ECE 25500: Homework III

Diodes and Diode Circuits

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Due on: Sep. 13th, 2019 *by* 5:00 PM

Note: Scan your work (there is a scanner in the EE computer lab for student use) and submit it on Blackboard by the deadline indicated above. Late homework is **not** accepted. Make sure that the scan is readable. Please email the course GTA at `rchatrie@purdue.edu` if you have any questions about this assignment.

The reason this homework appears lengthy is that a lot of expository information has been included in order to motivate the problems. The problem themselves are not lengthy, and they are of standard difficulty for a homework assignment in this course.

Problem 1 (15 pts) : Before you can confidently design and build circuits around diodes, you must be able to analyze circuits (i.e. calculate circuit voltages and currents) that contain them. In class, you have seen the following three diode models, listed in order of accuracy: the ideal diodes model which assumes the diode drops no voltage in forward bias, the constant voltage drop model which assumes that the diode turns on at around 0.7 V and keeps this voltage fixed for any positive current, and the Shockley diode model which establishes the exponential i-v diode characteristic

$$I_D = I_S(e^{V_D/V_T} - 1).$$

In this problem, you will use each model successively to analyze simple diode circuits. While doing this, make sure to get an idea of how easy it is to apply each model, and at the end, compare the results you obtain for each one.

Consider the circuits shown in figure **Fig.1** on the next page. Assume that the diodes do not experience reverse bias breakdown at these voltages. For each of the circuits **(a)** and **(b)**, compute the values of the indicated voltages and currents (V_a, V_b, I_a and I_b)

(a) using the ideal diode model with $V_D = 0$ V in forward bias.

Solution. (Circuit **(a)**) Start by assuming both diodes are forward biased. Since D_1 is forward biased, it drops 0 V, and the anode of both diodes are at 0 V. We calculate the current through the 12 kΩ resistor

$$\frac{3-0}{12} = 0.25 \text{ mA}.$$

Since diode D_2 is forward biased as well, it drops 0 V and $V_a = 0$ V. Then, the current through the 6 kΩ resistor must be

$$\frac{0-(-3)}{6} = 0.5 \text{ mA}.$$

By Kirchhoff's current law, we compute

$$I_a = 0.25 - 0.5 = -0.25 \text{ mA}$$

which contradicts our assumption that D_1 is forward biased. As a revision, simply assume that D_1 is actually reverse biased while D_2 is forward biased. Then,

$$I_a = 0 \text{ A}$$

and

$$V_a = \frac{6}{12+6}(3-(-3)) - 3 = -1 \text{ V}.$$

Since D_2 drops no voltage, its anode is also at -1 V, and we have verified that D_1 is reverse biased. Hence,

$$\boxed{I_a = 0 \text{ mA} \quad \text{and} \quad V_a = -1 \text{ V}}$$

(Circuit **(b)**) Start by assuming that both diodes are forward biased. Both drop 0 V and both of their anodes are at 0 V. Then, the current through the 6 kΩ resistor is

$$\frac{3-0}{6} = 0.5 \text{ mA}.$$

We also have

$$V_b = 0 \text{ V}.$$

Then, the current through the 12 kΩ resistor is

$$\frac{0-(-3)}{12} = 0.25 \text{ mA}.$$

Using Kirchhoff's current law,

$$I_b = 0.5 - 0.25 = 0.25 \text{ mA}.$$

Hence, both diodes are indeed forward biased and the solution is

$$\boxed{V_b = 0 \text{ V} \quad \text{and} \quad I_b = 0.25 \text{ mA}}$$

(b) using the constant voltage drop diode model with $V_D = 0.7$ V in forward bias.

Solution. The analysis in the previous part using the ideal diode model does not produce very accurate results, but it should correctly determine which diodes conducts and which don't, in general. Hence, we should assume that the same diodes conduct in this part.

(Circuit **(a)**) Assume that D_1 is off and D_2 is on. Then, the voltage drop across D_2 is 0.7 V and D_1 is open. We can solve for V_a.

$$V_a = \frac{6}{12+6}(3-0.7-(-3)) - 3 \approx -1.23 \text{ V}.$$

We need to verify that D_1 is indeed reverse biased. Its anode is at

$$-1.23 + 0.7 = -0.53 \text{ V}$$

and must be reverse biased. Therefore, the solutions is

$$\boxed{V_a = -1.23 \text{ V} \quad \text{and} \quad I_a = 0 \text{ A}}$$

(Circuit **(b)**) Assume that both diodes conduct and are forward biased. Then, their anodes are at 0.7 V. Hence,

$$V_b = 0 \text{ V}.$$

The current through the 12 kΩ resistor is

$$\frac{0-(-3)}{12} = 0.25 \text{ mA}$$

The current through the 6 kΩ resistor is

$$\frac{3-0.7}{6} \approx 0.38 \text{ mA}.$$

Using Kirchhoff's current law

$$I_b = 0.38 - 0.25 = 0.13 \text{ mA}.$$

Since both diodes are indeed forward biased here, the solution is

$$\boxed{V_b = 0 \text{ V} \quad \text{and} \quad I_b = 0.13 \text{ mA}}$$

(c) using LTspice and selecting the `1N4148` diode model. To place a diode, simply press **D** on your keyboard. Right click on the diode and click on **Pick New Diode**. In the menu, select the desired diode model.

Solution. See the screenshots in **Fig.S1** and **Fig.S2** below for the DC Operating Point simulation results for both circuits.

