

ECE 25500: Homework I

Fundamentals of Semiconductor Device Modelling

Due on: Aug. 31st, 2019 by 5:00 PM

Note: Scan your work (there is a scanner in the EE computer lab for student use) and submit it on Blackboard by the deadline indicated above. Late homework is **not** accepted. Make sure that the scan is readable. Please email the course GTA at rchatric@purdue.edu if you have any questions about this assignment.

Problem 1 (Review) (10 pts) : Before attempting this problem or the next, you should review the basics of Thévenin's theorem and equivalent circuits. Note that our use of Thévenin's theorem will be **very frequent** throughout the semester. Consider the sub-circuit shown in **Fig.1** below. What is its Thévenin equivalent resistance R_{th} ?

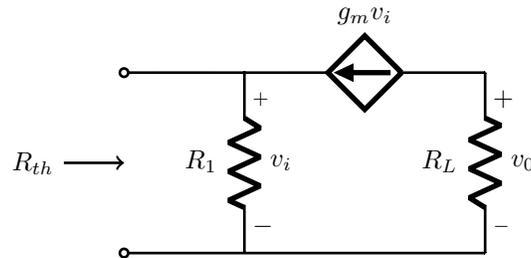
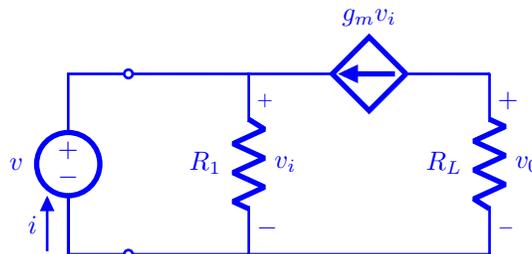


Fig.1 - A Sub-circuit with a Controlled Current Source.

Solution: Apply a test source v . Then, the desired resistance is given by

$$R_{th} = \frac{v}{i}.$$



Clearly,

$$v = R_1(i + g_m v) \implies v(1 - g_m R_1) = R_1 i$$

Therefore,

$$R_{th} = \frac{v}{i} = \frac{R_1}{1 - g_m R_1}$$

Problem 2 (Review) (10 pts) : Consider the sub-circuit shown in **Fig.2** below.

- (a) Suppose that $R_1 = R_2 = \dots = R_5 = 2 \text{ k}\Omega$. What is the Thévenin equivalent resistance R_{th} of this sub-circuit? (**Hint**: the desired value of R_{th} can be obtained by inspection in this symmetrical case).
- (b) Suppose that $R_1 = R_2 = R_3 = R_4 = 2 \text{ k}\Omega$ and $R_5 = 1 \text{ k}\Omega$. What is the Thévenin equivalent resistance R_{th} of this sub-circuit?

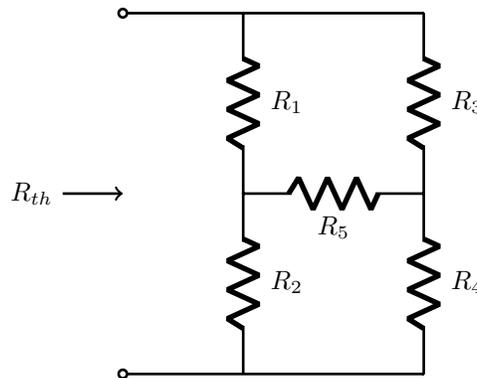


Fig.2 - A Passive Sub-circuit.

Solution: (a) This configuration of resistance values exhibits symmetry. There is no reason one branch $R_1 - R_2$ be any different from the other $R_3 - R_4$. Hence, there will be no voltage across resistor R_5 , and it can be ignored. Therefore,

$$R_{th} = (R_1 + R_2) \parallel (R_3 + R_4) = 2 \text{ k}\Omega$$

(b) The symmetry is still present. In fact, the result will be the same as above:

$$R_{th} = 2 \text{ k}\Omega$$

Problem 3 (15 pts) : The intrinsic carrier concentration parameter n_i is an important parameter for a semiconductor device. An approximate expression for n_i is given below.

$$n_i = BT^{(3/2)} e^{(-E_G/2k_B T)},$$

where T is the temperature of the semiconductor in Kelvins (K), E_G is the material's bandgap energy in electron-volt (eV), and k_B is Boltzmann's constant ($1.38 \times 10^{-23} \text{ J/K}$). Suppose a semiconductor device is made out of silicon (Si), for which $B = 4.87 \times 10^{15} \text{ cm}^{-3} \text{ K}^{-3/2}$ and $E_G = 1.12 \text{ eV}$ (assume that E_G is independent of temperature for this problem).

