ECE 25500: Homework II

Current Flow and PN Junctions

Due on: Sep. 6^{th}, 2019 *by* 5:00 PM

Note: Scan your work (there is a scanner in the EE computer lab for student use) and submit it on Blackboard by the deadline indicated above. Late homework is **not** accepted. Make sure that the scan is readable. Please email the course GTA at `rchatrie@purdue.edu` if you have any questions about this assignment.

Problem 1 (pts) : Suppose that a silicon semiconductor device has a p-type layer of width W. Suppose further that electron carriers are injected into the layer from one side resulting in the minority carrier distribution $n_p(x)$ shown in **Fig.** below. The excess minority carrier concentration at $x = 0$ is $n_p = 10^{14}$ cm^{-3}, and that at $x = W$ is $n_p = 0$ cm^{-3}. Let $\mu_n = 600$ cm^2/V · s, $T = 300$ K, and $W = 100$ nm.

(a) Compute the electron current density in A/cm^2 (with the correct sign).

Solution. Use Einstein's relationship to compute the diffusion coefficient.

$$J_n = qD_n \frac{dn(x)}{dx} = q\mu_n \frac{k_B T}{q} \frac{dn(x)}{dx}$$
$$= (1.6 \times 10^{-19} \text{ C}) \left(600 \ \frac{\text{cm}^2}{\text{V} \cdot \text{s}}\right) (0.026 \text{ V}) \left(\frac{-10^{14} \text{ cm}^{-3}}{100 \text{ nm}}\right)$$
$$\boxed{= -25 \text{ A/cm}^2}$$

Note that the electron are diffusing towards the $+x$ direction, hence the negative current.

(b) Compute the average velocity of an electron at $x = 0$ (with the correct sign. **Hint:** the current density if proportional to the average velocity $J_n(x) = -qn(x)\nu_n(x)$).

Solution. We simply equate the definition of current density with the result computed above (note that this possible only because we are assuming that diffusion current is the only form of current active in this situation). Then,

$$J_n = -qn(x)\nu_n(x) \implies \nu_n(x) = \frac{J_n}{-qn(x)}$$

At $x = 0$, we have

$$\nu_n(0) = \frac{-25}{-(1.6 \times 10^{-19} \text{ C})(10^{14} \text{ cm}^{-3})} \boxed{= 1.56 \times 10^6 \text{ cm/s}}$$

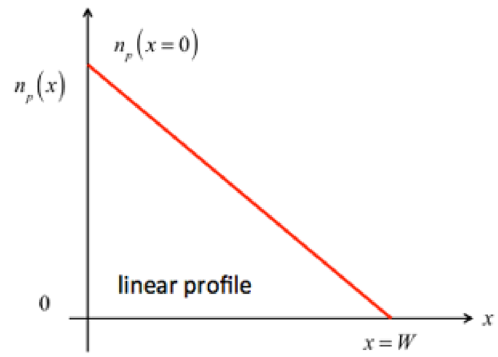

Fig.1

Problem 2 (pts) **:** Consider a uniformly doped n-type silicon resistor with $N_D = 3 \times 10^{17}\ \text{cm}^{-3}$. Suppose the resistor is 10 µm long, that the temperature is $T = 300$ K, and that the electron mobility is $\mu_n = 500\ \text{cm}^2/\text{V}\cdot\text{s}$. Suppose further that 1.5 V is applied at $x = 10$ µm and $x = 0$ µm is grounded.

(a) What the average electron velocity in the $+x$ direction in units of cm/s?

Solution. We assume that the electric field is uniform throughout the length of the resistor. Then,

$$\nu_n = -\mu_n \mathscr{E} = -\mu_n\left(-\frac{V}{L}\right) = -(500\ \text{cm}^2/\text{V}\cdot\text{s})\left(\frac{1.5\ \text{V}}{10 \times 10^{-4}\ \text{cm}}\right)$$
$$\boxed{= 7.5 \times 10^5\ \frac{\text{cm}}{\text{s}}}$$

(b) What is the direction of the electron drift current?

Solution. The current flows in the $-x$ direction.