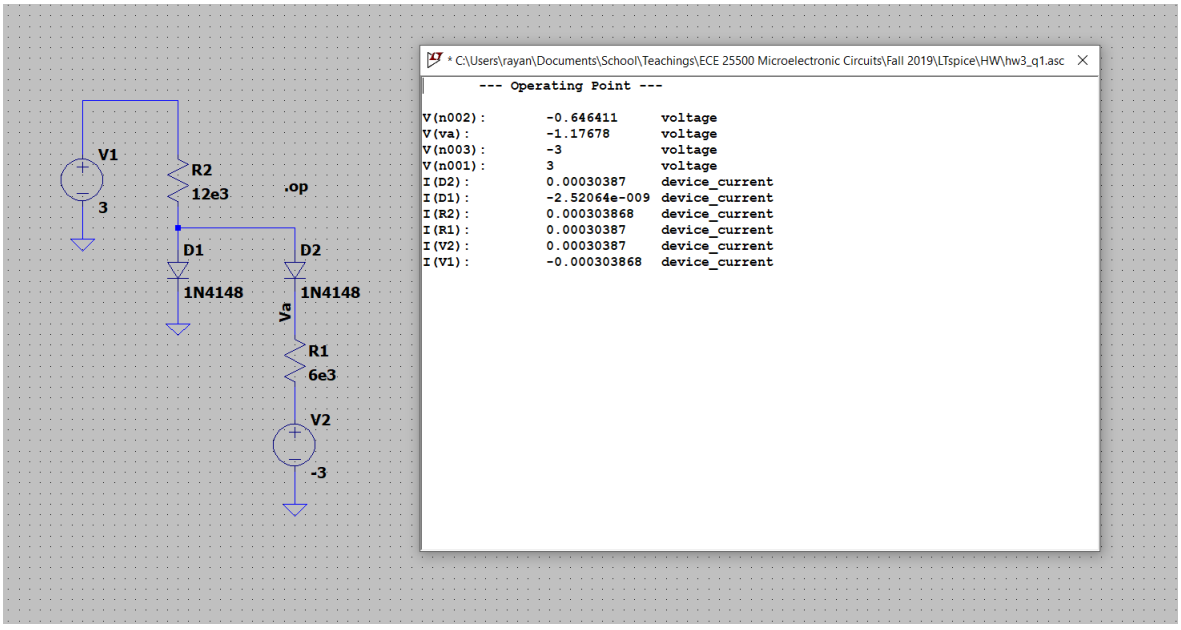
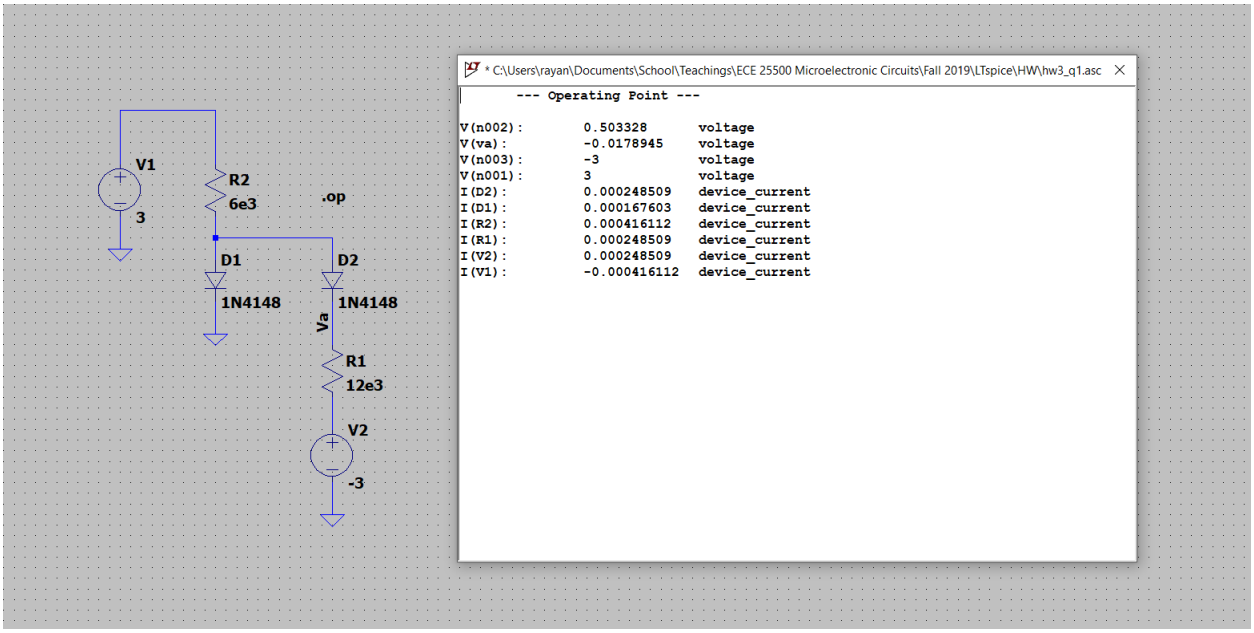


Fig.S1



Circuit **Fig.S2**

Conclusion. Let's take circuit **(a)**. Let's suppose that the LTspice simulation results are the most accurate ones, and compare the results from either hand analysis to them. Using the ideal diode model gives 0% error in I_a and 15% error in V_a, whereas the constant voltage drop model gives 0% error in I_a and 4.5% error in V_a. For no extra complexity, the constant voltage drop model significantly improves our predictions.

