

DC Motor Modeling



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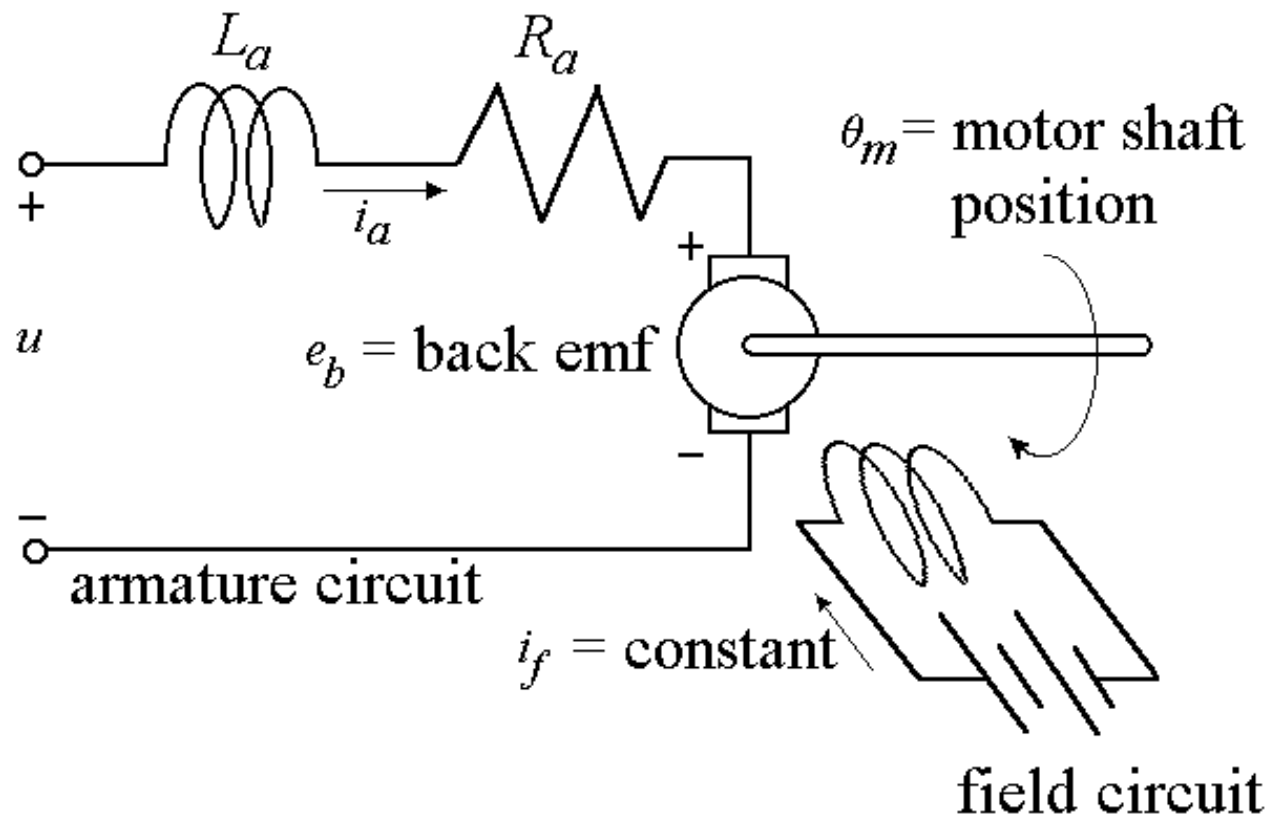
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DC Motor