Problem 3 (pts) **:** Consider the equilibrium energy band diagram shown in **Fig.2** below for a silicon semiconductor at $T = 300$ K and for which $n_i = 10^{10}\ \text{cm}^{-3}$. The semiconductor is doped p-type with $N_A = 10^{18}\ \text{cm}^{-3}$ so that $p_B = 10^{18}\ \text{cm}^{-3}$ in the bulk region where $x \gg 0$.

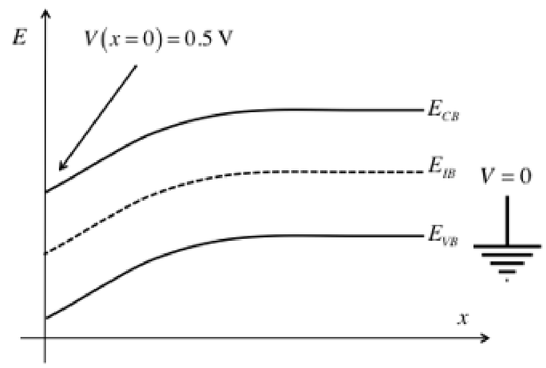

Fig.2

(a) What is the equilibrium electron density at $x = 0$?

Solution.

$$n(0) = n_B e^{qV/k_BT} = \frac{n_i^2}{p_B} e^{qV/k_BT} = (100\ \text{cm}^{-3})e^{0.5/0.026}\ \boxed{\approx 2.2 \times 10^{10}\ \text{cm}^{-3}}$$

(b) What is the equilibrium hole density at $x = 0$?

Solution.

$$p(0) = \frac{n_i^2}{n(0)}\ \boxed{\approx 4.5 \times 10^{9}\ \text{cm}^{-3}}$$

(c) What is the space charge density in C/cm^3 at $x = 0$?

Solution. Using the charge neutrality equation, we have

$$\rho(0) = q(p(0) - n(0) - N_A) \approx -qN_A\ \boxed{= -0.16\ \text{C/cm}^3}$$

Problem 4 (pts) : Consider a PN junction whose p-type side is doped with $N_A = 10^{15}\ \text{cm}^{-3}$ and whose n-type side is doped with $N_D = 10^{18}\ \text{cm}^{-3}$. Assume that the temperature $T = 300$ K.

(a) Suppose that the junction is made of silicon with $n_i = 10^{10}\ \text{cm}^{-3}$. Compute its built-in potential V_{bi}.

Solution.

$$V_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2} = (0.026 \text{ V}) \ln \frac{10^{15} 10^{18}}{10^{20}} \boxed{\approx 0.78 \text{ V}}$$

(b) Suppose that the junction is made of galium-arsenide with $n_i = 2 \times 10^6$ cm^{-3}. Compute its built-in potential V_{bi}.

Solution.

$$V_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2} = (0.026 \text{ V}) \ln \frac{10^{15} 10^{18}}{4^{12}} \boxed{\approx 1.22 \text{ V}}$$

(c) Compare your answers to the band-gap energy of the material.

Solution. The built-in voltage is in general slightly less than the band-gap energy ($E_{G,Si} = 1.12$ eV and $E_{G,GaAs} = 1.4$ eV).

Problem 5 (pts) : Consider a PN junction made of silicon with its p-type side doped with $N_A = 10^{17}$ cm^{-3} and its n-type side doped with $N_D = 10^{19}$ cm^{-3}. Assume room temperature conditions with $T = 300$ K so that $n_i = 10^{10}$ cm^{-3}. The bulk (not depleted) n-type and p-type regions are 100 nm and 200 nm long respectively. The junction area is 500×500 nm^2. The electron and hole mobilities are 1200 cm^2 V $\cdot$ s and 70 cm^2/V $\cdot$ s respectively.

(a) What the junction's saturation current I_S in A?

