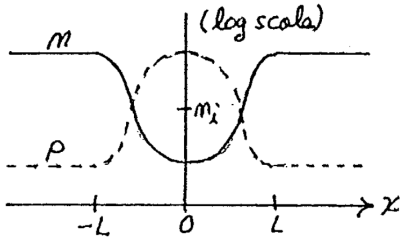
1 Problem 1

(a) The semiconductor is concluded to be in equilibrium because the Fermi level is invariant as a function of position.

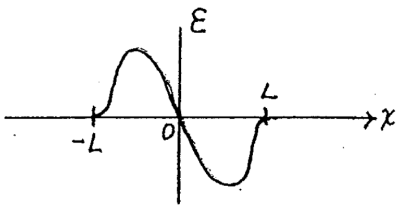
(b) $J_N = 0$ and $J_p = 0$ at both $x = -L/2$ and $x = L/2$. The net electron and hole currents are always zero everywhere under equilibrium conditions.



(c)

(d) There is an electron diffusion current at both $x = -L/2$ and $x = L/2$. From the answer to part (c), we note that $\frac{dn}{dx} \neq 0$ at the cited points. With $\frac{dn}{dx} < 0$, $J_{N|diff}$ at $x = -L/2$ flows in the $-x$ direction. Conversely, $\frac{dn}{dx} > 0$ at $x = L/2$ and $J_{N|diff}$ at this point flows in the $+x$ direction.

(e) The electric field, $\mathcal{E}$, is proportional to the slope of the bands.



(f) There is an electron drift current at both $x = -L/2$ and $x = L/2$. Since both n and $\mathcal{E}$ are non-zero, it follows that $J_{N|drift} = q\mu_n n\mathcal{E} \neq 0$. The drift component of the current has the same direction as the electric field. Thus, $J_{N|drift}$ is in the $+x$ direction at $x = -L/2$ and in the $-x$ direction at $x = L/2$. (Note that the diffusion and drift currents are in the opposite directions at the cited points, as must be the case since the net carrier currents are zero.)

Problem 2 (a)

Given that the material is n-doped, the holes are the minority carriers. As such, the minority carrier continuity equations must be used to determine the hole density at steady-state. The minority carrier continuity equation is as follows:

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial p}{\partial t}\right|_{\text{diff}} + \left.\frac{\partial p}{\partial t}\right|_{\text{R-G}} + \left.\frac{\partial p}{\partial t}\right|_{\text{other}} \tag{1}$$

Given that the semiconductor is in the dark, the other generation processes will be negligible.

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - r_p \tag{2}$$

Evaluating the portion related to the current density J_p:

$$J_p = q\mu_p(p_0 + \Delta p)\mathcal{E} - qD_p\frac{\partial(p_0 + \Delta p)}{\partial x} \tag{3}$$

$$\frac{\partial J_p}{\partial x} = q\mu_p\mathcal{E}\frac{\partial \Delta p}{\partial x} - qD_p\frac{\partial^2 \Delta p}{\partial x^2} \tag{4}$$

Which results in a time dependent equation based on the current density and low-level injection recombination of

$$\frac{\partial p}{\partial t} = -\mu_p\mathcal{E}\frac{\partial \Delta p}{\partial x} + D_p\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} \tag{5}$$

Under steady-state conditions this becomes a homogeneous second order differential equation with respect to the excess carriers

$$0 = -\mu_p\mathcal{E}\frac{\partial \Delta p}{\partial x} + D_p\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} \tag{6}$$

$$0 = \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\mu_p\mathcal{E}}{D_p}\frac{\partial \Delta p}{\partial x} - \frac{1}{D_p\tau_p}\Delta p \tag{7}$$

The solution of this equation is found by solving for the roots of the characteristic equation $r^2 - \frac{\mu_p\mathcal{E}}{D_p}r - \frac{1}{D_p\tau_p} = 0$. This yields

$$r_{1,2} = \frac{\mu_p\mathcal{E}}{2D_p} \pm \frac{1}{2}\sqrt{\left(\frac{\mu_p\mathcal{E}}{D_p}\right)^2 + \frac{4}{D_p\tau_p}} \tag{8}$$

and

$$\Delta p(x) = Ae^{r_1 x} + Be^{r_2 x} \tag{9}$$

Using the boundary conditions for $x = 0$ and $x = \infty$ the leading coefficients can be found:

$$\Delta p(0) = \Delta p_0 = A + B \tag{10}$$

$$\Delta p(L) = 0 = Ae^{r_1 L} + Be^{r_2 L} \tag{11}$$

$$0 = \Delta p_0 e^{r_1 L} - B e^{r_1 L} + B e^{r_2 L} \tag{12}$$

$$B = \frac{\Delta p_0 e^{r_1 L}}{e^{r_1 L} - e^{r_2 L}} \tag{13}$$

$$A = \Delta p_0 - \frac{\Delta p_0 e^{r_1 L}}{e^{r_1 L} - e^{r_2 L}} \tag{14}$$

$$\Delta p(x) = \Delta p_0 \left[\left(1 - \frac{e^{r_1 L}}{e^{r_1 L} - e^{r_2 L}} \right) e^{r_1 x} + \left(\frac{e^{r_1 L}}{e^{r_1 L} - e^{r_2 L}} \right) e^{r_2 x} \right] \tag{15}$$

$$\Delta p(x) = \Delta p_0 \left[\left(\frac{-e^{r_2 L}}{e^{r_1 L} - e^{r_2 L}} \right) e^{r_1 x} + \left(\frac{e^{r_1 L}}{e^{r_1 L} - e^{r_2 L}} \right) e^{r_2 x} \right] \tag{16}$$

$$\Delta p(x) = \Delta p_0 \left[\left(\frac{-1}{e^{(r_1 - r_2)L} - 1} \right) e^{r_1 x} + \left(\frac{1}{1 - e^{(r_2 - r_1)L}} \right) e^{r_2 x} \right] \tag{17}$$

Problem 2 (b)

Assuming room temperature, the diffusion coefficient can be found through the Einstein relation: $D_p = kT\mu_p/q$. Using MATLAB, the roots can be calculated as $r_1 = 1196$ cm^{-1} and $r_2 = -809$ cm^{-1}. The drift current is related the excess minority carrier concentration through the following relation:

$$J_{\text{drift}} = q\mu_p \mathcal{E} p \tag{18}$$

$$= q\mu_p \mathcal{E}(p_0 + \Delta p) \tag{19}$$

$$J_{\text{drift}}(x) = q\mu_p \mathcal{E} \Delta p(x) \tag{20}$$

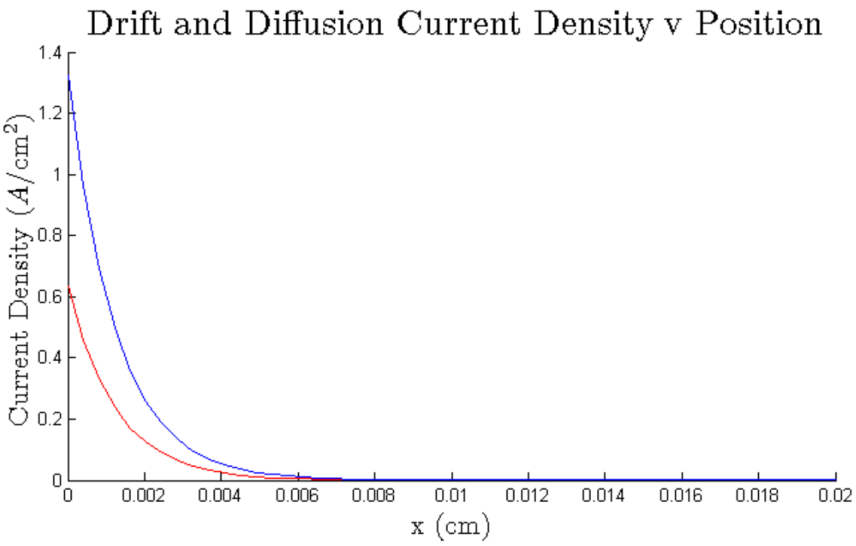
given the $\Delta p \gg p_0$. The diffusion current is related via the gradient of the excess carriers:

$$J_{\text{diff}}(x) = -qD_p \frac{\partial p}{\partial x} \tag{21}$$

$$= -qD_p \frac{\partial p_0 + \Delta p}{\partial x} \tag{22}$$

$$J_{\text{diff}}(x) = -qD_p \frac{\partial \Delta p(x)}{\partial x} \tag{23}$$

These two current densities were plotted as a function of position as shown in Figure 1.



