrix}SD_{1,i} & -\begin{bmatrix} 0 & I_s \end{bmatrix}SG_{1,i} \end{bmatrix}
\end{aligned}$$

$$= \begin{bmatrix} -I_{p-s} & 0 & 0 \\ 0 & 0_s & I_s \\ 0 & -I_s & 0 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_{1,i} & G_{1,i} \\ 0 & 0 \end{bmatrix}.$$

Thus,

$$\text{rank}\begin{bmatrix} D_{1,i} & G_{1,i} \end{bmatrix} = \text{rank}\begin{bmatrix} \mathcal{D}_{1,i} & \mathcal{G}_{1,i} \end{bmatrix} = q+d,$$

which contradicts Assumption 2 and the proof is complete. □

Remark 4: It should be noted that Assumption 2 is not necessary for the observer design, but rather for the actuator fault detection.

The observation error equation for the case when a fault occurred, that is, $f_a(k) \neq 0$, is (12). It then follows from Lemma 2 that the proposed observer is robust with respect to the uncertainty $\eta(k)$ but sensitive to actuator faults $f_a(k)$. We proceed by constructing a residual as the output error

$$e_y(k) = y(k) - \hat{y}(k),$$

where, $\hat{y}(k) = C\hat{x}(k)$. Then, by checking the residual $e_y(k)$ to determine if a fault has occurred or not, we can use the following scheme,

$$\begin{cases} \|e_y(k)\| \leq \lambda, & \text{No fault}, \\ \|e_y(k)\| > \lambda, & \text{Fault alarm!}, \end{cases}$$

where $\lambda > 0$ is the user-defined alarm threshold.

4. OBSERVER WITH VARIABLE GAINS

In the case when a given system does not satisfy conditions for the observer existence given in Theorem 1, we provide an alternative. Herein we present a method for constructing an observer-based system for the actuator fault detection and sensor fault reconstruction where the observer has time-varying gains that are computed on-line at each iteration.

4.1. Preliminary transformation

We begin our discussion with an equivalent state transformation,

$$\begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ -E_2 - L_k E_1 & I_{n+s-p} \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix},$$

where the matrices $L_k \in \mathbb{R}^{(n+s-p)\times p}$ are computed on-line and a method to compute them will presented later in this subsection. The inverse of the above transformation has the form,

$$\begin{aligned} z_1(k) &= \theta_1(k), \\ z_2(k) &= (L_k E_1 + E_2)\theta_1(k) + \theta_2(k) \\ &= \begin{bmatrix} L_k & I \end{bmatrix} N\tilde{E}\theta(k), \end{aligned}$$

where, $\theta_1(k), z_1(k) \in \mathbb{R}^p$. Then the system (5a) is transformed into the form,

$$z_1(k) = \theta_1(k), \tag{17a}$$

$$\begin{aligned} z_2(k+1) = \begin{bmatrix} L_k & I \end{bmatrix} N \sum_{i=1}^{r} \mu_i(\xi(k)) \big[&\tilde{A}_i\theta(k) + \bar{B}_i u(k) \\ &+ \bar{D}_i f_a(k) + \bar{G}_i \eta(k) \big], \end{aligned} \tag{17b}$$

let

$$\tilde{\mathcal{A}}_k = N\sum_{i=1}^{r} \mu_i(\xi(k))\tilde{A}_i = \begin{bmatrix} \tilde{\mathcal{A}}_{k1} & \tilde{\mathcal{A}}_{k2} \\ \tilde{\mathcal{A}}_{k3} & \tilde{\mathcal{A}}_{k4} \end{bmatrix},$$

$$\bar{\mathcal{B}}_k = N\sum_{i=1}^{r} \mu_i(\xi(k))\bar{B}_i = \begin{bmatrix} \bar{\mathcal{B}}_{k1} \\ \bar{\mathcal{B}}_{k2} \end{bmatrix},$$

$$\bar{\mathcal{D}}_k = N\sum_{i=1}^{r} \mu_i(\xi(k))\bar{D}_i = \begin{bmatrix} \bar{\mathcal{D}}_{k1} \\ \bar{\mathcal{D}}_{k2} \end{bmatrix},$$

$$\bar{\mathcal{G}}_k = N\sum_{i=1}^{r} \mu_i(\xi(k))\bar{G}_i = \begin{bmatrix} \bar{\mathcal{G}}_{k1} \\ \bar{\mathcal{G}}_{k2} \end{bmatrix},$$

