Observers for Systems With Unknown Inputs

ECE 680, Fall 2019

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You can observe a lot by watching

—Yogi Berra
Outline

Uncertain systems and their modeling
Outline

- Uncertain systems and their modeling
- Observer development
Outline

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- Observer development
- Unknown input observer (UIO) design algorithm
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- Observer development
- Unknown input observer (UIO) design algorithm
- UIO application to fault detection and isolation
Outline

- Uncertain systems and their modeling
- Observer development
- Unknown input observer (UIO) design algorithm
- UIO application to fault detection and isolation
- Concluding remarks
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\dot{x} = Ax + Bu
\ \ \ \ \ \ \ \ \ \ \ \ \ y = Cx

Some or all of the inputs are unknown
\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]

Some or all of the inputs are unknown

Let

\[ B = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \]

where \( B_1 \in \mathbb{R}^{n \times m_1} \) and \( B_2 \in \mathbb{R}^{n \times m_2} \), and

\[ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]
Uncertain system model:

\[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]
Uncertain system model:

\[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]

The unknown input \( u_2 \) may model lumped uncertainties or nonlinearities in the plant.
Uncertain System Model

Uncertain system model:

\[ \dot{x} = Ax + B_1u_1 + B_2u_2 \]

The unknown input \( u_2 \) may model lumped uncertainties or nonlinearities in the plant.

We assume that \( u_2 \) is bounded, that is, there exists a nonnegative real number, \( \rho \), such that

\[ \|u_2\| \leq \rho, \]

where \( \| \cdot \| = \| \cdot \|_2 \) is standard Euclidean norm.
Using uncertain system model for nonlinear system modeling

Simple pendulum model: \[ Ml^2\ddot{\theta} + Mg l \sin \theta = \tau, \]
equivalently,
\[ \ddot{\theta} = -\frac{g}{l} \sin \theta + \frac{1}{I} \tau, \quad I = Ml^2 \]
Example contd.: Simple pendulum viewed as an uncertain system

Simple pendulum model:

\[ \ddot{\theta} = -\frac{g}{l} \sin(\theta) + \frac{1}{I} \tau \]
Example contd.: Simple pendulum viewed as an uncertain system

Simple pendulum model:

\[ \ddot{\theta} = -\frac{g}{l} \sin(\theta) + \frac{1}{I} \tau \]

Let \( u_1 = \frac{1}{I} \tau \)
Example contd.: Simple pendulum viewed as an uncertain system

Simple pendulum model:

\[ \ddot{\theta} = -\frac{g}{l} \sin(\theta) + \frac{1}{I} \tau \]

Let \( u_1 = \frac{1}{I} \tau \)

Define the state variables, \( x_1 = \theta \) and \( x_2 = \dot{\theta} \),

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
x_2 \\
-\frac{g}{l} \sin(x_1) + \frac{1}{I} \tau
\end{bmatrix}
= \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_1 + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_2
\]

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Example contd.: Simple pendulum as an uncertain system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u_1 +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u_2
\]

\[
= Ax + b_1 u_1 + b_2 u_2,
\]

where \(u_2 = -\frac{g}{l} \sin(x_1)\).
Example contd.: Simple pendulum as an uncertain system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_1 + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_2
\]

= \mathbf{A} \mathbf{x} + \mathbf{b}_1 u_1 + \mathbf{b}_2 u_2,

where \( u_2 = -\frac{g}{l} \sin(x_1) \).

Let \( y = x_1 \); another way to represent the model,

\[ \dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{b}_1 \mathbf{u}_1 \]

and
Example contd.: Simple pendulum as an uncertain system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2\end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 \\
= Ax + b_1 u_1 + b_2 u_2,
\]

where \( u_2 = -\frac{g}{l} \sin(x_1) \).