Problem 2 (c)

The general solution for the excess minority carrier concentration is:

$$\Delta p(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

The boundary conditions are:

$$\begin{cases} \Delta p(0) = \Delta p_0 & \text{(constant injection at } x = 0) \\ D_p \left.\frac{d\Delta p}{dx}\right|_{x=L} = S_p \Delta p(L) & \text{(surface recombination at } x = L) \end{cases}$$

$$\Delta p_0 = C_1 e^{r_1(0)} + C_2 e^{r_2(0)} \implies \Delta p_0 = C_1 + C_2$$

$$\implies C_2 = \Delta p_0 - C_1 \quad (1)$$

The derivative is $\frac{d\Delta p}{dx} = C_1 r_1 e^{r_1 x} + C_2 r_2 e^{r_2 x}$. Substituting into the condition:

$$D_p \left(C_1 r_1 e^{r_1 L} + C_2 r_2 e^{r_2 L}\right) = S_p \left(C_1 e^{r_1 L} + C_2 e^{r_2 L}\right) \quad (2)$$

Solve for C_1
Substitute equation (1) for C_2 into equation (2):

$$D_p \left[C_1 r_1 e^{r_1 L} + r_2(\Delta p_0 - C_1) e^{r_2 L}\right] = S_p \left[C_1 e^{r_1 L} + (\Delta p_0 - C_1) e^{r_2 L}\right]$$

Expand and group terms:

$$C_1 D_p r_1 e^{r_1 L} + D_p \Delta p_0 r_2 e^{r_2 L} - C_1 D_p r_2 e^{r_2 L} = S_p C_1 e^{r_1 L} + S_p \Delta p_0 e^{r_2 L} - S_p C_1 e^{r_2 L}$$

Move C_1 terms to the LHS and Δp_0 terms to the RHS:

$$C_1(D_p r_1 e^{r_1 L} - D_p r_2 e^{r_2 L} - S_p e^{r_1 L} + S_p e^{r_2 L}) = S_p \Delta p_0 e^{r_2 L} - D_p \Delta p_0 r_2 e^{r_2 L}$$

Factor C_1 and Δp_0:

$$C_1 \left[e^{r_1 L}(D_p r_1 - S_p) + e^{r_2 L}(S_p - D_p r_2) \right] = \Delta p_0 e^{r_2 L}(S_p - r_2 D_p)$$

The expression for C_1:

$$\mathbf{C_1} = \frac{\mathbf{\Delta p_0 e^{r_2 L}(S_p - r_2 D_p)}}{\mathbf{e^{r_1 L}(D_p r_1 - S_p) + e^{r_2 L}(S_p - D_p r_2)}}$$

Solve for C_2
Use $C_2 = \Delta p_0 - C_1$:

$$\mathbf{C_2} = \mathbf{\Delta p_0} - \frac{\mathbf{\Delta p_0 e^{r_2 L}(S_p - r_2 D_p)}}{\mathbf{e^{r_1 L}(D_p r_1 - S_p) + e^{r_2 L}(S_p - D_p r_2)}}$$

Problem 4

The minority carrier equation, similar to problem one, can be used for the holes noting that the minority carriers will not give rise to an electric field worth considering.

$$0 = D_p \frac{\partial^2 \Delta p}{\partial^2 x} - \frac{\Delta p}{\tau_p} + G \tag{21}$$

This second order non-homogeneous equation has a solution of the form

$$\Delta p(x) = Ae^{\frac{1}{\sqrt{D_p \tau_p}}x} + Be^{-\frac{1}{\sqrt{D_p \tau_p}}x} + G\tau_p \tag{22}$$

The coefficient can be found using the constraints specified in the problem. At $x = 0$, $\Delta p = 0$ presuming the extraction is via a metal contact. At $x = \infty$, the steady-state current is such that $\partial p/\partial t = 0 = -r_p + G = -\Delta p/\tau_p + G$ or simply $\Delta p = G\tau_p$:

$$0 = A + B + G\tau_p \tag{23}$$

$$A = -(B + G\tau_p) \tag{24}$$

and

$$G\tau_p = Ae^{\infty} + 0 + G\tau_p \tag{25}$$

Implies $A = 0$ must be true and therefore $B = -G\tau_p$. Combining these results, an equation for the excess minority holes as a function of position can be found:

$$\Delta p(x) = G\tau_p \left(1 - e^{-\frac{1}{\sqrt{D_p \tau_p}}x}\right) \tag{26}$$