- (a) Compute n_i at room temperature ($T = 27^\circ\text{C}$) using the expression above for this device.
- (b) Suppose that the temperature T in Celsius of the device is raised by 10%. By what percentage does the intrinsic carrier concentration n_i in cm^{-3} increase?
- (c) Suppose that the temperature is brought back down to room temperature, and that the bandgap energy E_G in Joules of the material is increased by 10%. By what percentage does the intrinsic carrier concentration n_i in cm^{-3} increase?
- (d) Doped semiconductor devices usually fail to operate as designed when n_i becomes comparable to the doping density. Suppose our silicon device is doped with $N_D = 10^{17} \text{ cm}^{-3}$. Around what temperature would the intrinsic carrier concentration reach a tenth of the doping concentration?

Solution: (a) The temperature is given as $T = 273.15 + 27 = 300.15 \text{ K}$. Since E_G is given in units of eV, we should convert k_B in those units as well.

$$k_B = 1.38 \times 10^{-23} \text{ J/K} = \left(\frac{1.38 \times 10^{-23}}{1.60 \times 10^{-19}} \right) \text{ eV/K} = 8.62 \times 10^{-5} \text{ eV/K}.$$

Then,

$$n_i = (4.87 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2}) \cdot (300.15 \text{ K})^{(3/2)} \cdot e^{-(1.12 \text{ eV})/(2(300.15 \text{ K})(8.62 \times 10^{-5} \text{ eV/K}))}$$

$$n_i \approx 1.02 \times 10^{10} \text{ cm}^{-3}$$

(b) The new temperature is $T = 273.15 + (27 \times 1.1) = 302.85 \text{ K}$. Hence,

$$n_i = (4.87 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2}) \cdot (302.85 \text{ K})^{(3/2)} \cdot e^{-(1.12 \text{ eV})/(2(302.85 \text{ K})(8.62 \times 10^{-5} \text{ eV/K}))}$$

$$n_i \approx 1.24 \times 10^{10} \text{ cm}^{-3}$$

Hence, the intrinsic carrier concentration increased by

$$\frac{n_i(29.7^\circ\text{C}) - n_i(27^\circ\text{C})}{n_i(27^\circ\text{C})} \times 100\% = 24\%$$

In case you increased the temperature in Kelvin by 10%, you get $T = 300.15 * 1.1 = 330.15 \text{ K}$. Then,

$$n_i = (4.87 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2}) \cdot (330.15 \text{ K})^{(3/2)} \cdot e^{-(1.12 \text{ eV})/(2(330.15 \text{ K})(8.62 \times 10^{-5} \text{ eV/K}))}$$

$$n_i = 8.31 \times 10^{10} \text{ cm}^{-3}$$

Therefore, the intrinsic carrier concentration increased by

$$\frac{n_i(330.15 \text{ K}) - n_i(300.15 \text{ K})}{n_i(300.15 \text{ K})} \times 100\% = 715\%$$

Both computations do show the simple fact that the intrinsic carrier concentration parameter n_i is extremely sensitive to temperature. This is an important reason as to why semiconductors are doped. Their properties (e.g. conductivity) are independent of n_i (as long as the temperature is kept low), and are, in general, much more robust against temperature variations.

(c) The new bandgap energy is $E_G = 1.12 * 1.1 = 1.23 \text{ eV}$. The temperature is $T = 300.15 \text{ K}$.

$$n_i = n_i = (4.87 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2}) \cdot (300.15 \text{ K})^{(3/2)} \cdot e^{-(1.23 \text{ eV})/(2(300.15 \text{ K})(8.62 \times 10^{-5} \text{ eV/K}))}$$

$$n_i = 1.20 \times 10^9 \text{ cm}^{-3}$$

The intrinsic carrier concentration has therefore **decreased** by

$$\frac{n_i(1.12 \text{ eV}) - n_i(1.23 \text{ eV})}{n_i(1.12 \text{ eV})} \approx 88\%$$

This decrease makes sense. Since the bandgap energy is higher, more thermal energy is required for a carrier to be created. Note that the intrinsic carrier concentration is also very sensitive to the bandgap energy parameter.