where all the partitioned matrices are of appropriate dimensions, then (17) can be rewritten as

$$\begin{aligned} z_1(k) &= \theta_1(k), \\ z_2(k+1) &= \begin{bmatrix} L_k & I \end{bmatrix} \big[\tilde{\mathcal{A}}_k\theta(k) + \bar{\mathcal{B}}_k u(k) \\ &\quad + \bar{\mathcal{D}}_k f_a(k) + \bar{\mathcal{G}}_k \eta(k)\big], \end{aligned}$$

or

$$z_1(k) = \theta_1(k), \tag{18a}$$

$$\begin{aligned} z_2(k+1) = {} & (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})z_2(k) \\ & + ((L_k\tilde{\mathcal{A}}_{k1} + \tilde{\mathcal{A}}_{k3}) \\ & - (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})(L_kE_1 + E_2))y(k) \\ & + (L_k\bar{\mathcal{B}}_{k1} + \bar{\mathcal{B}}_{k2})u(k) + (L_k\bar{\mathcal{D}}_{k1} + \bar{\mathcal{D}}_{k2})f_a(k) \\ & + (L_k\bar{\mathcal{G}}_{k1} + \bar{\mathcal{G}}_{k2})\eta(k)\,. \end{aligned} \tag{18b}$$

4.2. The variable-gain observer design

The proposed robust observer for system (18) has the form,

$$\hat{z}_1(k) = \theta_1(k) = y(k), \tag{19a}$$

$$\begin{aligned} \hat{z}_2(k+1) = {} & (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})\hat{z}_2(k) \\ & + ((L_k\tilde{\mathcal{A}}_{k1} + \tilde{\mathcal{A}}_{k3}) \\ & - (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})(L_kE_1 + E_2))y(k) \\ & + (L_k\bar{\mathcal{B}}_{k1} + \bar{\mathcal{B}}_{k2})u(k)\,. \end{aligned} \tag{19b}$$

Theorem 2: Let $f_a(k) = 0$ in (18), then (19) is a robust observer for system (18) if there exist symmetric positive definite matrices $P_k \in \mathbb{R}^{(n+s-p)\times(n+s-p)}$ and matrices $Q_k \in \mathbb{R}^{(n+s-p)\times p}$, such that the following LMIs,

$$\min\delta, \tag{20a}$$

$$\begin{bmatrix} -\delta I & \begin{bmatrix} Q_k & P_{k+1} \end{bmatrix}\bar{\mathcal{G}}_k \\ (\begin{bmatrix} Q_k & P_{k+1} \end{bmatrix}\bar{\mathcal{G}}_k)^\top & -\delta I \end{bmatrix} \prec 0, \tag{20b}$$

$$\begin{bmatrix} -P_{k+1} & P_{k+1}\tilde{\mathcal{A}}_{k4} + Q_k\tilde{\mathcal{A}}_{k2} \\ \tilde{\mathcal{A}}_{k4}^\top P_{k+1} + \tilde{\mathcal{A}}_{k2}^\top Q_k^\top & -P_k \end{bmatrix} \prec 0, \tag{20c}$$

$$P_k \succ 0, \tag{20d}$$

are feasible, and the observer gain matrices are given by $L_k = P_{k+1}^{-1}Q_k$.

Proof: Note that (20a) and (20b) implies that,

$$\begin{bmatrix} Q_k & P_{k+1} \end{bmatrix}\begin{bmatrix} \bar{\mathcal{G}}_{k1} \\ \bar{\mathcal{G}}_{k2} \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} L_k & I \end{bmatrix}\begin{bmatrix} \bar{\mathcal{G}}_{k1} \\ \bar{\mathcal{G}}_{k2} \end{bmatrix} = 0$$
$$\Leftrightarrow L_k\bar{\mathcal{G}}_{k1} + \bar{\mathcal{G}}_{k2} = 0.$$

Subtracting (19b) from (18b) yields the error dynamic system,

$$\begin{aligned}\tilde{z}_2(k+1) &= (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})\tilde{z}_2(k) \\ &\quad + (L_k\bar{\mathcal{D}}_{k1} + \bar{\mathcal{D}}_{k2})f_a(k),\end{aligned} \tag{21}$$

where $\tilde{z}_2(k) = z_2(k) - \hat{z}_2(k)$. When $f_a(k) = 0$, the above takes the form,

$$\tilde{z}_2(k+1) = (L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4})\tilde{z}_2(k). \tag{22}$$