The current density from diffusion is found by solving $J_p\big|_{\text{diff}} = -qD_p \partial p/\partial x$ noting that $p = p_0 + \Delta p$ where p_0 is intrinsic and does not vary as a function of position:

$$J_p(x) = -qD_p \left(\frac{G\tau_p}{\sqrt{D_p \tau_p}} e^{-\frac{1}{\sqrt{D_p \tau_p}}x}\right) \tag{27}$$

$$J_p(x) = -qG\sqrt{D_p \tau_p} e^{-\frac{1}{\sqrt{D_p \tau_p}}x} \tag{28}$$

At $x = 0$, the current density will be maximal and is given by $-qG\sqrt{D_p \tau_p}$ (holes flowing left is consistent with bar being in the positive x direction). This current is from excess minority carriers diffusing into the contact due to generation via illumination.

Problem 4

(a)/(b)/(c) To first order, the sketches here should be essentially identical to Fig. 6.1(a)–(c), except the Fermi level on the p-side is close to or actually in the valence band. Although it is perhaps unreasonable to expect from someone who has read only through Subsection 6.1.1 in the text, the majority carrier hole concentration on the p-side of the junction should also be visualized as much larger than the majority carrier electron concentration on the n-side of the junction. Likewise, the hole minority carrier concentration on the n-side is much greater than the electron minority carrier concentration on the p-side in a p^+-n diode. Consequently, one would expect the hole component of the current to dominate over the electron component of the current under both forward and reverse biases.—The hole current arrows in the part (b) and (c) diagrams should be significantly larger than the electron current arrows. Note that the qualitative conclusion here is consistent with the formula-based observations made in Subsection 6.1.4.

Problem 5 (a)

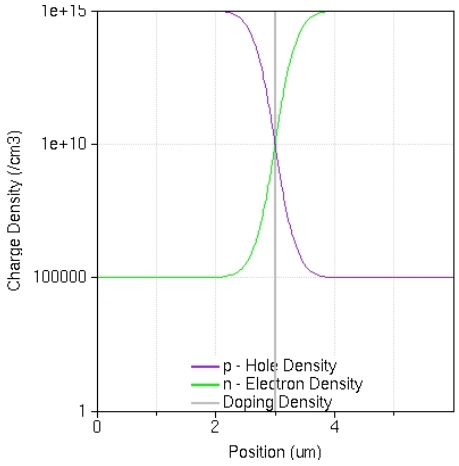


Figure 1: Doping, Electron, and Hole Density at Equilibrium

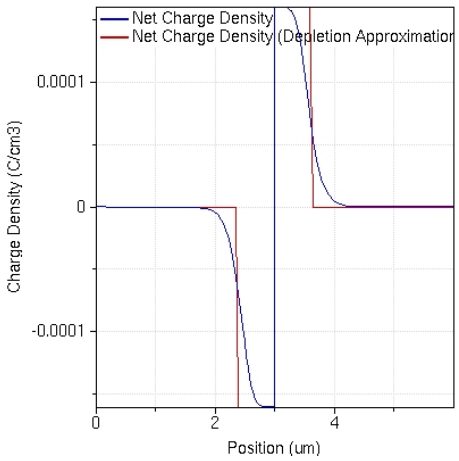


Figure 2: Net Charge Density at Equilibrium

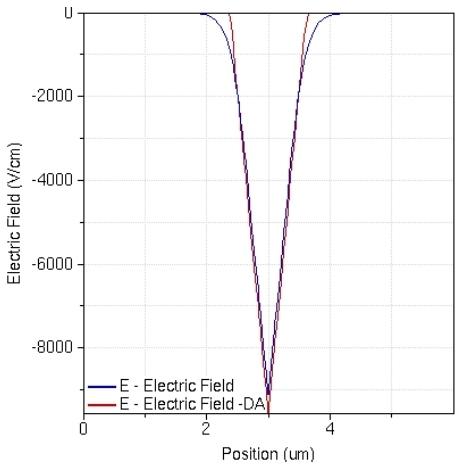


Figure 3: Electric Field at Equilibrium

As long as a p-n junction is formed, holes in the p-region (left-side) diffuse across the junction and combine with electrons in the n-region (right-side). They leave behind negatively charged acceptor ions. Similarly, electrons flow from left to right and leave behind positively charged donor ions. Therefore, the net charges in the region near the junction do not cancel out and eventually build up an electric field.