Let \( y = x_1 \); another way to represent the model,

\[
\dot{x} = Ax + b_1 u_1
\]

and

\[
u_1 = \frac{1}{I} \tau - \frac{g}{l} \sin(y)\]
A Practical Example of a System With Unknown Input

http://www.vcc-10.org/
Hypothalamic-pituitary-adrenal axis

The hypothalamic-pituitary-adrenal—HPA
The hypothalamic-pituitary-adrenal—HPA

The HPA axis is a set of interactions between the hypothalamus (a part of the brain), the pituitary gland (also part of the brain) and the adrenal or suprarenal glands (at the top of each kidney)
Basic Structure of HPA

Observers for Systems With Unknown Inputs
The HPA axis helps regulate temperature, digestion, immune system, mood, sexuality and energy usage.
HPA axis

- The HPA axis helps regulate temperature, digestion, immune system, mood, sexuality and energy usage.
- Also a major part of the system that controls reaction to stress, trauma and injury.
HPA Axis Modeling

http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2613527
Many controllers call for the complete availability of the state vector of the controlled system.
Need for the System State Estimate

- Many controllers call for the complete availability of the state vector of the controlled system.
- It is frequently impossible to sense all the elements of the system state vector.
Need for the System State Estimate

- Many controllers call for the complete availability of the state vector of the controlled system.
- It is frequently impossible to sense all the elements of the system state vector.
- To retain the many useful properties of the state feedback control, one needs to overcome the problem of incomplete state vector information.
Beginnings of the observer

The observer provides a solution to the problem of incomplete state vector information.
Beginnings of the observer

- The observer provides a solution to the problem of incomplete state vector information.
- Observer—a dynamic system that estimates the system state based on the system inputs and outputs.
Beginnings of the observer

The observer provides a solution to the problem of incomplete state vector information.

Observer—a dynamic system that estimates the system state based on the system inputs and outputs.

A trivial observer (open-loop observer)—a system copy as an observer
A trivial observer (open-loop observer)—a system copy as an observer

Observation error, $e = x - \hat{x}$
Problems with the open-loop observer

Observation error dynamics,

\[(\dot{x} - \dot{\hat{x}}) = A(x - \hat{x})\]
Problems with the open-loop observer

Observation error dynamics,

\[
(\dot{x} - \dot{\hat{x}}) = A(x - \hat{x})
\]

The observation error tends to zero only if the observed system is stable.
Problems with the open-loop observer

- Observation error dynamics,

\[
\dot{(x - \hat{x})} = A(x - \hat{x})
\]

- The observation error tends to zero only if the observed system is stable

- There is no control over the observation error dynamics
Problems with the open-loop observer

- Observation error dynamics,

\[
(\dot{x} - \dot{\hat{x}}) = A(x - \hat{x})
\]

- The observation error tends to zero only if the observed system is stable

- There is no control over the observation error dynamics

- There is a fix—add observer innovation to get the closed-loop observer
Closed-loop observer

\[ \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \]

Luenberger’s Innovation

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Luenberger’s closed-loop observer

Observation error dynamics,

\[(\dot{x} - \dot{\hat{x}}) = (A - LC)(x - \hat{x})\]
Combined observer-controller compensator

Works well for systems without uncertainties

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Combined observer-controller compensator

Works well for systems without uncertainties

What about systems with uncertainties?
Objective: Design an observer innovation so that the resulting observer can estimate asymptotically the state vector of the uncertain system.
What is a sliding-mode observer?
What is a sliding-mode observer?

State estimator where the state estimation error exhibits sliding-mode behavior in the estimation space.
What is a sliding-mode observer?

State estimator where the state estimation error exhibits sliding-mode behavior in the estimation space

What is sliding-mode behavior?
What is a sliding-mode observer?

State estimator where the state estimation error exhibits sliding-mode behavior in the estimation space

What is sliding-mode behavior?

Non-linear phenomena in which system trajectory is forced to remain on a subspace (or manifold) of the state space
Plant: $\dot{x} = Ax + B_1 u_1 + B_2 u_2$
Plant: \[
\dot{x} = Ax + B_1 u_1 + B_2 u_2
\]

Observer: \[
\dot{\hat{x}} = A\hat{x} + B_1 u_1 + T_U^{-1} \begin{bmatrix} \bar{L} \\ I_p \end{bmatrix} v,
\]

where

\[v = M \text{sign}(y - \hat{y})\]
Plant: \( \dot{x} = Ax + B_1 u_1 + B_2 u_2 \)
Plant: \( \dot{x} = Ax + B_1u_1 + B_2u_2 \)