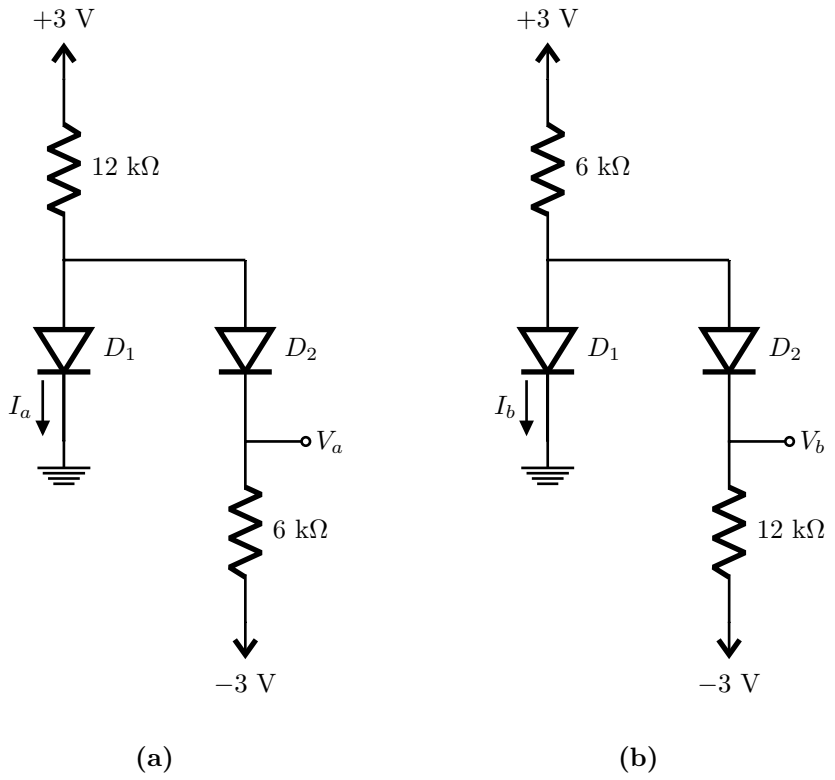


Fig.1 - Simple Diode Circuits.

Problem 2 (20 pts) : Many circuit applications of the diode are related to power regulation and signal rectification in which voltages are usually quite high (e.g. 120 V AC at the output of the wall socket in the U.S.). In these applications, the ideal diode model, which ignores any voltage drop across the diode, is adequate. However, as you have seen in lecture and in the previous problem, the constant voltage drop model is not much harder to use, yet the gains in accuracy when we are dealing with low voltages are significant. In fact, the constant voltage drop model is the recommended model to use for pen and paper analyses. In this problem, you will practice using this model a little more.

Consider the circuit shown in **Fig.2** on the next page. Assume that the diodes do not experience reverse bias breakdown at these voltages. Using the constant voltage drop model for the diode with $V_D = 0.7$ V in forward bias, answer the following two questions.

(a) Let R be a 5 kΩ resistor. What is the value of V?

Solution. Given that $R = 5$ kΩ and that diode D_1's anode is at potential V, it is most sensible to guess that D_1 is off while D_2 and D_3 are on. We can already see that if R is increased, the voltage at node V will increase, and there will some point at which diode D_1 will have to turn on. With our assumptions, we have the equivalent circuit shown in **Fig.S3** below. Hence, (I am using superposition here)

$$V = \frac{5||10}{5||10 + 10} \cdot (10 - 0.7) \cdot (2) = 4.65 \text{ V}$$

Indeed, diode D_1 is off. Therefore, the result is

$$\boxed{V = 4.65 \text{ V}}$$

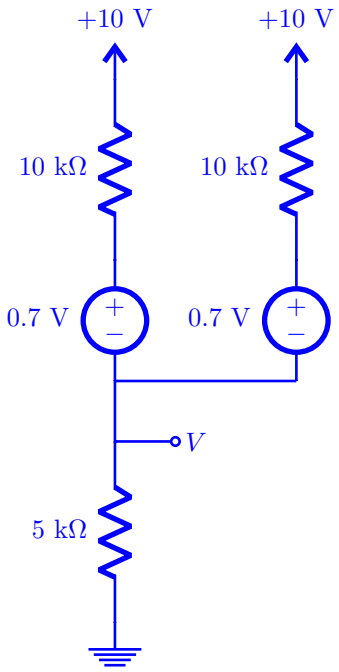


Fig.S3

(b) For what range of resistance values on R is the diode D_1 forward biased? Before going through any math, you should think about the question and attempt to visualize the answer. For example, starting from the situation in the previous question, where $R = 5$ kΩ, think about what happens when R is increased or decreased. Being able to understand and predict a circuit's operation without using anything but your mind should be one of your goals out of this course (although we won't test you on that).

Solution. As we have already established, V will increase as R increases. Hence, the diode is off from 0 Ω up to some upper bound. This upper bound is the minimum value of R such that diode D_1 is conductive. Diode D_1 becomes conductive when 0.7 V is applied in forward bias across it. As long as the diode is off, its cathode stays at +5 V. We need to figure out the value of R such that the anode is at 5.7 V. To visualize what needs to be done, in **Fig.S3**, suppose that V is known to be 5.7 V, but R is not known. What value for R satisfies the circuit? We have the following equation (again, using superposition here)

$$5.7 = \frac{R||10}{R||10 + 10} \cdot (10 - 0.7) \cdot (2)$$

Hence,

$$R||10 \approx 4.42 \text{ kΩ}$$

And then, we need

$$R \approx 7.92 \text{ kΩ}$$

Therefore, the range of resistance values such that the diode D_1 is forward biased is

$$\boxed{R > 7.92 \text{ kΩ}}$$

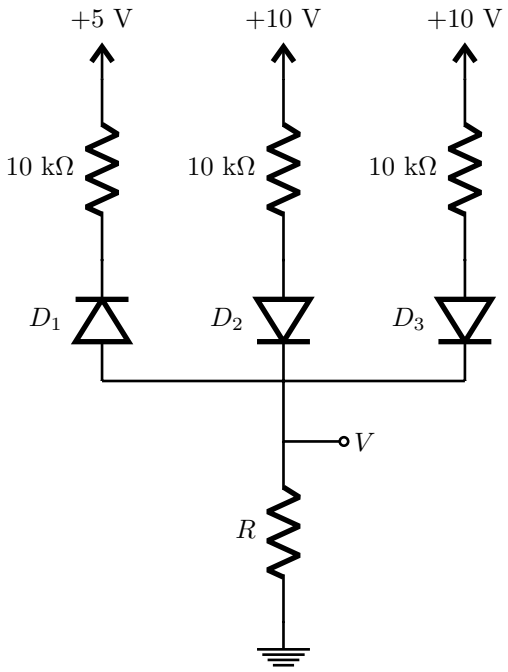


Fig.2 - A More Complex Diode Circuit.