According to Figure 2, the area above the blue curve on the left-side of the vertical red line (depletion approximation) is roughly equal to the one under the blue curve on the right-side of the same line. Though the depletion approximation cannot precisely describe the charge distribution, it is a good simplified model for the electrostatic analysis.

Problem 5 (b)

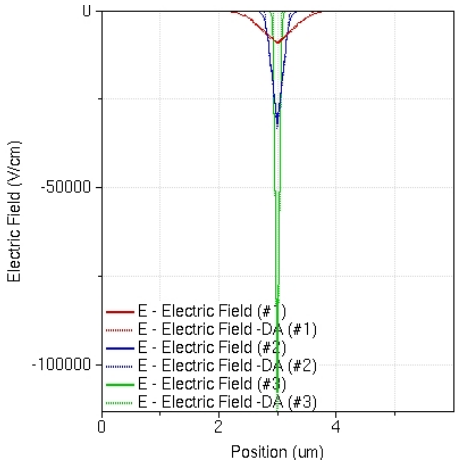


Figure 4: Electric Field at Equilibrium

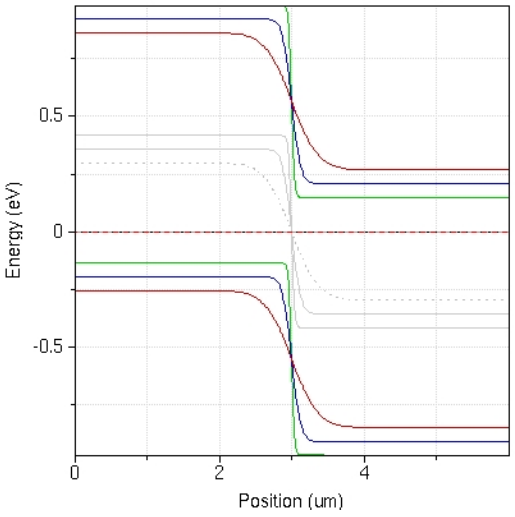


Figure 5: Energy Band Diagram at Equilibrium (green: 1E17, blue: 1E16, red: 1E15)

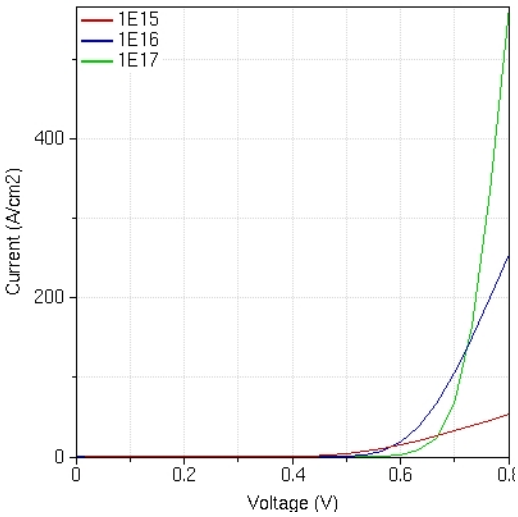


Figure 6: I-V Characteristics

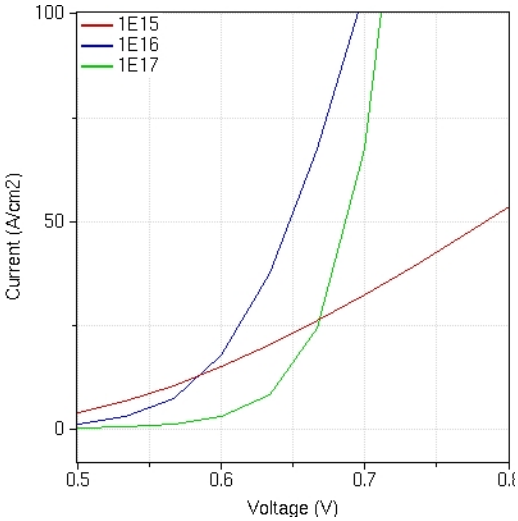


Figure 7: I-V Characteristics

The dopants are much closer to each other when the doping level is higher. The concentration of dopant ions in the depletion region increases for the same width. In other words, the junction width reduces accordingly to maintain the field that is sufficient to prevent the diffusion of charge carriers from one side of the junction to other. As a result, the electric field becomes stronger when the doping level increases. From the energy band diagram, we also can observe the differences of doping levels among these three cases from the energy separations between the band edges and the fermi level. In addition, the potential barrier becomes higher when the doping level increases, which implies that larger external bias is needed to turn on the device. In other words, the turn on voltage of the device with lower doping level is earlier.