Now, consider the Lyapunov function candidate $V(k) = \tilde{z}_2^\top(k)P_k\tilde{z}_2(k)$. The first forward time difference of $V(k)$ along the trajectories of the error dynamic system (22) is

$$\begin{aligned}\Delta V &= \tilde{z}_2^\top(k+1)P_{k+1}\tilde{z}_2(k+1) - \tilde{z}_2^\top(k)P_k\tilde{z}_2(k) \\ &= \tilde{z}_2^\top(k)\left(\Omega_k^\top P_{k+1}\Omega_k - P_k\right)\tilde{z}_2(k),\end{aligned}$$

where $\Omega_k = L_k\tilde{\mathcal{A}}_{k2} + \tilde{\mathcal{A}}_{k4}$.

We obtain $\Delta V < 0$ if

$$\Omega_k^\top P_{k+1}\Omega_k - P_k < 0.$$

We satisfy (20c) using the Schur complement selecting $Q_k = P_{k+1}L_k$. This completes the proof. □

An asymptotic state estimation of system (4) is obtained as,

$$\hat{\bar{x}}(k) = M\begin{bmatrix} I_p & 0 \\ -E_2 - L_kE_1 & I_{n+s-p} \end{bmatrix}\hat{z}(k).$$

Remark 5: The proposed observer rejects the uncertainty when its gains are obtained by solving the LMIs (20). This in turn makes it possible to deal with actuator fault detection problem as discussed in the following subsection.

Remark 6: The existing design methods use a constant common quadratic Lyapunov function or multiple constant Lyapunov functions However, a common symmetric positive definite P may not always exist, as shown in LMIs (16). On the other hand, time-varying symmetric positive definite gain matrices P_k determined LMIs (20) can be easily computed on-line. The advantage of on-line computation of the observer gains is shown in Example 2. The proposed observer is of reduced-order, so it can make the structure of the overall system simpler, especially for high-dimensional plants.

4.3. Actuator fault detection

To proceed, we need the following lemma.

Lemma 3: Suppose Assumption 2 holds, then if a matrix L_k satisfies $L_k\bar{\mathcal{G}}_{k1} + \bar{\mathcal{G}}_{k2} = 0$, then it must satisfy $L_k\bar{\mathcal{D}}_{k1} + \bar{\mathcal{D}}_{k2} \neq 0$.

Proof: The proof is similar to that of Lemma 2. □

The error observation equation in the presence of an actuator fault, that is, when $f_a(k) \neq 0$, is given by (21). Applying Lemma 3 to (21) implies that the proposed observer is robust with respect to the uncertainty $\eta(k)$ but sensitive to the actuator fault $f_a(k)$. We then can proceed as in the previous case to construct a residual output error $e_y(k) = y(k) - \hat{y}(k)$ and then use it to decide if an actuator fault occurred or not.

5. EXAMPLES

In this section, we test our proposed observer-based fault detection and sensor fault reconstruction systems on two numerical examples. In the first example, we start with a nonlinear model. We then construct a T-S fuzzy design model and use the fuzzy model to compute the observer gains. Then we perform simulations on the nonlinear model to demonstrate the validity of the method. We compare the performance of two proposed methods.

5.1. Example 1

We consider a truck-trailer system model from [19, 27] subjected to disturbance, actuator and sensor faults described as

$$\begin{aligned}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} &= \begin{bmatrix} (1-\frac{vt_s}{L})x_1(k) \\ (\frac{vt_s}{L})x_1(k) + x_2(k) \\ x_3(k) + vt_s\sin(x_2(k) + \frac{vt_s}{2L}x_1(k)) \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{vt_s}{l} \\ 0 \\ 0 \end{bmatrix}u(k) + \begin{bmatrix} 0.1 \\ -0.05 \\ -0.05 \end{bmatrix}f_a(k) \\ &\quad + \begin{bmatrix} 0.1 \\ -0.05 \\ 0.1 \end{bmatrix}\eta(k),\end{aligned}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} x_1(k) + f_s(k) \\ x_2(k) + 1.2 f_s(k) \end{bmatrix},$$

where $x_1(k)$ is the angle difference between truck and trailer, $x_2(k)$ is the angle of trailer, $x_3(k)$ is the vertical position of the rear end of the trailer, $u(k)$ is the steering angle, $\eta(k)$ is the uncertainty of the system, $f_a(k)$ and $f_s(k)$ are actuator and sensor faults, respectively. The length of the trailer is $L = 5.5$ m, l is the length of the truck where $l = 2.8$ m, $t_s = 2$ sec is the sampling time, and $v = 1$ m/sec is the constant speed of backing up.