Problem 3 (15 pts) : As is the case with any mathematical model used in engineering, it is important to be able to recognize when the model should and should not be applied. Although the constant voltage drop model is the preferred model for almost all diode circuits, it can fail to produce meaningful, even yet, accurate, results when applied onto some circuits. One example of a pathological circuit is that shown in **Fig.3** below. Generally, when a specific current value is forced through a diode, you should not use the constant voltage drop model. The reason for this is that this model gives no information about the current through the diode, and simply let's it be anything. It gives us no understanding of how current flows through the circuit. If all we know about a diode in a circuit is the value of the current through it, we need to use the more informative Shockley diode model.

Consider once again the circuit shown in **Fig.3** below. Suppose that the junction area of diode D_1 is 10 times that of diode D_2.

(a) Using the constant voltage drop model with $V_D = 0.7$ V, determine the value V. Do you think this is an accurate result, given the different currents that flow through each diode, and the fact that they differ in junction area?

Solution. Clearly, both diodes must be conducting in this circuit. Using the constant voltage drop model, we quickly establish that the anode of both diodes are at 0.7 V and that $V = 0$ V. However, this analysis says that regardless of the diodes' junction areas and drive current, they will have the exact same potential drop across them. This is clearly uninformative.

(b) Determine the value V this time using the Shockley diode model.

Solution. Using Kirchhoff's current law, the current through diode D_1 is

$$I_{D_1} = 10 - 3 = 7 \text{ mA}.$$

Then, using the Shockley diode model,

$$V_{D_1} = \frac{k_B T}{q} \ln \frac{I_{D_1}}{I_{S_1}} = V_T \ln \frac{I_{D_1}}{I_{S_1}}$$

and

$$V_{D_2} = \frac{k_B T}{q} \ln \frac{I_{D_2}}{I_{S_2}} = V_T \ln \frac{I_{D_2}}{I_{S_2}}$$

We establish the value of V with

$$V = V_{D_1} - V_{D_2} = V_T \ln \frac{I_{D_1}}{I_{S_1}} - V_T \ln \frac{I_{D_2}}{I_{S_2}}$$

$$\implies V = V_T \ln \frac{I_{D_1}/I_{S_1}}{I_{D_2}/I_{S_2}}$$

Now, since the junction area of diode D_1 is 10 times larger than that of diode D_2, its saturation current must also be about 10 times larger. Therefore, we can compute the value of V using the above equation.

$$\boxed{V = (26 \text{ mV}) \ln \frac{7}{3 \cdot 10} \approx -38 \text{ mV}}$$

(c) In order to obtain a value of V of 60 mV, what should be the current of the 3 mA source set to instead? Once again, you should think about and visualize the answer in your head, and the mathematics should be performed as a verification of your intuition, which you are building.

Solution. In order to bring V to 60 mV, we need to significantly reduce the voltage across diode D_2. We expect that the current will have to be reduced. We can start with the equation used in the previous part.

$$V = V_T \ln \frac{I_{D_1}/I_{S_1}}{I_{D_2}/I_{S_2}}$$

We solve it for I_{D_2} over I_{D_1}.

$$\frac{I_{D_2}}{I_{D_1}} = \frac{I_{S_2}}{I_{S_1}} \cdot \frac{1}{e^{V/V_T}}$$

and therefore,

$$\frac{I_{D_2}}{I_{D_1}} = 0.01$$

Since

$$I_{D_1} + I_{D_2} = 10,$$

We have

$$\boxed{I_{D_2} \approx 0.1 \text{ mA}}$$

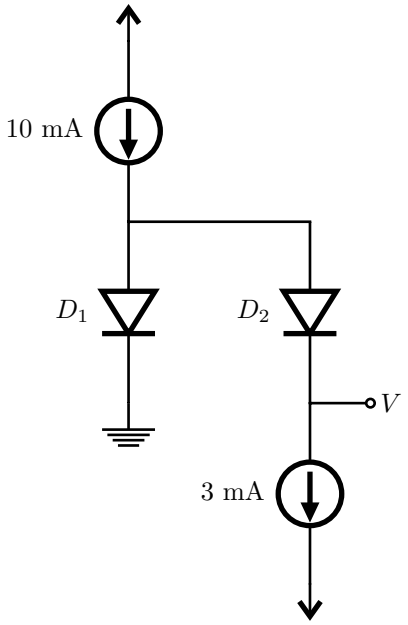


Fig.3 - Circuit for which $V_D = 0.7$ V Cannot be Assumed.

Problem 4 (20 pts) : The most prevalent application of the diode in today's circuits is that of constraining a given voltage signal to a certain voltage range. A popular and easily understood example is the electrostatic discharge (ESD) protection circuit. ESD occurs when a charged object is brought in contact with a circuit lead and quickly discharges through it. Although ESDs never carry any large amounts of power, the short duration, high current pulse that results from them is usually enough to destroy any IC that isn't adequately protected. In fact, if you look at the circuit schematic for any I/O pin of a microcontroller for example, you'll see at least two diodes there that serve as ESD protection. Diodes circuits used for the purpose of ESD protection are usually called **voltage clippers** (the diodes "clip" the voltage pulses to prevent any large current from flowing). Another voltage constraining application of the diode is in the **voltage clamp circuits**. The voltage clamp prevents the voltage from rising above (or below) a certain value, the same way voltage clippers do. The different name arises from the fact that clamps are usually used to prevent failure of one part of circuit from affecting another, especially when that failure results in a short circuit to the supply rail. Finally, another popular application is that of **voltage limiter circuits**. Once again, the operating principle is identical to that in the voltage clamps and voltage clippers, but the application is different. With voltage limiters, we want to keep the amplitude of a voltage signal below some threshold. We usually want to do this at the input of a high gain amplifier which drives sensitive components. Since the gain is high, to avoid failure, we must keep the input voltage swing low.

