ECE 382 Fall 2018

Frequency response

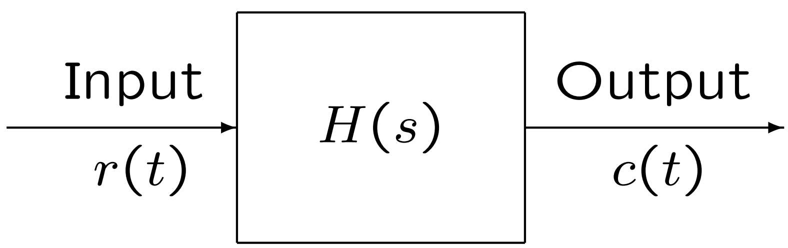
The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input.

The transfer function describing the sinusoidal steady-state behavior is obtained by replacing s with $j\omega$ in the system transfer function, that is,

$$H(j\omega) = H(s)|_{s=j\omega}$$

$H(j\omega)$ is called the *sinusoidal transfer function*.

The sinusoidal steady-state response



The sinusoidal steady-state response of a BIBO stable system to an input $r(t) = X \sin(\omega t)$ is given by

$$c_{ss} = X|H(j\omega)| \sin(\omega t + \Phi),$$

where $|H(j\omega)|$ is the magnitude of $H(j\omega)$ and $\Phi = \angle H(j\omega)$ is the argument of $H(j\omega)$.

The system frequency response

The sinusoidal steady-state response of a BIBO stable system to an input $r(t) = X\,\mathsf{sin}(\omega t)$ is given by

$$c_{ss} = X|H(j\omega)|\,\mathsf{sin}(\omega t + \Phi),$$

where $|H(j\omega)|$ is the magnitude of $H(j\omega)$ and $\Phi = \angle H(j\omega)$ is the argument of $H(j\omega)$

The steady-state output has the same frequency as the input and can be obtained by multiplying the input $r(t) = X\,\mathsf{sin}(\omega t)$ by $|H(j\omega)|$ and shifting the phase angle by $\angle H(j\omega)$.

The magnitude $|H(j\omega)|$ and the angle $\angle H(j\omega)$ for all ω constitute the *system frequency response*.

The magnitude and phase responses

The magnitude $|H(j\omega)|$ represents the **gain** of the system for sinusoidal inputs with frequency ω.

A plot of $|H(j\omega)|$ versus ω is called the magnitude, or amplitude, response.

The angle $\angle H(j\omega)$ represents the **phase** of the system for sinusoidal inputs with frequency ω.

A plot of $\angle H(j\omega)$ versus ω is called the phase response.

Octave and decade

An octave is a frequency band from ω_1 to ω_2 such that $\frac{\omega_2}{\omega_1} = 2$.

There is an increase in decade from ω_1 to ω_2 when $\frac{\omega_2}{\omega_1} = 10$.

Logarithmic plots

Logarithmic plots of $H(j\omega)$, or Bode diagrams of $H(j\omega)$, are two graphs:

1. A plot of $20\log_{10}|H(j\omega)|$ versus the frequency in log scale, that is, versus $\log_{10}\omega$

2. The phase angle $\angle H(j\omega)$ versus $\log_{10}\omega$

The standard representation of the logarithmic magnitude of $H(j\omega)$ is

$$20\log_{10}|H(j\omega)| \ \text{dB}$$

Historical Comment: Researchers for the telephone company first defined the unit of power gain as a "bel." However, this unit proved to be too large, and hence a decibel, that is, one-tenth of a bell, was selected as the unit, which is, of course, named after Alexander Graham Bell, the founder of the company

Motivation for using $20\log_{10}|H(j\omega)|$

In communications it is standard to measure power gain in decibels,

$$|H|_{\text{dB}} = 10\log_{10}\frac{P_2}{P_1}$$

Since power is the square of voltage, the voltage gain is

$$|H|_{\text{dB}} = 20\log_{10}\frac{V_2}{V_1}$$

From now on, we will drop the base of the logarithm; it is understood to be 10.

Advantages of working with frequency response in terms of Bode plots

- Multiplication of magnitudes converted into addition

- A much wider range of the behavior of the circuit can be displayed; that is, both low- and high-frequency behavior can be displayed in one plot

- Bode plots can be determined experimentally

Example

$$H(s) = \frac{1}{1+2s}$$

Hence,

$$H(j\omega) = \frac{1}{1+2j\omega}$$

Hence the log magnitude, Lm, is

$$\begin{aligned}
Lm\frac{1}{1+2j\omega} &= 20\log\left|\frac{1}{1+2j\omega}\right| \\
&= 20\log 1 - 20\log\sqrt{1+(2\omega)^2} \\
&= -20\log\sqrt{1+(2\omega)^2} \\
&= \begin{cases}
\text{for very small values} \\
\text{of } \omega,\ \text{that is, } 2\omega \ll 1, \\
Lm = \log 1 = 0 \\
\\
\text{for very large values} \\
\text{of } \omega,\ \text{that is, } 2\omega \gg 1, \\
Lm = -20\log(2\omega)
\end{cases}
\end{aligned}$$