The fuzzy rules are,

Rule 1: IF $x_2(k) + \frac{vt_s}{2L} x_1(k)$ is 0, THEN,

$$\begin{cases} x(k+1) = A_1 x(k) + B_1 u(k) + D_1 f_a(k) + G_1 \eta(k), \\ y(k) = Cx(k) + F f_s(k). \end{cases}$$

Rule 2: IF $x_2(k) + \frac{vt_s}{2L} x_1(k)$ is $\pm\pi$, THEN,

$$\begin{cases} x(k+1) = A_2 x(k) + B_2 u(k) + D_2 f_a(k) + G_2 \eta(k), \\ y(k) = Cx(k) + F f_s(k). \end{cases}$$

We obtain a T-S fuzzy model as in [19], where the activation functions are selected to be

$$\mu_1(y(k)) = \frac{\sin(y_2(k) + \frac{vt_s y_1(k)}{2L})}{y_2(k) + \frac{vt_s y_1(k)}{2L}},$$

and

$$\mu_2(y(k)) = 1 - \mu_1(y(k)).$$

The local models' matrices are

$$A_1 = \begin{bmatrix} 1 - \frac{vt_s}{L} & 0 & 0 \\ \frac{vt_s}{L} & 1 & 0 \\ \frac{(vt_s)^2}{2L} & vt_s & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 - \frac{vt_s}{L} & 0 & 0 \\ \frac{vt_s}{L} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} \frac{vt_s}{l} \\ 0 \\ 0 \end{bmatrix}, \; D_1 = D_2 = \begin{bmatrix} 0.1 \\ -0.05 \\ -0.05 \end{bmatrix},$$

$$G_1 = G_2 = \begin{bmatrix} 0.1 \\ -0.05 \\ 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$F = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}.$$

The control input $u(k)$, the actuator fault $f_a(k)$, the sensor fault $f_s(k)$, and the uncertainty of the system $\eta(k)$ are

$$u(k) = 0.3\cos(0.2k + 2.5),$$

$$f_s(k) = \begin{cases} -0.8 & 0 \le k < 40 \\ 2.5\sin(0.2k) & 40 \le k < 80, \end{cases}$$

$$f_a(k) = \begin{cases} 0 & 0 \le k < 40 \\ 2\sin(0.2) & 40 \le k < 55 \\ 0 & 55 \le k < 80, \end{cases}$$

$$\eta(k) = 0.1\cos(0.1k + 2).$$

We compute the observer-based system constant gain matrices by solving the LMIs (16) to obtain

$$L_1 = \begin{bmatrix} -0.0298 & 0 \\ 4.5045 & 0 \end{bmatrix}, \; L_2 = \begin{bmatrix} 0.3334 & 0 \\ 0.2625 & 0 \end{bmatrix}.$$

We also compute time-varying gain matrices of the time-varying observer based system by solving the LMIs (20). Plots of time history of the gains of L_k are shown in Fig. 1. We compare the performance of the system with constant and time-varying gains in Figs. 2-4. In Fig. 2 we show the state $x_3(k)$ and its estimate for the constant gain and the time-varying gain observers, respectively. Sensor fault and its reconstruction are shown in Fig. 3. Fig. 4 shows the actuator fault detection. It follows from our analysis that the actuator fault detection plots are the same for the two methods.

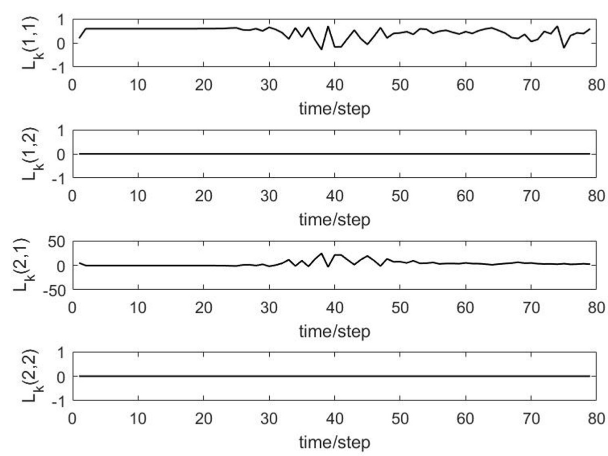

Fig. 1. Time-varying gains of the gain matrices L_k.