(d) There are of course no exact temperature at which the device starts to fail. The device performance will instead get progressively worse as temperature is increased. To get an idea of what temperatures we are talking about, let's suppose that the device begins to fail once the intrinsic carrier concentration n_i is at least a tenth of the dopant concentration. Thus, we are interested in the temperature at which $n_i = 0.1 \times 10^{17} = 10^{16} \text{ cm}^{-3}$. Looking back at the equation we have for n_i , you should not attempt to solve it for T analytically. You should use an **iterative method** to get T in this case (the details as to why this method works are left to a math course). We start the iterative process by guessing a sensible value for T and plugging it into one of the T literal to compute the other. The expression for n_i is much more sensitive to the T parameter in the exponent rather than the one in front of the exponent. In order to converge to a solution more rapidly, we should guess the value of the T in front of the exponent. Let's guess that $T = 300.15 \text{ K}$, so chosen because this was our initial condition in the first part of this question. Let's solve the expression of n_i for T in the exponent.

$$T = \frac{-E_G}{2k_B \ln(n_i/BT^{(3/2)})}$$

The iteration proceeds as follows.

$$T(300.15 \text{ K}) = \frac{-1.12}{2(8.62 \times 10^{-5}) \ln((10^{16})/(4.87 \times 10^{15})(300.15)^{(3/2)})} \approx 829 \text{ K}$$

$$T(829 \text{ K}) = \frac{-1.12}{2(8.62 \times 10^{-5}) \ln((10^{16})/(4.87 \times 10^{15})(829)^{(3/2)})} \approx 694 \text{ K}$$

$$T(694 \text{ K}) = \frac{-1.12}{2(8.62 \times 10^{-5}) \ln((10^{16})/(4.87 \times 10^{15})(694)^{(3/2)})} \approx 714 \text{ K}$$

$$T(714 \text{ K}) = \frac{-1.12}{2(8.62 \times 10^{-5}) \ln((10^{16})/(4.87 \times 10^{15})(714)^{(3/2)})} \approx 711 \text{ K}$$

$$T(711 \text{ K}) = \frac{-1.12}{2(8.62 \times 10^{-5}) \ln((10^{16})/(4.87 \times 10^{15})(711)^{(3/2)})} \approx 712 \text{ K}$$

The solution seems to converge around $T \approx 712 \text{ K}$. Therefore, we conclude that our device starts to fail around a temperature of

$$T \approx 712 \text{ K}$$

Problem 4 (15 pts) : Suppose there are two silicon semiconductor devices, one at a temperature of 300 K, and the other at a temperature of 600 K. You know that

$$n_i(300 \text{ K}) = 1.0 \times 10^{10} \text{ cm}^{-3} \quad \text{and} \quad n_i(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}.$$

For each of the following doping configurations, compute the corresponding electron and hole carrier concentrations (n_0 and p_0) in each of the two silicon devices. Assume that the dopants are fully ionized in the silicon crystal. The units will always be cm^{-3} in the following.

(a) $N_D = N_A = 0$ (intrinsic material).

- (b) $N_D = 1.00 \times 10^{13}$ and $N_A = 0$.
(c) $N_D = 5.00 \times 10^{15}$ and $N_A = 0$.
(d) $N_D = 0$ and $N_A = 5.00 \times 10^{15}$.
(e) $N_D = 1.00 \times 10^{18}$ and $N_A = 3.00 \times 10^{18}$.

Solution: (a) $n_0(300 \text{ K}) = p_0(300 \text{ K}) = n_i(300 \text{ K}) = 1.0 \times 10^{10} \text{ cm}^{-3}$ and $n_0(600 \text{ K}) = p_0(600 \text{ K}) = n_i(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}$.

$n_0(300 \text{ K}) = 1.0 \times 10^{10} \text{ cm}^{-3}$ $p_0(300 \text{ K}) = 1.0 \times 10^{10} \text{ cm}^{-3}$ $n_0(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}$ $p_0(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}$
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(b) $N_D \gg N_A$. At 300 K, $N_D \gg n_i(300 \text{ K})$, thus the material is extrinsic n -type. Hence, $n_0(300 \text{ K}) \approx N_D = 1.00 \times 10^{13} \text{ cm}^{-3}$ and $p_0(300 \text{ K}) = n_i^2(300 \text{ K})/n_0(300 \text{ K}) = 1 \times 10^7 \text{ cm}^{-3}$. At 600 K, $n_i(600 \text{ K}) \gg N_D$, hence, the device is intrinsic, and $n_0(600 \text{ K}) = p_0(600 \text{ K}) = n_i = 4.0 \times 10^{15} \text{ cm}^{-3}$.