Problem 5 (c)

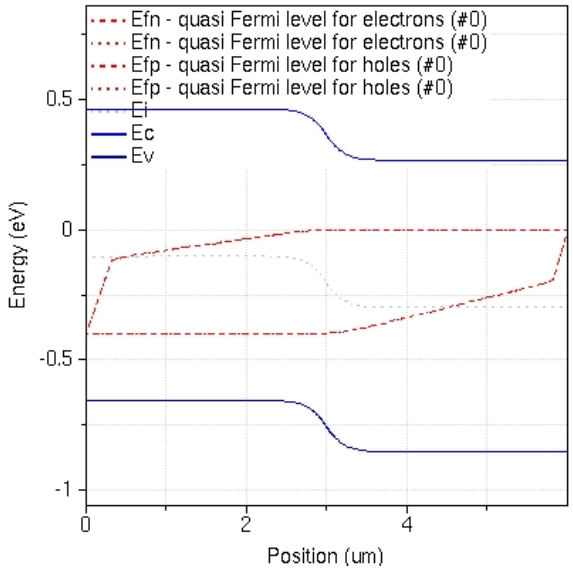


Figure 8: Energy Band Diagram at a Bias of 0.4V (1E15)

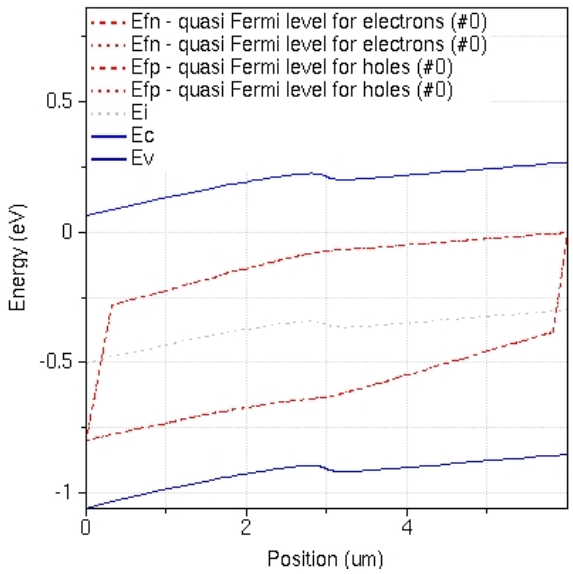


Figure 9: Energy Band Diagram at a Bias of 0.8V (1E15)

The potential barrier at the junction can be changed by applying external bias. At the equilibrium condition, the potential barrier is high enough so that carriers cannot transport from one side of the junction to other. However, once the barrier is reduced by applying bias, positive bias lowers the band but negative bias lifts the band, which makes the asymmetry in the p and n side. With external bias, the device is out of equilibrium, there are more carriers able to overcome the reduced barrier, so the current is getting larger.

Problem 5 (d)

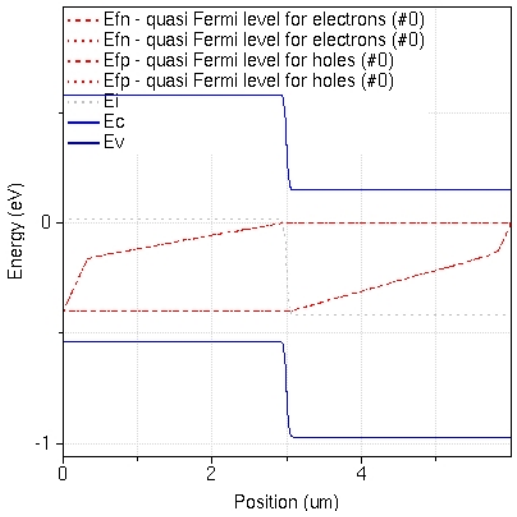


Figure 10: Energy Band Diagram at a Bias of 0.4V (1E17)