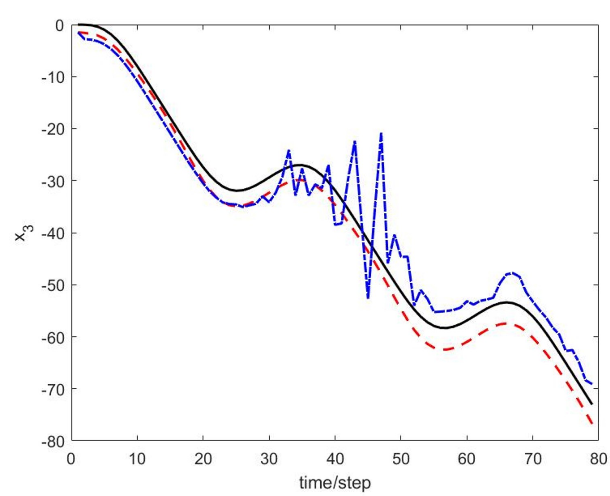

Fig. 2. States $x_3(k)$ (black solid) and its estimates for constant gain method (red dotted), for time-varying gains (blue dot dash).

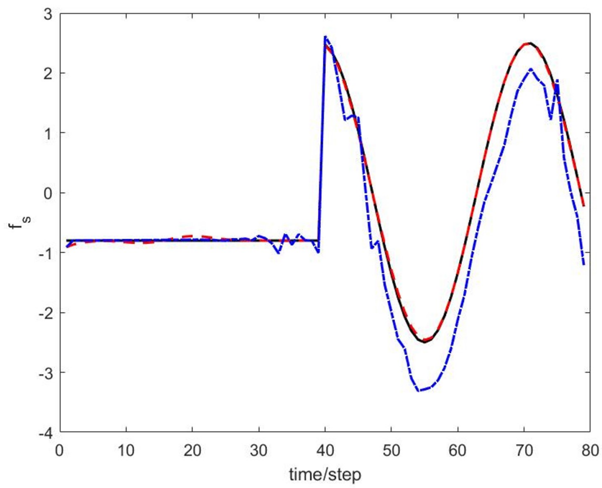

Fig. 3. Sensor fault (black solid) and its reconstructions for constant gain method (red dotted), for time-varying gains (blue dot dash).

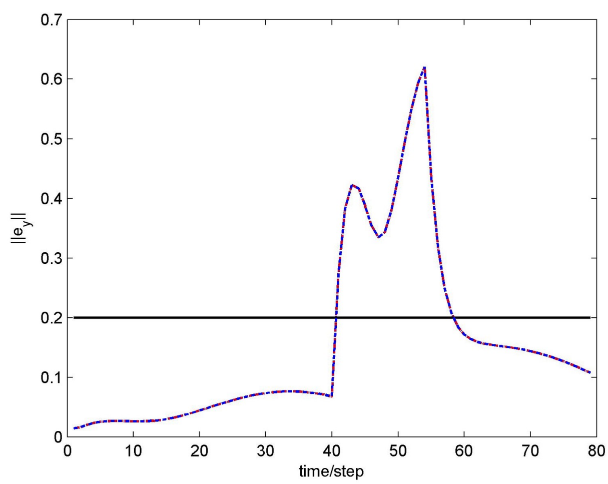

Fig. 4. Actuator fault detection for constant gain method (red dotted), for time-varying gains (blue dot dash).

