

An Introduction to Proportional-Integral-Derivative (PID) Controllers

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Motivation

- Growing gap between "real world" control problems and the theory for analysis and design of linear control systems
- Design techniques based on linear system theory have difficulties with accommodating nonlinear effects and modeling uncertainties
- Increasing complexity of industrial process as well as household appliances
 - Effective control strategies are required to achieve high performance for uncertain dynamic systems

Usefulness of PID Controls

- Most useful when a mathematical model of the plant is not available
- Many different PID tuning rules available
- Our sources
 - K. Ogata, Modern Control Engineering, Fifth Edition, Prentice Hall, 2010, Chapter 8
 - IEEE Control Systems Magazine, Feb. 2006, Special issue on PID control

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Proportional-integral-derivative (PID) control framework is a method to control uncertain systems

Type A PID Control

Transfer function of the type A PID controller

$$G_{PID}(s) = \frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s\right)$$

□ The three term control signal,

$$U(s) = K_p E(s) + K_i \frac{1}{s} E(s) + K_d s E(s)$$

In the time domain,

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(\tau) d\tau + K_p T_d \frac{de(t)}{dt}$$

PID-Controlled System

PID controller in forward path



PID Tuning

- Controller tuning---the process of selecting the controller parameters to meet given performance specifications
- PID tuning rules---selecting controller parameter values based on experimental step responses of the controlled plant
- The first PID tuning rules proposed by Ziegler and Nichols in 1942
- The Ziegler-Nichols tuning rules provide a starting point for fine tuning
- Our exposition is based on K. Ogata, *Modern Control Engineering*, Prentice Hall, Fifth Edition, 2010, Chapter 8

PID Tuning---First method (open-loop method)

Start with obtaining the step response



The S-shaped Step Response

Parameters of the S-shaped step response



Transfer Function of System With S-Shaped Step Response

- The S-shaped curve may be characterized by two parameters: lag (delay) time L, and time constant T
- The transfer function of such a plant may be approximated by a first-order system with a transport delay

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}$$

PID Tuning---First method (open-loop method)

Table 10-1Ziegler-Nichols Tuning Rule Based on StepResponse of Plant (First Method)

Type of Controller	K_p	T_i	T_d
Р	$\frac{T}{L}$	∞	0
PI	$0.9 \frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2 \frac{T}{L}$	2L	0.5L

Transfer Function of PID Controller Tuned Using the First Method

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$
$$= 1.2 \frac{T}{L} \left(1 + \frac{1}{2Ls} + 0.5Ls \right)$$
$$= 0.6T \frac{\left(s + \frac{1}{L}\right)^2}{s}$$

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Ziegler-Nichols PID Tuning---Second method (closed-loop method)

Use the proportional controller to force sustained oscillations



PID Tuning---Second method (closed-loop method)

Measure the period of sustained oscillation



PID Tuning Rules---Second method (closed-loop method)

Table 10-2Ziegler-Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T _d
Р	$0.5K_{\rm cr}$	s ∞	0
PI	$0.45K_{\rm cr}$	$\frac{1}{1.2} P_{\rm cr}$	0
PID	$0.6K_{\rm cr}$	0.5 <i>P</i> _{cr}	0.125P _{cr}

Transfer Function of PID Controller Tuned Using the Second Method

$$G_{c}(s) = K_{p} \left(1 + \frac{1}{T_{i}s} + T_{d}s \right)$$

= $0.6K_{cr} \left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s \right)$
= $0.075K_{cr}P_{cr} \frac{\left(s + \frac{4}{P_{cr}}\right)^{2}}{s}$

Example 1---PID Controller for DC Motor

- Plant---Armature-controlled DC motor; MOTOMATIC system produced by Electro-Craft Corporation
- Design a Type A PID controller and simulate the behavior of the closed-loop system; plot the closed-loop system step response
- Fine tune the controller parameters so that the max overshoot is 25% or less

Armature-Controlled DC Motor Modeling



Physics---The Magnetic Field

Oersted (1820): A current in a wire can produce magnetic effects; it can change the orientation of a compass needle



Force Acting on a Moving Charge in a Magnetic Field

□ Force

$$\vec{F} = q_0 \vec{v} \times \vec{B}$$

Magnitude

$$F = q_0 v B \sin \theta$$

■ The unit of B (flux density)---1Tesla, where $1 \text{ Tesla} = \frac{1 \text{ Weber}}{1 \text{ m}^2} = 10^4 \text{ Gauss}$

Torque on a Current Loop

The force F_4 has the same magnitude as F_2 but points in the opposite direction



An End View of the Current Loop



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Building a Motor From a Current Loop



DC Motor Construction

- To keep the torque in the same direction as the loop rotates, change the direction of the current in the loop---do this using slip rings at 0 and п (pi) or - п
- The brushes are fixed and the slip rings are connected to the current loop with electrical contact made by the loop's slip rings sliding against the brushes

Modeling Equations

Kirchhoff's Voltage Law to the armature circuit

$$U(s) = (L_a s + R_a) I_a(s) + E_b(s)$$

■ Back-emf (equivalent to an "electrical friction") $E_b(s) = K_b \Omega_m(s)$

Torque developed by the motor

$$T_m(s) = \left(J_m s^2 + B_m s\right)\Theta_m(s)$$

Electromechanical coupling $T_m(s) = K_t I_a(s)$

Relationship between K_t and K_b

Mechanical power developed in the motor armature (in watts)

$$p = e_b(t)i_a(t)$$

Mechanical power can also be expressed as

$$p = T_m(t)\omega_m(t)$$

Combine

$$p = T_m \omega_m = e_b i_a = K_b \omega_m \frac{T_m}{K_t}$$

In SI Units $K_t = K_b$

The back-emf and the motor torque constants are equal in the SI unit system

$$K_t \left(\frac{\mathrm{V}}{\mathrm{rad}/\mathrm{sec}} \right) = K_b \left(\mathrm{N} \cdot \mathrm{m}/\mathrm{A} \right)$$