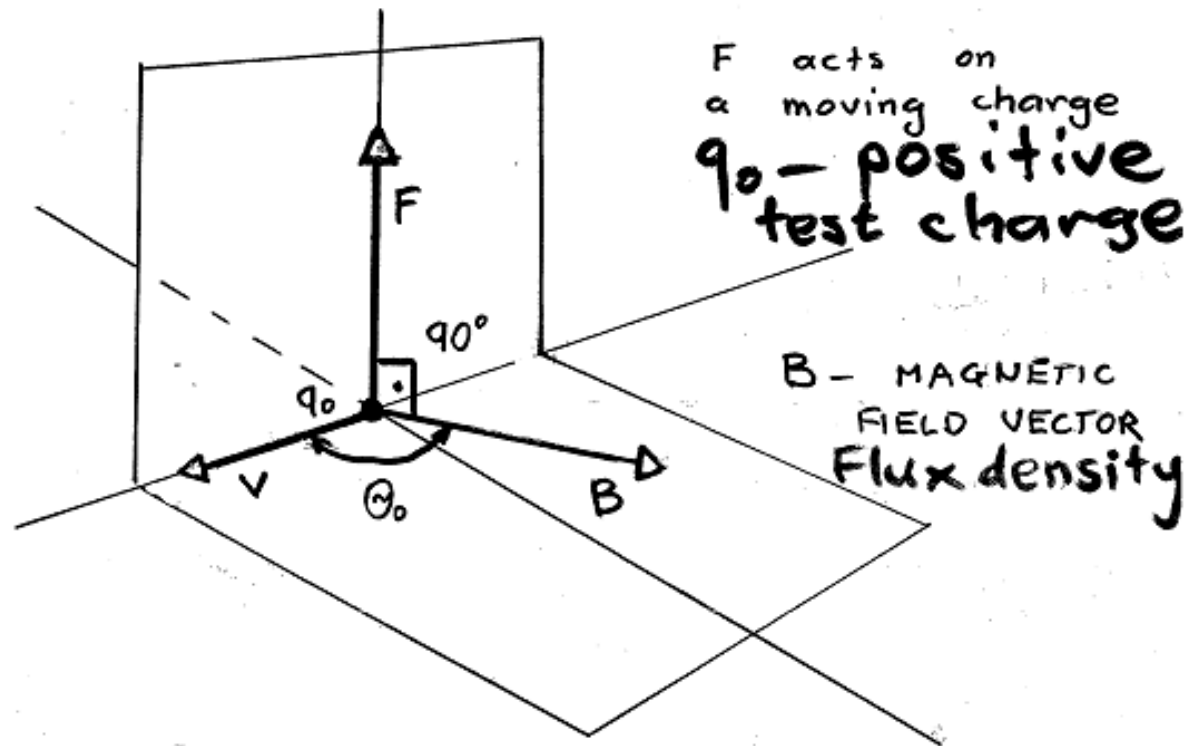
- Armature-controlled DC motor; MOTOMATIC system produced by Electro-Craft Corporation

Modeling the Armature Controlled DC Motor



Physics---The Magnetic Field

Oersted (1820): A current in a wire can produce magnetic effects; it can change the orientation of a compass needle