Solution. Because $N_D \gg N_A$, we have

$$\begin{aligned} I_S &= qA\left(\frac{D_n n_i^2}{W_n N_A} + \frac{D_p n_i^2}{W_p N_D}\right) \approx qA\frac{D_n n_i^2}{W_n N_A} \\ &= (1.6 \times 10^{-19})(500 \times 10^7)^2 \frac{(1200)(0.026)}{(100 \times 10^7)(10^{17})} \\ &\boxed{\approx 6.24 \times 10^{-19} \text{ A}} \end{aligned}$$

(b) What potential must be applied across the junction so that it conducts exactly 10^{-12} A of current?

Solution. Use the Schockley equation.

$$I_D = I_S\left(e^{qV_A/k_B T} - 1\right) \approx I_S e^{qV_A/k_B T} \implies V_A = \frac{k_B T}{q} \ln \frac{I_D}{I_S}$$

Hence, the voltage that needs to be applied is

$$V_A = (0.026\text{ V}) \ln \frac{10^{-12}}{6.24 \times 10^{-19}} \boxed{\approx 0.37\text{ V}}$$

(c) What potential must be applied across the junction so that it conducts exactly 10^{-5} A of current?

Solution. We repeat part **(b)** with the new current.

$$\boxed{V_A \approx 0.79\text{ V}}$$

Problem 6 (pts) : A diode with saturation current $I_S = 2.32 \times 10^{-15}$ A conducts 0.36 mA when forward biased by 0.67 V. What current will it conduct if the bias is increased to 0.79 V? Assume room temperature ($T = 300$ K).

Solution. Use the Schockley diode equation.

$$I_D \approx I_S e^{qV_A/k_BT}$$

If the applied voltage V_A is increased by $0.79 - 0.67 = 0.12$ V, then the current is increased by a factor of

$$e^{q(0.12)/k_BT} \approx 101.$$

Therefore, the new current will be

$$I_{S,new} = 101 \times I_S \boxed{\approx 36\text{ mA}}$$

Problem 7 (pts) : Consider the diode test circuit shown in **Fig.3** below. The load-line of the circuit and the diode's i-v characteristic are plotted together in **Fig.4**.

(a) What is the value of R?

Solution. To find the value of R, we need only look at the load-line.

$$R = \frac{\delta V}{\delta I} = \frac{3\text{ V}}{1.5\text{ mA}} \boxed{= 2\text{ k}\Omega}$$

(b) What is the value of I_D?

Solution. We need to look at the interection of the load-line and the i-v characteristic of the diode. From the figure, we read off $I_D \approx 1.2$ mA.

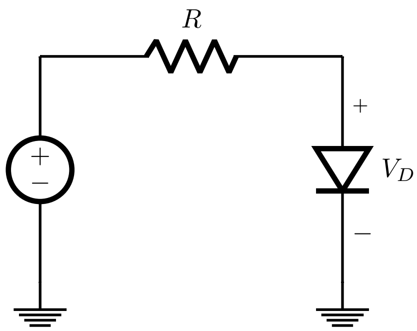

Fig.3

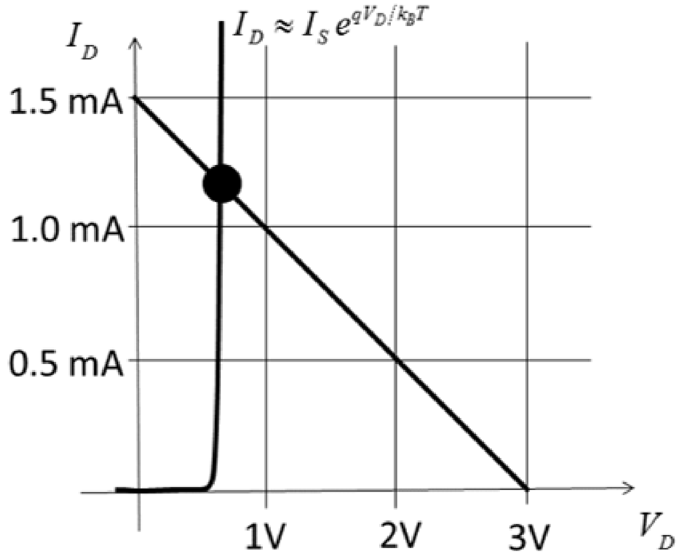

Fig.4