Transfer Function of the DC Motor System

Transfer function of the DC motor

$$G_{p}(s) = \frac{Y(s)}{U(s)} = \frac{0.1464}{7.89 \times 10^{-7} s^{3} + 8.25 \times 10^{-4} s^{2} + 0.00172s}$$

where Y(s) is the angular displacement of the motor shaft and U(s) is the armature voltage

Tuning the Controller Using the Second Method of Ziegler and Nichols

Use the Routh-Hurwitz stability test; see page 212 of the Text

Determine K_{cr}

 \square Determine P_{cr}

Compute the controller parameters

Generating the Step Response

```
t = 0: 0.00005: .017;
K_cr = 12.28; P_cr = 135;
K = 0.075 * K_cr * P_cr; a = 4/P_cr;
num1 = K^{1} 2^{a} a^{2}; den1 = [0 1 0];
tf1 = tf(num1, den1);
num2 = [0 \ 0 \ 0 \ 0.1464];
den2 = [7.89e-007 8.25e-004 0.00172 0];
tf2=tf(num2,den2);
tf3 = tf1 * tf2;
sys=feedback(tf3,1);
y=step(sys,t); m=max(y);
```

Closed-Loop System Performance



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Example 2 (Based on Ex. 10-3 in Ogata, 2002)

Use a computational approach to generate an optimal set of the DC motor PID controller's parameters

$$G_c(s) = K \frac{(s+a)^2}{s}$$

Generate the step response of the closedloop system

Optimizing PID Parameters

```
t=0:0.0002:0.02;
font=14;
for K=5:-0.2:2%Outer loop to vary the values of
  %the gain K
  for a=1:-0.01:0.01; %Outer loop to vary the
  %values of the parameter a
     num1 = K^{1} 2^{a} a^{2}; den1 = [0 1 0];
     tf1 = tf(num1, den1);
     num2 = [0 \ 0 \ 0 \ 0.1464];
     den2=[7.89e-007 8.25e-004 0.00172 0];
     tf2=tf(num2,den2);
     tf3 = tf1 * tf2;
     sys=feedback(tf3,1);
     y=step(sys,t); m=max(y);
```

Finishing the Optimizing Program

```
if m<1.1 & m>1.05;
        plot(t,y);grid;set(gca,'Fontsize',font)
sol=[K;a;m]
        break % Breaks the inner loop
     end
  end
  if m<1.1 & m>1.05;
     break; %Breaks the outer loop
  end
end
```

Closed-Loop System Performance



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Modified PID Control Schemes

- If the reference input is a step, then because of the presence of the derivative term, the controller output will involve an impulse function
- The derivative term also amplifies higher frequency sensor noise
- Replace the pure derivative term with a derivative filter---PIDF controller
- Set-Point Kick---for step reference the PIDF output will involve a sharp pulse function rather than an impulse function

The Derivative Term

- Derivative action is useful for providing a phase lead, to offset phase lag caused by integration term
- Differentiation increases the highfrequency gain
- Pure differentiator is not proper or causal
- 80% of PID controllers in use have the derivative part switched off
- Proper use of the derivative action can increase stability and help maximize the integral gain for better performance

Remedies for Derivative Action---PIDF Controller

Pure differentiator approximation

$$\frac{T_d s}{\gamma T_d s + 1}$$

- where γ is a small parameter, for example, 0.1
- Pure differentiator cascaded with a firstorder low-pass filter

The Set-Point Kick Phenomenon

- If the reference input is a step function, the derivative term will produce an impulse (delta) function in the controller action
- Possible remedy---operate the derivative action only in the feedback path; thus differentiation occurs only on the feedback signal and not on the reference signal

Eliminating the Set-Point Kick

PID controller revisited



Eliminating the Set-Point Kick---Finding the source of trouble

More detailed view of the PID controller



Eliminating the Set-Point Kick---PI-D Control or Type B PID

Operate derivative action only in the feedback



I-PD---Moving Proportional and Derivative Action to the Feedback

I-PD control or Type C PID



I-PD Equivalent to PID With Input Filter (No Noise)

Closed-loop transfer function Y(s)/R(s) of the I-PD-controlled system



PID-Controlled System

Closed-loop transfer function Y(s)/R(s) of the PIDcontrolled system with input filter

$$\frac{Y(s)}{R(s)} = \frac{1}{1 + T_i s + T_i T_d s^2} \frac{K_p \left(1 + \frac{1}{T_i s} + T_d s\right) G_p(s)}{1 + K_p \left(1 + \frac{1}{T_i s} + T_d s\right) G_p(s)}$$

After manipulations it is the same as the transfer function of the I-PD-controlled closed-loop system PID, PI-D and I-PD Closed-Loop Transfer Function---No Ref or Noise

In the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the system output is the same for the three types of PID control



The Three Terms of Proportional-Integral-Derivative (PID) Control

- Proportional term responds immediately to the current tracking error; it cannot achieve the desired setpoint accuracy without an unacceptably large gain. Needs the other terms
- Derivative action reduces transient errors
- Integral term yields zero steady-state error in tracking a constant setpoint. It also rejects constant disturbances



Proportional-Integral-Derivative (PID) control provides an efficient solution to many real-world control problems

Summary

- PID control---most widely used control strategy today
- Over 90% of control loops employ PID control, often the derivative gain set to zero (PI control)
- The three terms are intuitive---a nonspecialist can grasp the essentials of the PID controller's action. It does not require the operator to be familiar with advanced math to use PID controllers
- Engineers prefer PID controls over untested solutions