Observer: \( \dot{\hat{x}} = A\hat{x} + B_1u_1 + G_l(y - \hat{y}) + G_nv, \) where

\[
G_l = T_{ES}^{-1} \begin{bmatrix}
\hat{A}_{12} \\
\hat{A}_{22} - \hat{A}_s^{22}
\end{bmatrix}
\]

and

\[
v = -\rho(t, y, u) \frac{P_2(y - \hat{y})}{\|P_2(y - \hat{y})\|}
\]
Walcott and Žak (1987) and Hui and Žak (2005)

\[
\dot{x} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta)
\]
Sliding-mode observer

\[ \dot{\hat{x}} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta) \]
Sliding-mode observer

\[
\dot{\hat{x}} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta)
\]

For \( \eta \geq \rho \), \( \hat{x} \) is an asymptotic estimate of the state vector, \( x \), of the uncertain system, where

\[
E(y, \hat{y}, \eta) = \begin{cases} 
\eta \frac{F(y - \hat{y})}{\|F(y - \hat{y})\|_2} & \text{for } F(y - \hat{y}) \neq 0 \\
0 & \text{for } F(y - \hat{y}) = 0 
\end{cases}
\]
\[
\dot{\hat{x}} = A\hat{x} + B_1u_1 + L(y - \hat{y}) + B_2E(y, \hat{y}, \eta)
\]

For \(\eta \geq \rho\), \(\hat{x}\) is an asymptotic estimate of the state vector, \(x\), of the uncertain system, where

\[
E(y, \hat{y}, \eta) = \begin{cases} 
\eta \frac{F(y - \hat{y})}{\|F(y - \hat{y})\|_2} & \text{for } F(y - \hat{y}) \neq 0 \\
0 & \text{for } F(y - \hat{y}) = 0
\end{cases}
\]

For single-input single-output plant,

\[
E(y, \hat{y}, \eta) = \eta \text{ sign } (F(y - \hat{y}))
\]
Plant: \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]
Convergence of Observation Error

Plant: \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]

Observer:
\[ \dot{\hat{x}} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta) \]
Convergence of Observation Error

Plant: \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]

Observer: \[ \dot{\hat{x}} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta) \]

Observation error: \[ \dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC) e + B_2 u_2 - B_2 E(e, \eta) \]
Convergence of Observation Error

- **Plant:** \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]
- **Observer:**
  \[ \dot{\hat{x}} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta) \]
- **Observation error:**
  \[ \dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC)e + B_2 u_2 - B_2 E(e, \eta) \]
- Can show \[ \frac{d}{dt} (e^\top P e) = -e^\top Q e < 0 \], which implies
  \[ \lim_{t \to \infty} e(t) = 0 \]
For SISO system

\[ G(s) = c(sI_n - A)^{-1}b = \frac{\det \begin{bmatrix} sI_n - A & -b \\ c & 0 \end{bmatrix}}{\det(sI_n - A)} \]
For SISO system

\[ G(s) = c(sI_n - A)^{-1}b = \frac{\det \begin{bmatrix} sI_n - A & -b \\ c & 0 \end{bmatrix}}{\det(sI_n - A)} \]

To see this, note that

\[
\begin{bmatrix}
I_n & -0 \\
-c(sI_n - A)^{-1} & 1
\end{bmatrix}
\begin{bmatrix}
sI_n - A & -b \\
c & 0
\end{bmatrix}
= \begin{bmatrix}
sI_n - A & -b \\
0 & c(sI_n - A)^{-1}b
\end{bmatrix}
\]
Hence

\[
\begin{vmatrix}
 sI_n - A & -b \\
 c & 0 \\
\end{vmatrix}
\begin{align*}
&= \det(sI_n - A) G(s) \\
&= \det(sI_n - A) \frac{c \operatorname{adj}(sI_n - A)b}{\det(sI_n - A)}
\end{align*}
\]
Hence

\[
\begin{vmatrix}
sI_n - A & -b \\
c & 0
\end{vmatrix}
= \det(sI_n - A) G(s)
\]

\[
= \det(sI_n - A) \frac{c \adj(sI_n - A)b}{\det(sI_n - A)}
\]

That is,

\[
G(s) = c(sI_n - A)^{-1}b = \frac{\det \begin{vmatrix}
sI_n - A & -b \\
c & 0
\end{vmatrix}}{\det(sI_n - A)}
\]
Linear time invariant plant model:

\[
\dot{x} = Ax + Bu, \quad x(0) = x_0 \\
y = Cx
\]
Linear time invariant plant model:

\[ \dot{x} = Ax + Bu, \quad x(0) = x_0 \]
\[ y = Cx \]