5.2. Example 2

In this Example, we consider a discrete-time T-S model that is a modification of the example system from [25],

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{2} \mu_i(\xi(k))(A_i x(k) + B_i u(k) \\ &\quad + D_i f_a(k) + G_i \eta(k)), \\ y(k) &= Cx(k) + F f_s(k), \end{aligned}$$

where

$$A_1 = \begin{bmatrix} -0.4 & 0.2 & 0.3 \\ 0.3 & -0.6 & 0.3 \\ 0.4 & 0.2 & 0.6 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.45 & 0.375 & 0.375 \\ 0.15 & -0.45 & 0 \\ 0.75 & 0.75 & -0.45 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \end{bmatrix}, \ B_2 = \begin{bmatrix} -0.5 \\ 1 \\ -0.5 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \end{bmatrix}, \ D_2 = \begin{bmatrix} -0.5 \\ 1 \\ 0.5 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.4 \\ 0.2 \\ -0.4 \end{bmatrix}, \ G_2 = \begin{bmatrix} 0.2 \\ -0.4 \\ -0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ F = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}.$$

The control input $u(k)$, the actuator fault $f_a(k)$, the sensor fault $f_s(k)$, and the uncertainty of the system $\eta(k)$ are

$$u(k) = \begin{cases} 2, & 0 \le k < 20, \\ 2\sin(0.2k+1), & 20 \le k < 40, \\ 5, & 40 \le k < 80, \end{cases}$$

$$f_a(k) = \begin{cases} 0, & 0 \le k < 20, \\ 2, & 20 \le k < 40, \\ 0, & 40 \le k < 80, \end{cases}$$

$$f_s(k) = \begin{cases} 0, & 0 \le k < 20, \\ 2\sin(0.5k+5), & 20 \le k < 40, \\ 2, & 40 \le k < 80, \end{cases}$$

$$\eta(k) = \begin{cases} 2\cos(0.1k+2), & 0 \le k < 20, \\ 2, & 20 \le k < 80, \end{cases}$$

with $\mu_1(y(k)) = 1 - 0.02y_2^2(k)$ and $\mu_2(y(k)) = 0.02y_2^2(k)$. We obtained the following gain matrices using the LMI toolbox of MATLAB to solve the LMIs (16),

$$L_1 = \begin{bmatrix} 0.4638 & 0 \\ -0.2556 & 0 \end{bmatrix}, L_2 = \begin{bmatrix} 0.4545 & 0 \\ -0.2260 & 0 \end{bmatrix}.$$

The time-varying gain matrices can be obtained by solving the LMIs (20) and the time histories of the elements of L_k are shown in Fig. 5. The performance of the constant gain and time-varying gain systems are shown in Figs. 6–8. We show plots of the state $x_3(k)$ and its estimates in Fig. 6. Sensor fault and its reconstruction are given in Fig. 7. Fig. 8 illustrates the actuator fault detection. In this example, the time-varying gain system clearly outperforms the constant gain system. From the two examples, we find that the performances of the two methods are different in different situations. In Example 1, the performance of the constant gain system is somewhat better than that of the time-varying gain system. But it is exactly opposite in Example 2. The cause of differences might be the solutions of the LMIs for the two systems. In Example 2,

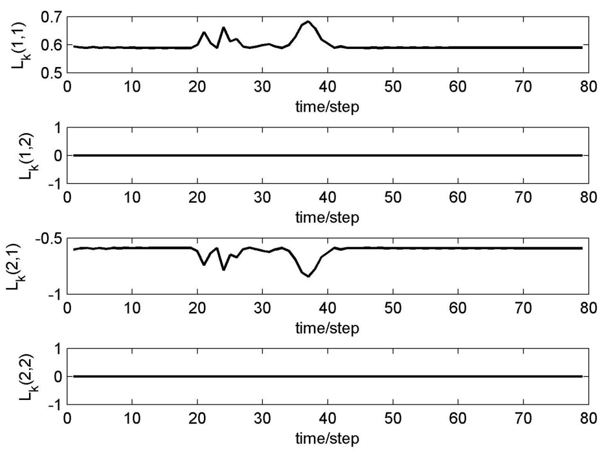

Fig. 5. The variation of gain L_k for the time-varying gain.

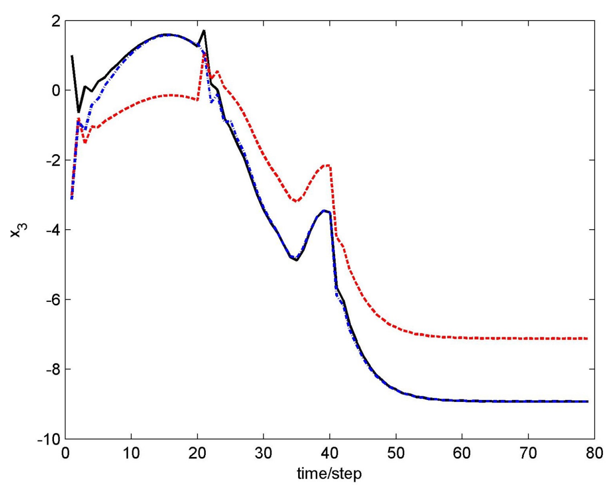

Fig. 6. State $x_3(k)$ (black solid) and its estimates for constant gain method (red dotted), for time-varying gains (blue dot dash).