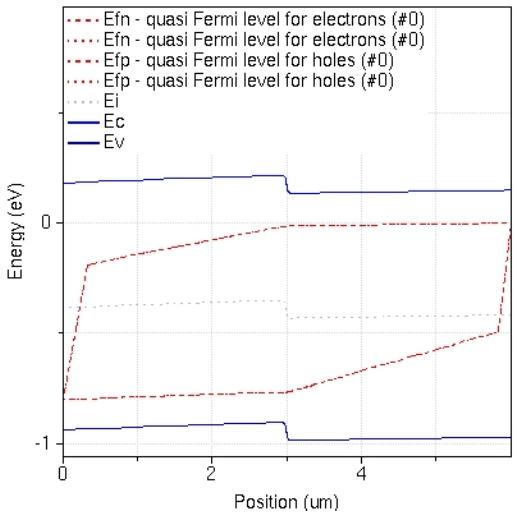


Figure 11: Energy Band Diagram at a Bias of 0.8V (1E17)

Apparently, higher doping level makes the potential barrier at the junction become higher, so larger bias is needed to turn on the device.

Problem 5 (e)

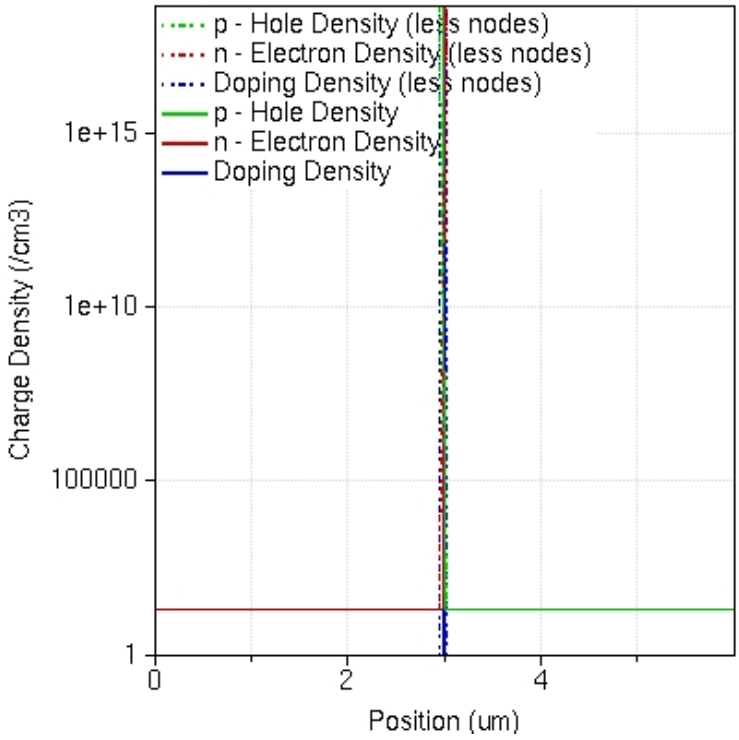


Figure 12: Doping, Electron, and Hole Density at Equilibrium