Take the Laplace transforms

\[ sX(s) - x(0) = AX(s) + BU(s) \]
\[ Y(s) = CX(s) \]
Linear time invariant plant model:

\[
\dot{x} = Ax + Bu, \quad x(0) = x_0 \\
y = Cx
\]

Take the Laplace transforms

\[
sX(s) - x(0) = AX(s) + BU(s) \\
Y(s) = CX(s)
\]

\[
\begin{bmatrix}
  sI_n - A & -B \\
  C & O
\end{bmatrix}
\begin{bmatrix}
  X(s) \\
  Y(s)
\end{bmatrix}
= 
\begin{bmatrix}
  x(0) \\
  Y(s)
\end{bmatrix}
\]
Rosenbrock’s system matrix

\[ P(s) = \begin{bmatrix} sI_n - A & -B \\ C & O \end{bmatrix} \]
Normal Rank

Rosenbrock’s system matrix

\[ P(s) = \begin{bmatrix}
  sI_n - A & -B \\
  C & O
\end{bmatrix} \]

The normal rank, denoted \textit{normalrank}, of a polynomial matrix is the maximally possible rank of this matrix for at least one \( s \).
Rosenbrock’s system matrix

\[ P(s) = \begin{bmatrix} \ M sI_n - A & -B \\ C & O \end{bmatrix} \]

The normal rank, denoted \textbf{normal rank}, of a polynomial matrix is the maximally possible rank of this matrix for at least one \( s \).

When we say that a polynomial matrix is of rank \( r \), we refer to its normal rank.
A complex number $z_0$ is an invariant zero of the system $(A, B, C)$ if

$$\text{rank} \begin{bmatrix} z_0 I_n - A & -B \\ C & O \end{bmatrix} < \text{normalrank} \begin{bmatrix} s I_n - A & -B \\ C & O \end{bmatrix}$$
Plant:

\[
\begin{align*}
\dot{x} &= Ax + B_1u_1 + B_2u_2 \\
y &= Cx
\end{align*}
\]
Existence conditions of sliding-mode observer

Plant:
\[
\dot{x} = Ax + B_1 u_1 + B_2 u_2 \\
y = Cx
\]

Observer:
\[
\dot{x} = A\hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta)
\]
Existence conditions of sliding-mode observer

Plant:
\[
\dot{x} = Ax + B_1 u_1 + B_2 u_2 \\
y = C x
\]

Observer:
\[
\hat{x} = A \hat{x} + B_1 u_1 + L(y - \hat{y}) + B_2 E(y, \hat{y}, \eta)
\]

The observer can be constructed if and only if
(i) \( \text{rank } B_2 = \text{rank } CB_2 = r \), and
(ii) the system zeros of the triple \((A, B_2, C)\) are in the open left-hand complex plane.
Discussion of existence conditions

Plant: \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]
Discussion of existence conditions

Plant: \[ \dot{x} = Ax + B_1 u_1 + B_2 u_2 \]

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Discussion of existence conditions

Plant: \[ \dot{x} = Ax + B_1u_1 + B_2u_2 \]

(i) \( \text{rank } B_2 = \text{rank } CB_2 = r \), and

(ii) the system zeros of the triple \((A, B_2, C)\) are in the open left-hand complex plane, that is,

\[ \text{rank} \begin{bmatrix} sI_n - A & B_2 \\ C & O \end{bmatrix} = n + r \]

for all \( s \) such that \( \text{Re}(s) \geq 0 \)
Comment on the existence conditions for sliding-mode observer

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\[
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sI_n - A & B_2 \\
C & O
\end{bmatrix} = n + r
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Comment on the existence conditions for sliding-mode observer

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\text{rank} \begin{bmatrix} sI_n - A & B_2 \\ C & O \end{bmatrix} = n + r
\]

for all \( s \) such that \( \text{Re}(s) \geq 0 \)

The above conditions are identical to the existence conditions for all unknown input observers (UIOs) available on the market today!
Constructing the sliding mode observer

Given a system model, \((A, B_1, B_2, C)\)
Constructing the sliding mode observer

- Given a system model, \( (A, B_1, B_2, C) \)
- Find a triple of matrices \( (P, F, L) \)
Constructing the sliding mode observer

Given a system model, \((A, B_1, B_2, C)\)