$n_0(300 \text{ K}) = 1.00 \times 10^{13} \text{ cm}^{-3}$ $p_0(300 \text{ K}) = 1.00 \times 10^7 \text{ cm}^{-3}$ $n_0(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}$ $p_0(600 \text{ K}) = 4.0 \times 10^{15} \text{ cm}^{-3}$

(c) $N_D \gg N_A$. At 300 K, $N_D \gg n_i(300 \text{ K})$, hence, $n_0 \approx N_D = 5.00 \times 10^{15} \text{ cm}^{-3}$ and $p_0(300 \text{ K}) = n_i^2(300 \text{ K})/n_0(300 \text{ K}) = 2.00 \times 10^4 \text{ cm}^{-3}$. At 600 K, $N_D > n_i(600 \text{ K})$, but still comparable. Hence, we should use the more accurate quadratic relationship.

$$n_0(600 \text{ K}) = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2(600 \text{ K})}$$

$$n_0 = 7.3 \times 10^{15} \text{ cm}^{-3}$$

Then, $p_0(600 \text{ K}) = n_i^2(600 \text{ K})/n_0(600 \text{ K}) = 2.2 \times 10^{15} \text{ cm}^{-3}$.

$n_0(300 \text{ K}) = 5.00 \times 10^{15} \text{ cm}^{-3}$ $p_0(300 \text{ K}) = 2.00 \times 10^4 \text{ cm}^{-3}$ $n_0(600 \text{ K}) = 7.3 \times 10^{15} \text{ cm}^{-3}$ $p_0(600 \text{ K}) = 2.2 \times 10^{15} \text{ cm}^{-3}$

(d) By symmetry, we can extract the result for this configuration from those of the last in part (c).

$n_0(300 \text{ K}) = 2.00 \times 10^4 \text{ cm}^{-3}$ $p_0(300 \text{ K}) = 5.00 \times 10^{15} \text{ cm}^{-3}$ $n_0(600 \text{ K}) = 2.2 \times 10^{15} \text{ cm}^{-3}$ $p_0(600 \text{ K}) = 7.3 \times 10^{15} \text{ cm}^{-3}$

(e) N_A and N_D might seem comparable, but their difference $N_A - N_D = 2.00 \times 10^{18} \text{ cm}^{-3} \gg n_i$ for both temperature. Hence, the devices are both extrinsic and p -type since $N_A \gg N_D$. We have $p_0(300 \text{ K}) =$

$p_0(600 \text{ K}) = 2.00 \times 10^{18} \text{ cm}^{-3}$, and $n_0(300 \text{ K}) = n_i^2(300 \text{ K})/p_0 = 5.0 \times 10^1 \text{ cm}^{-3}$, while $n_0(600 \text{ K}) = n_i^2(600 \text{ K})/p_0 = 8.0 \times 10^{12} \text{ cm}^{-3}$.

$n_0(300 \text{ K}) = 5.0 \times 10^1 \text{ cm}^{-3}$ $p_0(300 \text{ K}) = 2.00 \times 10^{18} \text{ cm}^{-3}$ $n_0(600 \text{ K}) = 8.0 \times 10^{12} \text{ cm}^{-3}$ $p_0(600 \text{ K}) = 2.00 \times 10^{18} \text{ cm}^{-3}$

Problem 5 (10 pts) : Suppose a region in a silicon device is uniformly doped with $N_D = 10^{17} \text{ cm}^{-3}$ (assume that the temperature is low enough that the hole carrier concentration in the material is negligible). Let the electron mobility be $1000 \text{ cm}^2/\text{V} \cdot \text{s}$. Suppose further that an electric field of magnitude 100 V/cm is uniformly applied across this region in a given direction (call it the $+x$ direction). What is the magnitude of the resulting drift current density? Include the correct units in your answer.

Solution: Drift current density within the material in the $+x$ direction, ignoring the hole current, is given by

$$\mathbf{J}_{drift} = \mathbf{J}_{p,drift} + \mathbf{J}_{n,drift} = \mathbf{J}_{n,drift} = q\mu_n n \mathcal{E}.$$

Thus,

$$\mathbf{J}_{drift} = (1.6 \times 10^{-19} \text{ C})(1000 \text{ cm}^2/\text{V} \cdot \text{s})(10^{17} \text{ cm}^{-3})(100 \text{ V/cm})$$

$\mathbf{J}_{drift} = 1600 \text{ A/cm}^2$
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Problem 6 (10 pts) : Suppose that a piece of semiconductor material is at room temperature such that $k_B T/q = 26 \text{ mV}$, and that its electron mobility is $1000 \text{ cm}^2/\text{V} \cdot \text{s}$. What is its electron diffusion coefficient D_n ?