Let's investigate the behavior of a few voltage clamps and limiters. The **DC transfer characteristic** plot is a very useful tool to understand the behavior these types of circuits. In this problem, you'll be asked to draw one for each of the following circuits.

(a) Consider the voltage limiter circuit shown in **Fig.4** below. Model each diode with the constant voltage drop model using $V_D = 0.7$ V. Plot (using a straight edge) the DC voltage transfer characteristic from V_{in} to V_{out} where V_{in} ranges from -10 V to $+10$ V. Clearly annotate the plot. Write one sentence describing the limiting action of the circuit.

Solution. See **Fig.S4**. The limiter truncates the output voltage such that it remains within the $(-0.7\text{ V}, 0.7\text{ V})$ range.

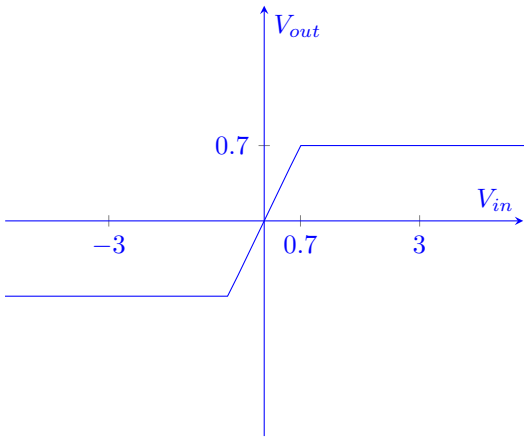


Fig.S4

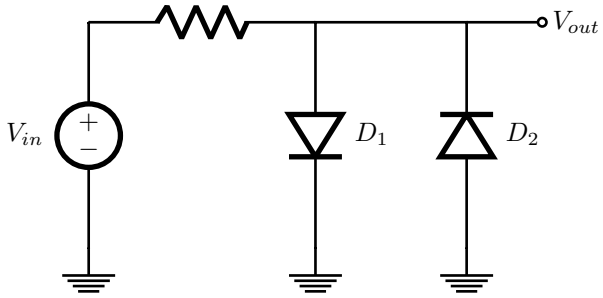


Fig.4 - A simple voltage limiter.

(b) Consider the voltage clamp circuit shown in **Fig.5** below. Model the diode with the constant voltage drop model using $V_D = 0.7$ V. Again, plot (using a straight edge) the DC voltage transfer characteristic from V_{in} to V_{out} where V_{in} ranges from -10 V to $+10$ V. Clearly annotate the plot. Write one sentence describing the clamping action of this circuit. Why do you think the resistor shown in series with the voltage V_{in} is necessary here?

Solution. See **Fig.S5**. The clamp clamps any voltage higher than 5.7 V down to 5.7 V. The resistor is necessary to limit the current flowing through the clamping diode. If it were not here, increasing the input voltage beyond 5.7 V would lead to an exponential increase in the diode current.

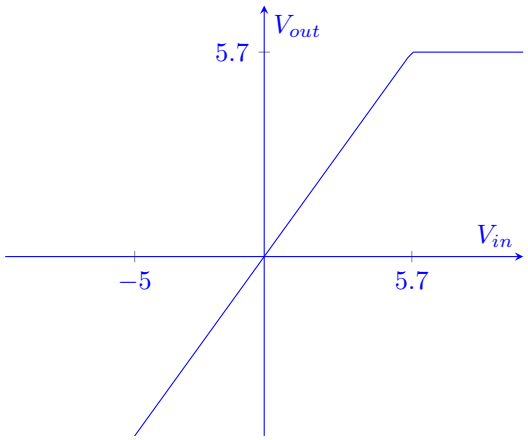


Fig.S5

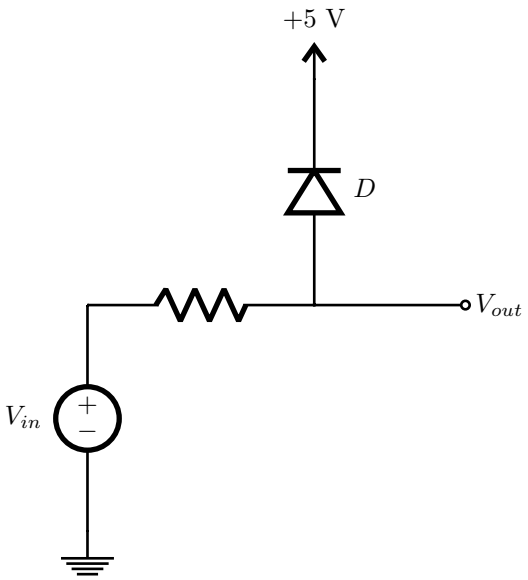


Fig.5 - A simple voltage clamp.

(c) The voltage clamp in **Fig.5** above ensures that the voltage at V_{out} stays below about 5.7 V. Note that we had to use a voltage reference of +5 V to achieve this clamping voltage. Sometimes, we want another voltage reference, one that is not immediately available from the voltage supplies in the circuit. We can create a reference (not a great one) using a simple voltage divider, as shown in **Fig.6** below. Suppose that we want a voltage reference of 3 V and that we want to keep the current flowing through the voltage divider (when the diode is off) at 1 mA. What value for R_1 and R_2 are needed? Plot (using a straight edge) the DC voltage transfer characteristic from V_{in} to V_{out} where V_{in} ranges from −10 V to +10 V. Clearly annotate the plot. Also, what are the slopes of each lines in your plot?