Find a triple of matrices \((P, F, L)\)

such that

\[
(A - LC)^\top P + P (A - LC) < 0
\]
Constructing the sliding mode observer

Given a system model, \((A, B_1, B_2, C)\)

Find a triple of matrices

\((P, F, L)\)

such that

\[(A - LC)^\top P + P (A - LC) < 0\]

and

\[FC = B_2^\top P\]
Observer for the simple pendulum

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

\[
= Ax + b_1 u_1 + b_2 u_2,
\]

where \( u_2 = -\frac{g}{l} \sin(x_1) \)

Let \( y = \begin{bmatrix} 0 & 1 \end{bmatrix} x \)
Observer for the simple pendulum

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_1 + \begin{bmatrix}
0 \\
1
\end{bmatrix} u_2
\]

\[= A \underline{x} + b_1 u_1 + b_2 u_2,\]

where \( u_2 = -\frac{g}{l} \sin(x_1) \)

- Let \( y = \begin{bmatrix}
0 & 1
\end{bmatrix} \underline{x} \)

- The rank condition satisfied,

\[\text{rank } b_2 = \text{rank } cb_2 = 1\]
The rank condition satisfied,

\[ \text{rank } b_2 = \text{rank } cb_2 = 1 \]
The rank condition satisfied,

\[ \text{rank } b_2 = \text{rank } cb_2 = 1 \]

The system zero not in the open LHP,

\[
\det \begin{bmatrix}
    sI_2 - A & b_2 \\
    c & 0
\end{bmatrix} = -s
\]
The rank condition satisfied,

\[ \text{rank } b_2 = \text{rank } cb_2 = 1 \]

The system zero not in the open LHP,

\[ \det \begin{bmatrix} sI_2 - A & b_2 \\ c & 0 \end{bmatrix} = -s \]

Sliding mode observer cannot be constructed
Observer for the simple pendulum; second attempt

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
0 & 0 \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
\end{bmatrix} u_1 = A\mathbf{x} + b_1 u_1,
\]

where \( y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} \) and

\[ u_1 = \frac{1}{I} \tau - \frac{g}{l} \sin(x_1) = \frac{1}{I} \tau - \frac{g}{l} \sin(y) \]

The pair \((A, c)\) is observable and \(b_2 = [\ ]\); the existence conditions for the sliding mode observer are trivially satisfied.
Observer design for the simple pendulum

The pair \((A, c)\) is observable and \(b_2 = [ ]\).
The pair \((A, c)\) is observable and \(b_2 = [ \] \)

The existence conditions for the sliding mode observer are trivially satisfied
The pair \((A, c)\) is observable and \(b_2 = [\quad]\)

The existence conditions for the sliding mode observer are trivially satisfied

Sliding mode observer becomes a Luenberger observer,

\[
\dot{\hat{x}} = A\hat{x} + b_1 u_1 + l(y - \hat{y})
\]
Fault Definitions

- Fault—an unexpected change of the system function that may not necessarily represent physical failure or breakdown


Fault Definitions

- Fault—an unexpected change of the system function that may not necessarily represent physical failure or breakdown
- An unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition
Fault Definitions

- Fault—an unexpected change of the system function that may not necessarily represent physical failure or breakdown

- An unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition

- Fault implies a failure, not necessarily culpable, to reach some standard of perfection in disposition, action, or habit (Merriam-Webster)
Fault diagnosis system—a monitoring system used to detect faults and diagnose their location and significance.
Fault diagnosis system—a monitoring system used to detect faults and diagnose their location and significance

Fault detection—to make a binary decision; either that something has gone wrong or that everything is fine
Fault diagnosis system—a monitoring system used to detect faults and diagnose their location and significance

Fault detection—to make a binary decision; either that something has gone wrong or that everything is fine

Fault isolation—to determine the location of the fault, for example, which sensor or actuator has become faulty
Fault diagnosis system—a monitoring system used to detect faults and diagnose their location and significance

Fault detection—to make a binary decision; either that something has gone wrong or that everything is fine

Fault isolation—to determine the location of the fault, for example, which sensor or actuator has become faulty

Fault identification—to estimate the size and type or nature of the fault
Importance of Fault Diagnosis

- Early indications which faults are developing can help avoid the system breakdown—especially important in safety-critical systems such as in a nuclear reactor, chemical plant, or an aircraft.
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- On-line fault diagnosis to improve plant efficiency, maintainability, and reliability.
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- On-line fault diagnosis to improve plant efficiency, maintainability, and reliability.

