

Logarithmic plots

Logarithmic plots of $H(j\omega)$, or Bode diagrams of $H(j\omega)$, are two graphs:

1. A plot of $20 \log_{10} |H(j\omega)|$ versus the frequency in log scale, that is, versus $\log_{10} \omega$
2. The phase angle $\angle H(j\omega)$ versus $\log_{10} \omega$

The standard representation of the logarithmic magnitude of $H(j\omega)$ is

$$20 \log_{10} |H(j\omega)| \text{ dB}$$

Historical Comment: Researchers for the telephone company first defined the unit of power gain as a “bel.” However, this unit proved to be too large, and hence a decibel, that is, one-tenth of a bell, was selected as the unit, which is, of course, named after Alexander Graham Bell, the founder of the company

Motivation for using $20 \log_{10} |H(j\omega)|$

In communications it is standard to measure power gain in decibels,

$$|H|_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1}$$

Since power is the square of voltage, the voltage gain is

$$|H|_{\text{dB}} = 20 \log_{10} \frac{V_2}{V_1}$$

From now on, we will drop the base of the logarithm; it is understood to be 10.

Advantages of working with frequency response in terms of Bode plots

- Multiplication of magnitudes converted into addition
- A much wider range of the behavior of the circuit can be displayed; that is, both low- and high-frequency behavior can be displayed in one plot
- Bode plots can be determined experimentally

Octave and decade

An octave is a frequency band from ω_1 to ω_2 such that

$$\frac{\omega_2}{\omega_1} = 2$$

There is an increase in decade from ω_1 to ω_2 when

$$\frac{\omega_2}{\omega_1} = 10$$

Example

$$H(s) = \frac{1}{1 + 2s}$$

Hence,

$$H(j\omega) = \frac{1}{1 + 2j\omega}$$

Hence the log magnitude, Lm , is

$$\begin{aligned} Lm \frac{1}{1 + 2j\omega} &= 20 \log \left| \frac{1}{1 + 2j\omega} \right| \\ &= 20 \log 1 - 20 \log \sqrt{1 + (2\omega)^2} \\ &= -20 \log \sqrt{1 + (2\omega)^2} \\ &= \begin{cases} \text{for very small values} \\ \text{of } \omega, \text{ that is, } 2\omega \ll 1, \\ Lm = \log 1 = 0 \\ \\ \text{for very large values} \\ \text{of } \omega, \text{ that is, } 2\omega \gg 1, \\ Lm = -20 \log(2\omega) \end{cases} \end{aligned}$$