Solution. We need a reference of +3 V. Hence,

$$\frac{R_2}{R_1 + R_2} = \frac{3}{5}.$$

Additionally, we need the current through the divider to be 1 mA. Hence,

$$\frac{5}{R_1 + R_2} = 1 \implies R_1 + R_2 = 5 \text{ k}\Omega$$

Then,

$$\frac{R_1}{5} = \frac{3}{5} \implies \boxed{R_1 = 3 \text{ k}\Omega \implies R_2 = 2 \text{ k}\Omega}$$

Note that the presence of the voltage divider affects the DC transfer characteristic in a significant way. V_{out} is no longer clamped at $3 + 0.7$ V. There is now another voltage division action that takes into account the 1 kΩ series resistance and that allows V_{out} to increase linearly passed the clamping voltage. See **Fig.S6** below. The slope of the overshoot line is simply the resistance divider ratio

$$\boxed{\frac{R_1||R_2}{R_1||R_2 + 1} \approx 0.55}$$

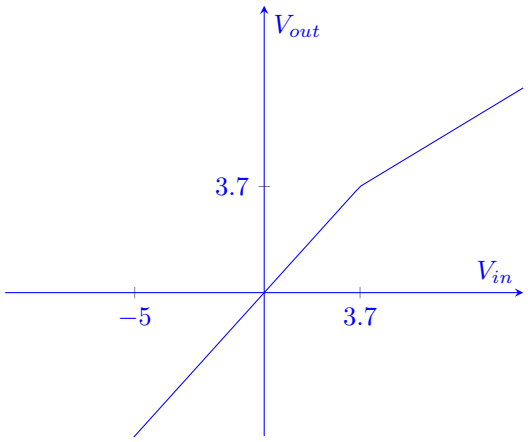


Fig.S6

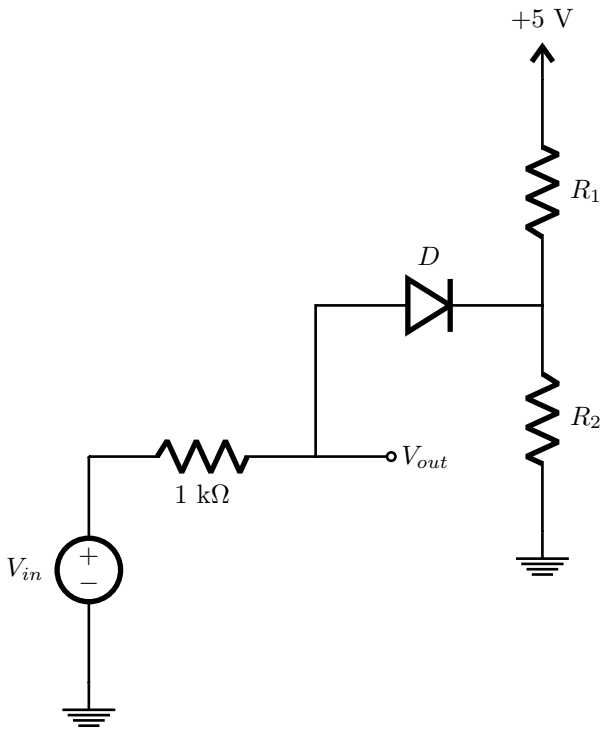


Fig.6 - A simple voltage clamp with modified reference voltage.

(d) Let's have a look at one last voltage limiter circuit which uses zener diodes this time. Consider the voltage limiter shown in **Fig.7** below. Model the diode with the constant voltage drop model in forward bias using $V_D = 0.7$ V. Suppose the zener diode has reverse breakdown voltage $V_{ZK} = 7$ V. Plot (using a straight edge) the DC voltage transfer characteristic of this limiter from V_{in} to V_{out} where V_{in} ranges from -10 V to 10 V. Clearly annotate your plot.

Solution. See **Fig.S7**. Note that as V_{in} becomes lower, the zener diode conducts in the forward direction at -2.7 V, before the regular diode.

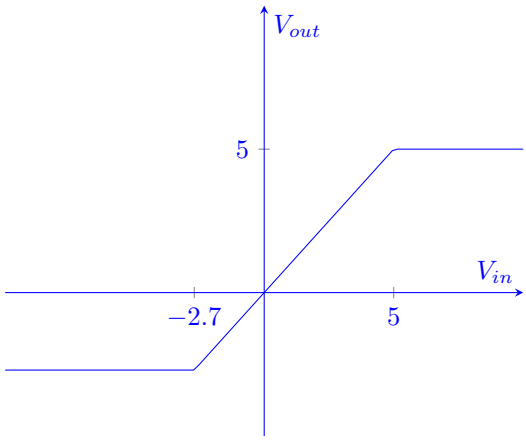


Fig.S7

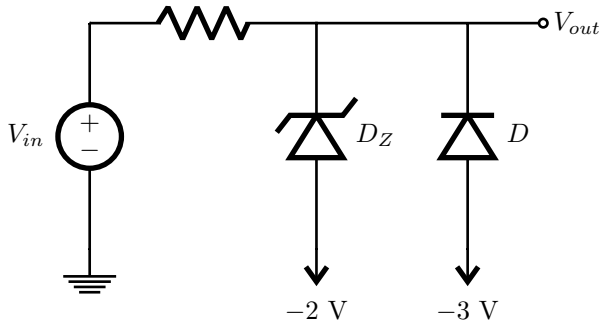


Fig.7 - A voltage limiter using zener diodes.