Solution: We use Einstein's relation

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q} \implies D_n = \frac{k_B T}{q} \mu_n$$

$D_n = (26 \text{ mV})(1000 \text{ cm}^2/\text{V} \cdot \text{s}) = 26 \text{ cm}^2/\text{s}$

Problem 7 (15 pts) : Consider an n -type semiconductor material with an excess non-uniform minority carrier hole concentration given by

$$p(x) = (10^{14})e^{-x/L_p}$$

where $L_p = 10^{-5} \text{ cm}$ and x represents the depth into the material from an edge situated at $x = 0$. Note that this hole concentration is added on top of the existing hole concentration p_0 within the material (these contribute no macroscopic diffusion current however). The hole diffusion coefficient is $2.6 \text{ cm}^2/\text{s}$. What is the sign (positive direction is towards increasing x) and magnitude of the hole diffusion current density at $x = 0$?

Solution: In this one dimensional problem, the expression for the desired hole diffusion current is

$$\mathbf{J}_{p,diff} = -qD_p \frac{dp(x)}{dx}$$

We first compute

$$\frac{dp(x)}{dx} = -\frac{10^{14}}{L_P} e^{-x/L_P}$$

Hence, at $x = 0$,

$$\left. \frac{dp(x)}{dx} \right|_{x=0} = -\frac{10^{14}}{L_P} = -10^{19}$$

And therefore (the units are already correct),

$$\mathbf{J}_{p,diff} = -(1.6 \times 10^{-19})(2.6)(-10^{19}) = 4.16 \text{ A/cm}^2$$

Problem 8 (15 pts) : Suppose that a semiconductor material has the energy band diagram profile shown in **Fig.3** below. The electron carrier concentration at x_1 is $n_0 = 10^{18} \text{ cm}^{-3}$. Assume that the material is at room temperature such that $k_B T/q = 26 \text{ mV}$ and that the intrinsic carrier concentration of the material is $n_i = 10^{10} \text{ cm}^{-3}$.

- (a) What is the equilibrium hole concentration at $x = x_1$?
- (b) What is equilibrium hole concentration at $x = 0$?

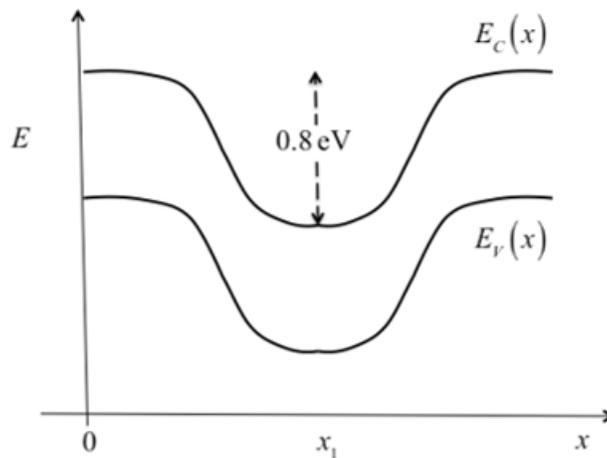


Fig.3 - Energy Band Diagram for Q8.

Solution: (a) At $x = x_1$, $n_0 = 10^{18} \text{ cm}^{-3}$. Hence,

$$p_0 = \frac{n_i^2}{n_0} = \frac{10^{20}}{10^{18}} = 100 \text{ cm}^{-3}$$

(b) The material is at equilibrium, and hence, its Fermi level must be constant. Recall the relationship between hole carrier concentration and the difference between the top of the valence band and the Fermi level

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

Going from $x = x_1$ to $x = 0$, the top of the valence band E_V increases by 0.8 eV. Hence,

$$p_0(x = 0) = N_V e^{(0.8 + E_V - E_F)/k_B T} = p_0(x = x_1) e^{0.8/k_B T}$$

Therefore, we compute the equilibrium hole concentration at $x = 0$ to be

$$p_0 = (2.31 \times 10^{13})(100) = 2.31 \times 10^{15} \text{ cm}^{-3}$$