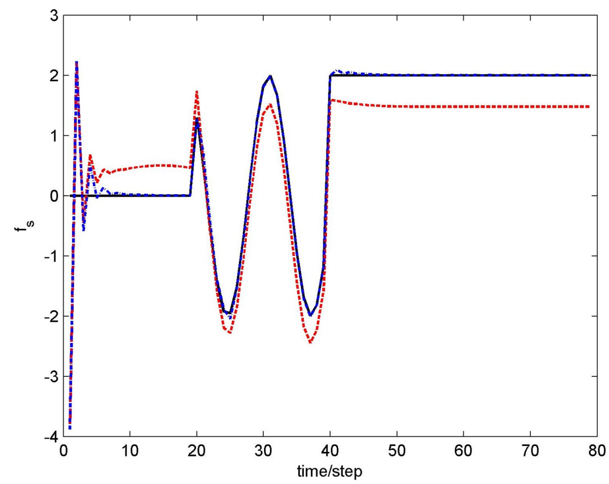

Fig. 7. Sensor fault (black solid) and its reconstructions for constant gain method (red dotted), for time-varying gains (blue dot dash).

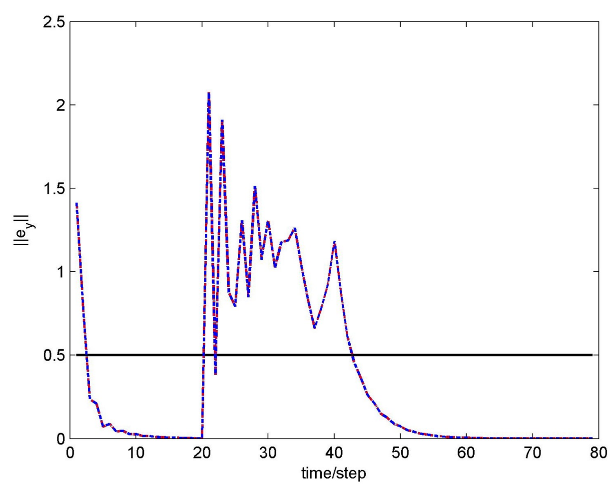

Fig. 8. Actuator fault detection for constant gain method (red dotted), for time-varying gains (blue dot dash).

the solutions of the LMIs (16) are close to 0. On the other hand, the solutions for the observer with the time-varying gains in this example are better conditioned.

Remark 7: On the one hand, the constant gain is obtained under the assumption that the LMIs (11) have a common positive definite matrix solution P which is actually a very strong constraint condition, so the total performance of the constant gain method is inferior to that of the time-varying gain method, which can be seen in Figs. 6-7. On the other hand, however, it is frequent varying of the gain in the time-varying gain method that leads to unsmoothed curves of the estimations, so it has sometimes an unsatisfied performance as shown in Figs. 2-3.

6. CONCLUSIONS

In this paper, observer-based actuator fault detection and sensor fault reconstruction systems are proposed for a class of discrete-time nonlinear systems using Takagi-Sugeno (T-S) models. In the design, the given plant model is first transformed into a descriptor model, and then novel robust observer design methods are developed by determining observers' gain matrices using linear matrix inequalities (LMIs). The proposed observer-based architectures can be used as sensor fault estimators because they can estimate the system states and sensor faults simultaneously. On the other hand, they can also be used as actuator fault detectors because the proposed observer-based systems are robust with respect to the uncertainty but sensitive to actuator faults. Numerical examples are given to show the validity of the proposed architectures.

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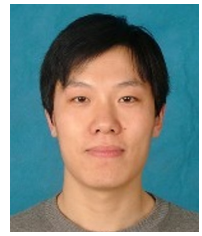

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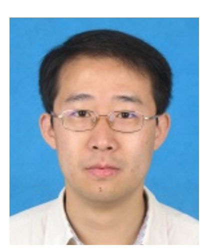

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