Problem 5 (15 pts) : This problem is taken from last semester's exam I. Consider the circuit shown in **Fig.8**. Model each diode with the constant voltage drop model using $V_D = 0.7$ V. What is the current I_1? Although this problem is multiple choice, you need to show your work.

(a) 0 mA

(b) 0.502 mA

(c) 1.00 mA

(d) 1.46 mA

(e) 1.60 mA

(f) 2.02 mA

(g) 12.1 mA

Solution. Given the supplies present in the circuit, it is sensible to guess that diode D_1 is on, while diode D_2 is off. Hence, the voltage at the anode of the diodes is $2 + 0.7 = 2.7$ V, and

$$I_1 = \frac{10 - 2.7}{5} = 1.46 \text{ mA}$$

Diode D_2's cathode is at 10 V, hence it is clearly off. The answer is **(d)**.

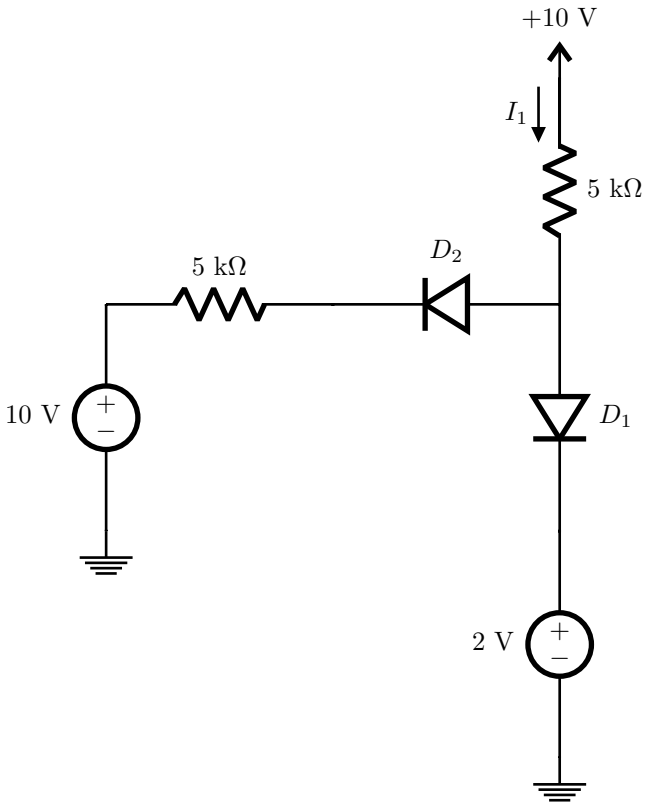


Fig.8 - Past exam I question.

Problem 6 (15 pts) : Let's continue with another easily understood application of diodes in circuits, namely, diode gates. As it turns out, one can very simply implement non-inverting OR and AND gates using diodes, commonly referred to as **Diode Logic** gates. In the early days of computing circuitry, when the bulky vacuum tube was still the main computing device, these diode logic gates were often incorporated into vacuum tube logic circuits to create **Diode-Transistor Logic** circuits. However, the use of these circuits is very limited, due mainly to the fact that voltage logic levels are significantly altered across diode logic gates, which leads to issues when one tries to cascade them. Rather soon after the invention of the bipolar junction transistor (BJT) and **Transistor-Transistor Logic** circuits, diode logic went out of use in computing.

Assume that the diodes do not experience reverse bias breakdown at these voltages. Use the constant voltage drop model with $V_D = 0.7$ V to answer the following questions.

(a) Consider the circuit shown in **Fig.9** below, in which two diode OR gates are cascaded (have a look at section 4.1.3 in the textbook for a quick overview of diode logic gates). Suppose, in this first scenario, that $V_{in} = +1$ V (`LOW`). What is the output voltage V_{out}?

Solution. In **Fig.9** I have labelled the internal node V_i. When $V_{in} = +1$ V, the internal node $V_i = 0.3$ V. In the second logic stage, only the diode on top conducts, and $\boxed{V_{out} = 0.3 \text{ V}}$

(b) Suppose now that $V_{in} = +5$ V (`HIGH`). What is the output voltage V_{out}? Note that this output voltage value represents logic level `HIGH`.

Solution. At $V_{in} = +5$ V, $V_i = 4.3$ V. Then, $\boxed{V_{out} = 3.6 \text{ V}}$

(c) Let's go one step further and take into account the series resistance R_s at the voltage source V_{in} that is surely present (not explicitly shown in the figure). What is the output voltage V_{out} in terms of R_s with $V_{in} = +5$ V? Again, this output value is supposed to be logic level `HIGH`.

Solution. The presence of the source series resistance adds a voltage division effect. Assuming correct operation of the diode gates, we have the equivalent circuit shown in **Fig.S8** below. Solving this circuit for V_{out} in terms of R_s, we have

$$V_{out} = \left(\frac{4.3}{R_s + 1} - 0.7\right)\left(\frac{1}{1 + R_s||1}\right)$$

You can obtain the above by computing the Thévenin equivalent of the first half of the circuit and then applying voltage division. As an example of the consequences of this, suppose that $R_s = 0.5$ kΩ. Then, we calculate $V_{out} \approx 1.63$ V.