- Predictive maintenance tools to ensure the system safety, while at the same time avoiding costly maintenance during the system down-time.
An increasing need for a control system to continue operating acceptably in spite of faults occurrence in the system.
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**Fault-tolerant** control system—a control system with the fault-tolerance capability.
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Fault-tolerant system should possess a graceful performance degradation to operate under a faulty condition
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**Fault-tolerant** control system—a control system with the fault-tolerance capability

Fault-tolerant system should possess a graceful performance degradation to operate under a faulty condition

Maintain the system operation to give sufficient time for the system repair or to use alternative measures to avoid catastrophes
Fault-Tolerant Control System Design

Retain the system control integrity for a set of possible components faults that resemble these faults.
Fault-Tolerant Control System Design

- Retain the system control integrity for a set of possible components faults that resemble these faults.
- Built into the system capability of automatic reconfiguration once a malfunction has been detected and isolated.
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- Built into the system capability of automatic reconfiguration once a malfunction has been detected and isolated.

- The fault must be reliably detected and isolated, and the info should be passed to the supervisor to make proper decisions.
Fault-tolerance is one of the characteristics of intelligent systems.
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“By design or implementation, failure-tolerant control systems are *intelligent* systems” R. F. Stengel (1991) in “Intelligent failure-tolerant control,” *IEEE Control Systems Magazine*, Vol 11, No. 4

“Fault diagnosis is an essential ingredient property of an intelligent control system” K. J. Åström (1991), *First European Control Conf*, ECC 91, Grenoble, France
Traditional Approach to Fault Diagnosis

- Hardware redundancy—use multiple lanes of sensors, actuators, and computers to measure and/or control a particular variable
Traditional Approach to Fault Diagnosis

- Hardware redundancy—use multiple lanes of sensors, actuators, and computers to measure and/or control a particular variable
- A voting scheme—to decide if and when a fault has occurred and its likely location amongst redundant system components
Disadvantages of Traditional Approach

• Major problem with hardware redundancy—extra equipment, maintenance costs and additional space
Disadvantages of Traditional Approach

- Major problem with hardware redundancy—extra equipment, maintenance costs and additional space
- Analytical redundancy or functional redundancy—cross check dissimilar measured values, that is, reconcile data rather than replicate hardware
Functional Redundancy

- Consistency checking—a comparison between an actual measured signal and its expected value
Functional Redundancy

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- The expected signal—generated using a math model of the system. Hence the term, the model-based approach.
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The residual is the difference between the measured signal and an expected signal
Functional Redundancy

- Consistency checking—a comparison between an actual measured signal and its expected value

- The expected signal—generated using a math model of the system. Hence the term, the model-based approach

- The residual is the difference between the measured signal and an expected signal

- The residual should be zero when there are no faults and nonzero when a fault occurs in the system
Model-Based Fault Diagnosis

- Fault detection from the comparison of available system output with the output from the system mathematical model
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A residual, the difference between the two signals, is a fault indicator.
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- A residual, the difference between the two signals, is a fault indicator

- Major advantage over the hardware redundancy approach—no additional hardware

- Need a robust math model of the monitored system to eliminate false and missed alarms
Observer-based fault detection and isolation

Observers for Systems With Unknown Inputs
Observation error:

\[ \dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC) e + B_2 u_2 - B_2 E(e, \eta) \]
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\[
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If

\[
\lim_{t \to \infty} e(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} \dot{e}(t) = 0,
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\[ \dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC) e + B_2 u_2 - B_2 E(e, \eta) \]

If

\[ \lim_{t \to \infty} e(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} \dot{e}(t) = 0, \]

then

\[ E(y, \hat{y}, \eta) \approx u_2 \]

for

\[ \eta \geq \rho \]
Observer-based actuator fault detection and isolation

Observers for Systems With Unknown Inputs
Observer-based sensor fault detection and isolation

Observers for Systems With Unknown Inputs
Concluding remarks

Observers can be used as “software” or “virtual” sensors as opposed to hardware sensors.
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Concluding remarks

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- Observers can augment the performance of low cost-sensors.

- Despite adding complexity to the system, observers can bring substantial benefits while reducing cost or increasing reliability.

- Unknown input observers (UIO) can also be used in decentralized control of large-scale systems.