Force Acting on a Moving Charge in a Magnetic Field

- Force

$$\vec{F} = q_0 \vec{v} \times \vec{B}$$

- Magnitude

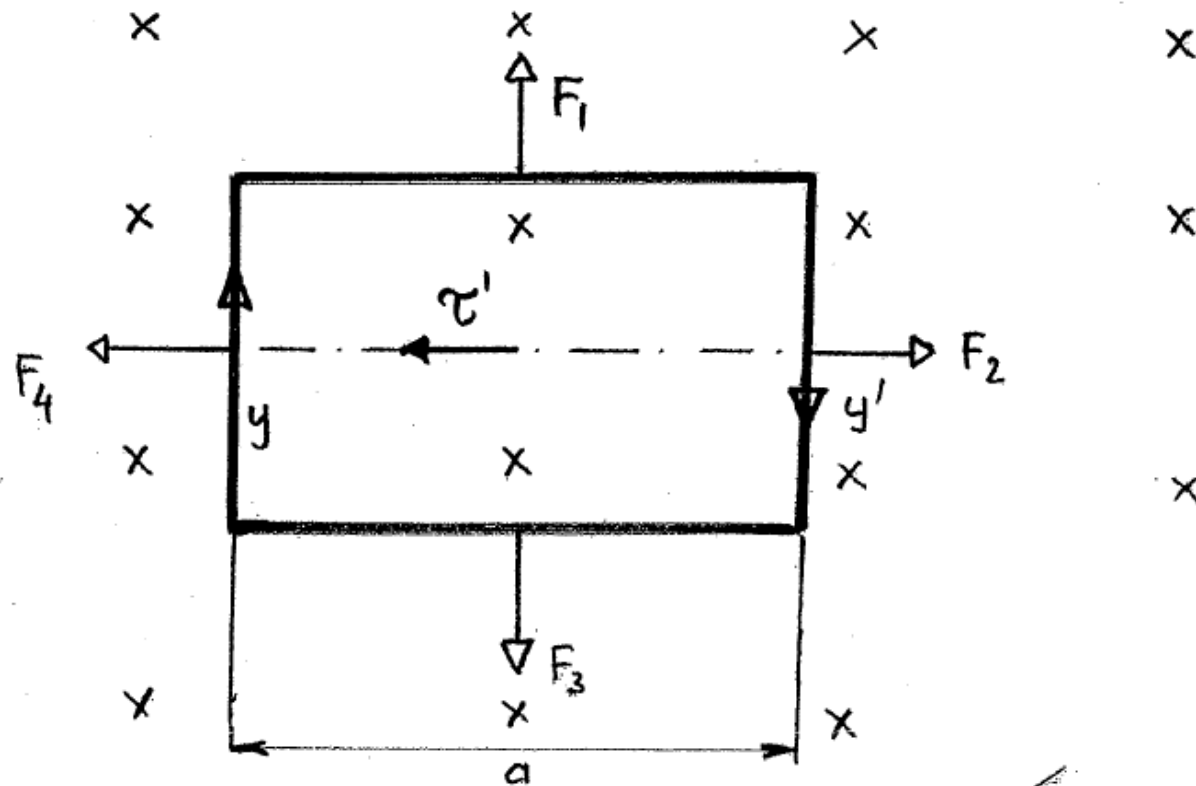
$$F = q_0 v B \sin \theta$$

- The unit of B (flux density)---1Tesla, where

$$1 \text{ Tesla} = \frac{1 \text{ Weber}}{1 \text{ m}^2} = 10^4 \text{ Gauss}$$

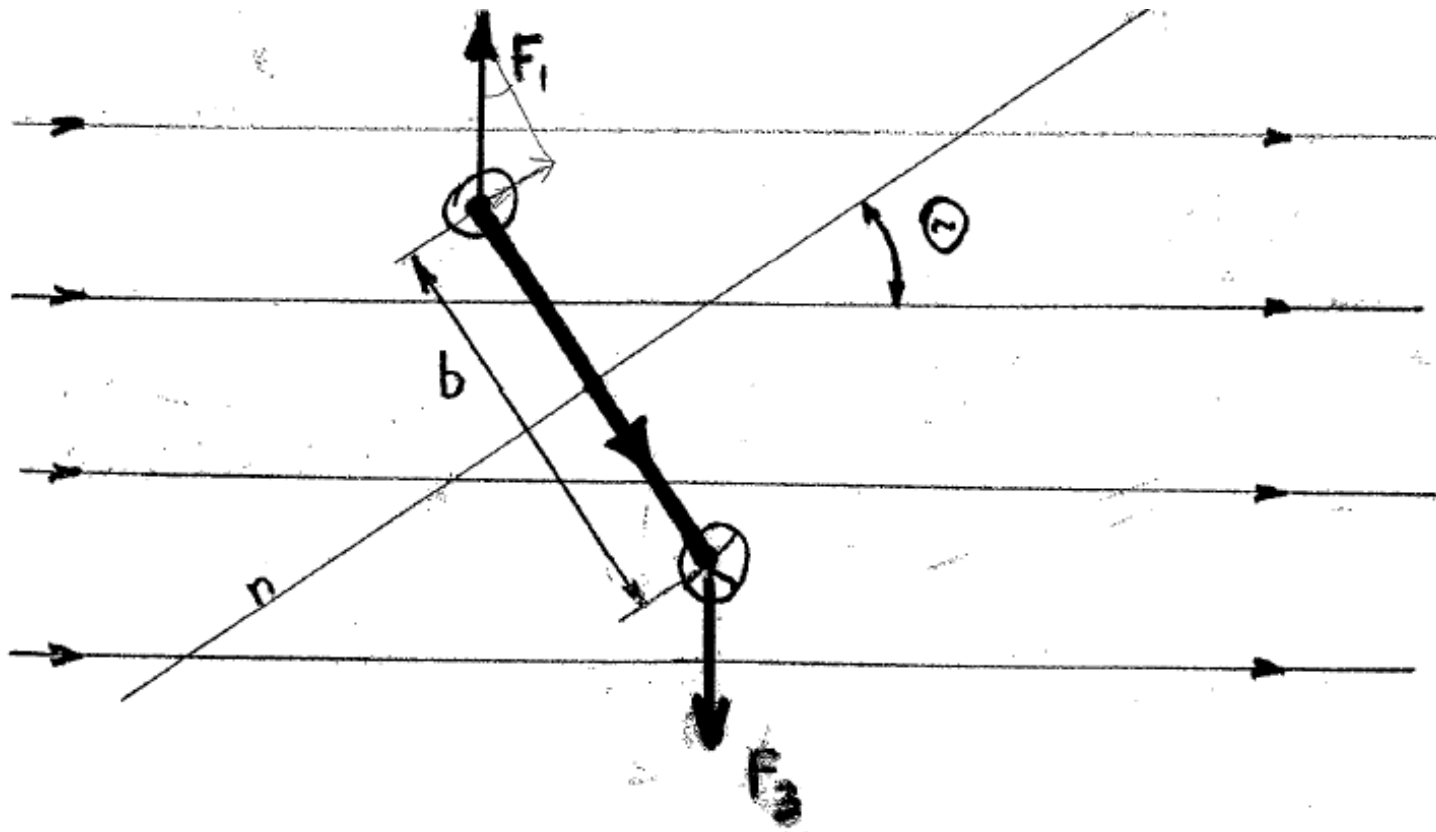
Torque on a Current Loop

The force F_4 has the same magnitude as F_2 but points in the opposite direction

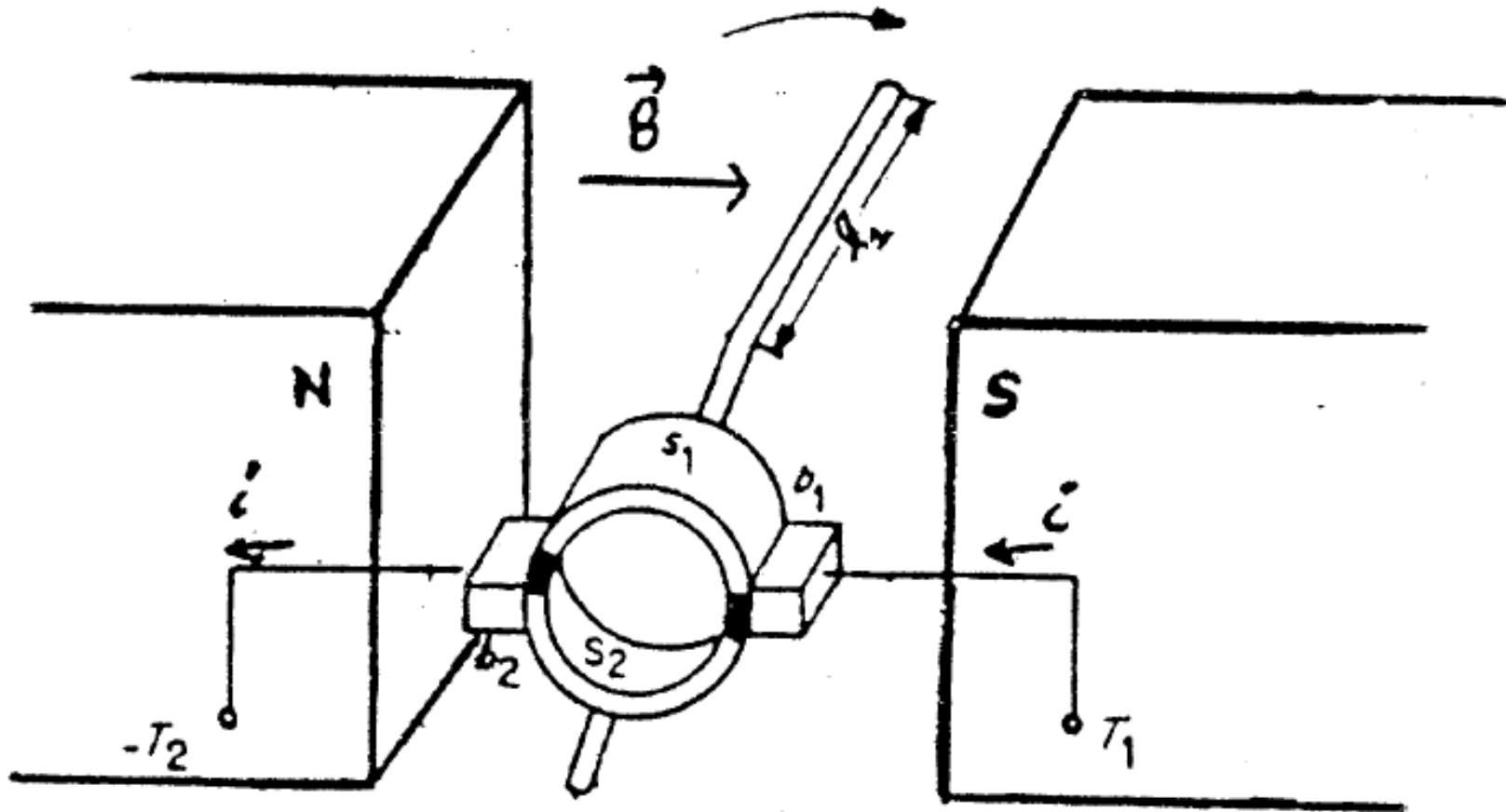


An End View of the Current Loop

The common magnitude of F_1 and F_3 is iaB



Building a Motor From a Current Loop



DC Motor Construction

- ❑ To keep the torque in the same direction as the loop rotates, change the direction of the current in the loop---do this using slip rings at 0 and π (pi) or $-\pi$