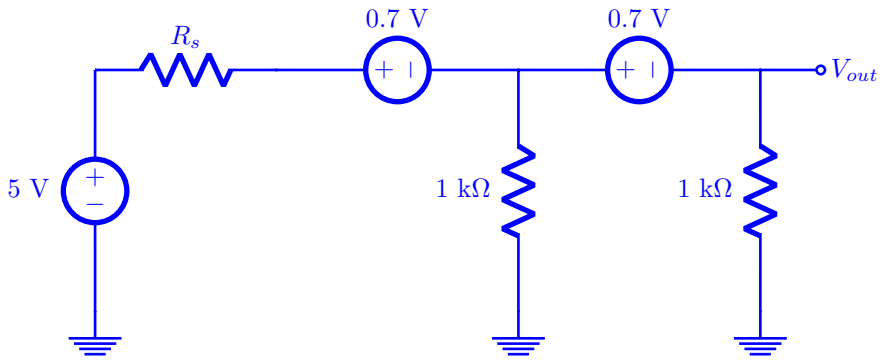


Fig.S8

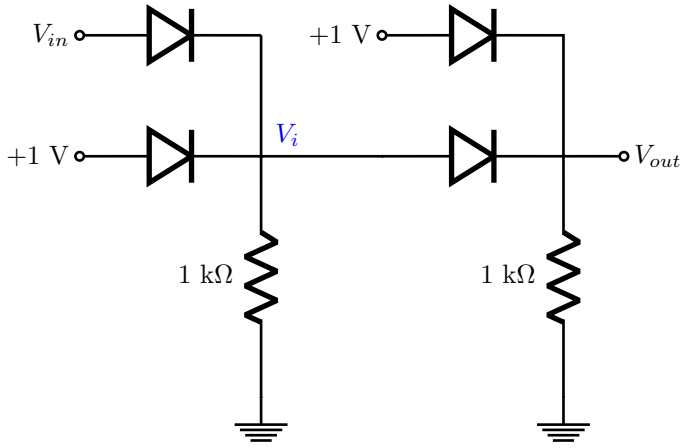


Fig.9 - Cascaded OR Diode Gates.

Bonus problem (+10 pts) : This problem is a bonus problem that will count as extra credit on this assignment. Although diode logic has fallen out of favor over half a century ago, diode gates still find some applications even in today's circuits. One example is the **battery backup** circuit shown in **Fig.10**. Any system that needs to keep track of real time, such as your laptop, is equipped with a **Real Time Clock (RTC)** on its motherboard. The RTC must always be powered, such that the system never loses track of real time (we understand this to mean "human" time here). When the system is active, the RTC is supplied with the system power supply (represented by the +5 V supply in **Fig.10**). When the system is inactive, the RTC is supplied by a battery cell (represented by the +3 V battery supply in **Fig.10**). The function of the diode gate in this case is to select the higher of the two voltages without affecting the lower one. Let's evaluate quickly how well this circuit performs. The real time clock shall be the `NXP PCF8563` IC, and the +3 V battery shall be `CR2032` coin cell. Suppose that the system is active on average 2 hours in a day. How long is the life span of the battery backup circuit? You will need to track down the necessary information (such as the battery's energy density) on the web.

Solution. The coin cell battery supplies the chip through a diode. Hence, there is a 0.7 V drop. We can expect the voltage at the supply terminal to be around 2.3 V when the battery is supplying the chip. We need to determine how much current the chip draws at that current. Assuming room temperature, we look at the **Static Characteristics** table in the RTC documentation and find that the chip draws 225 nA of current at 2.0 V. Having a look at the documentation for the coin cell battery, we find that its capacity is of 235 mA h. Thus, ignoring the battery's self discharge, the battery will be able to supply the desired current for $235/(225 \times 10^{-6}) \approx 1.04 \times 10^6$ h = 120 years. Since the system is active 2 hours in a day, the battery is in use 22 hours, or 92% of the time. Hence, the circuit will have a life span of $120 \times 1/0.92 =$ 130 years

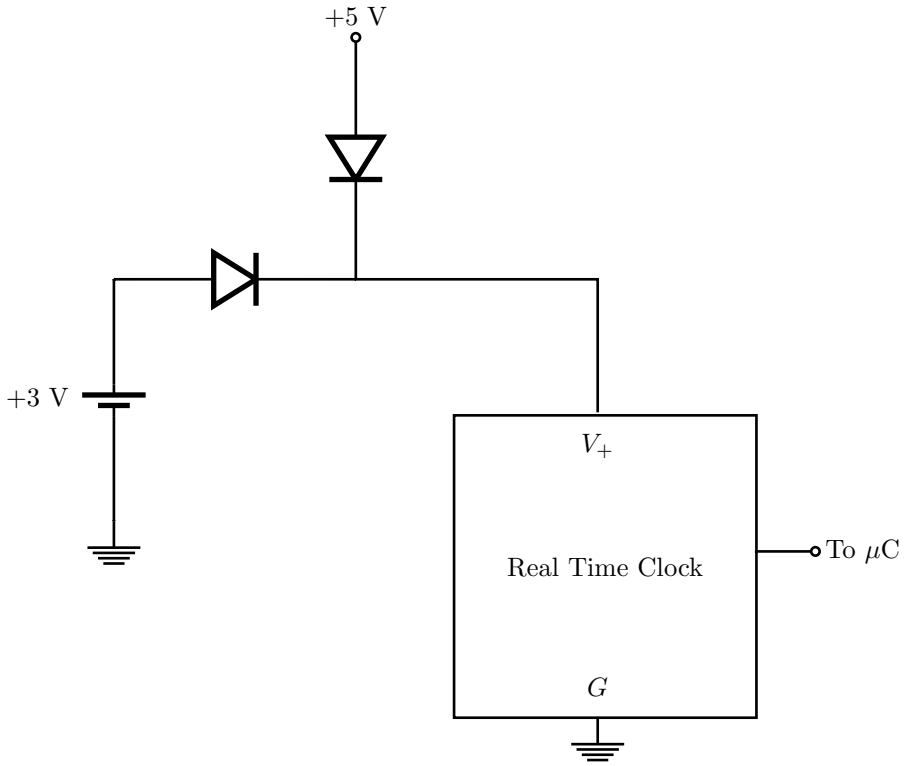


Fig.10 - Battery Backup Circuit.