- ❑ The brushes are fixed and the slip rings are connected to the current loop with electrical contact made by the loop's slip rings sliding against the brushes

Modeling Equations

- Kirchhoff's Voltage Law to the armature circuit

$$E_a(s) = (L_a s + R_a)I_a(s) + E_b(s)$$

- Back-emf (equivalent to an "electrical friction")

$$E_b(s) = K_b \Omega_m(s)$$

- Torque developed by the motor

$$T_m(s) = (J_m s + B_m)\Omega_m(s)$$

- Electromechanical coupling

$$T_m(s) = K_t I_a(s)$$

Relationship between K_t and K_b

- Mechanical power developed in the motor armature (in watts)

$$p = e_b(t)i_a(t)$$

- Mechanical power can also be expressed as

$$p = T_m(t)\omega_m(t)$$

- Combine

$$p = T_m\omega_m = e_b i_a = K_b \omega_m \frac{T_m}{K_t}$$

In SI Units $K_t = K_b$

- The back-emf and the motor torque constants are equal in the SI unit system

$$K_t \left(N \cdot m / A \right) = K_b \left(\frac{V}{rad / sec} \right)$$

- Note: sometimes the motor torque constant is also denoted as K_i

Transfer Function of the DC Motor System

- Transfer function of the DC motor

$$G_p(s) = \frac{C(s)}{U(s)} = \frac{0.1464}{7.89 \times 10^{-7} s^3 + 8.25 \times 10^{-4} s^2 + 0.00172s}$$

where $C(s)$ is the angular displacement of the motor shaft and $U(s)$ is the armature voltage

Generating the Step Response

```
t=0:0.00005:.017;  
num=[0 0 0 0.1464];  
den=[7.89e-007 8.25e-004 0.00172 0];  
dc_motor=tf(num,den);  
y=step(dc_motor,t);
```