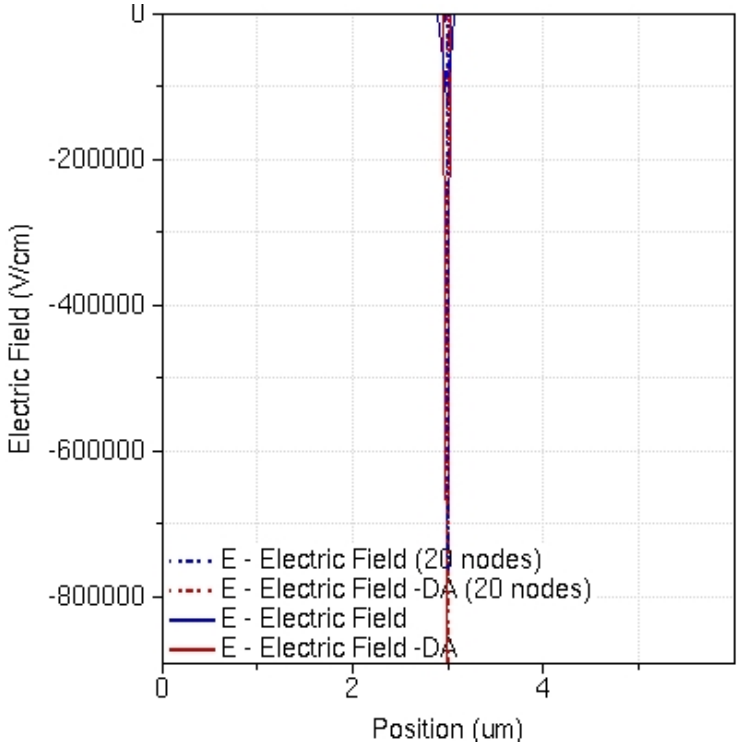


Figure 13: Electric Field at Equilibrium

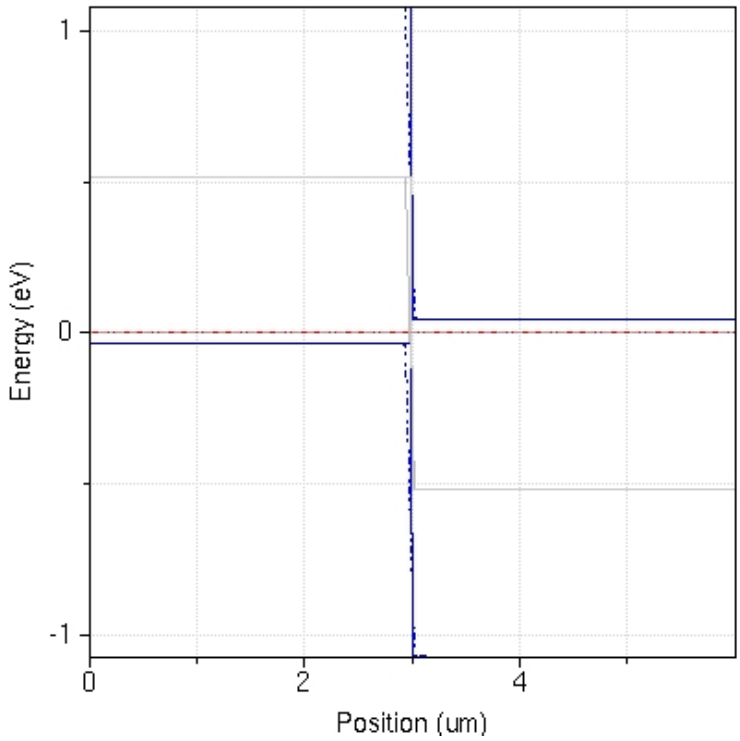


Figure 14: Energy Band Diagram at Equilibrium (The dashed lines are simulation with fewer nodes, and the solid lines are the simulation with default setting.)

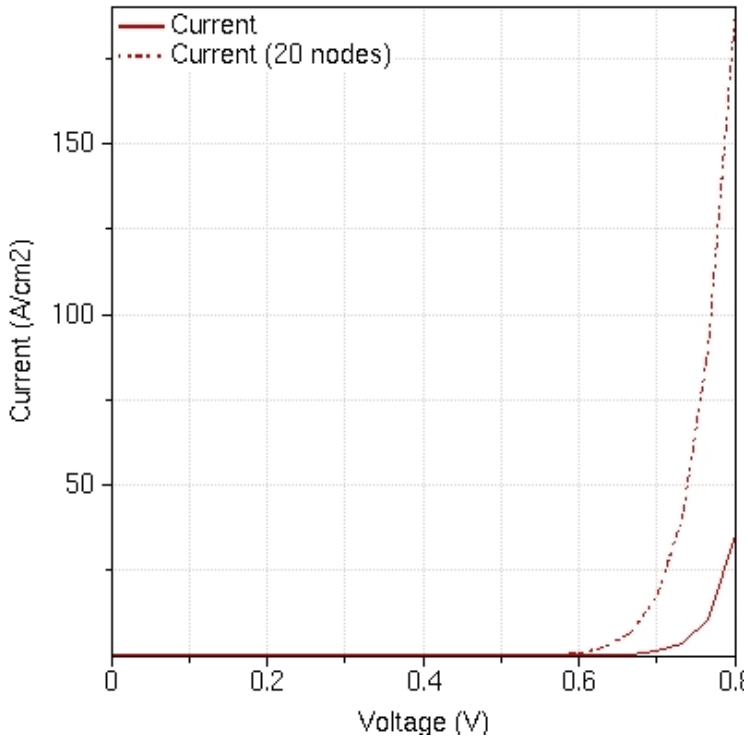
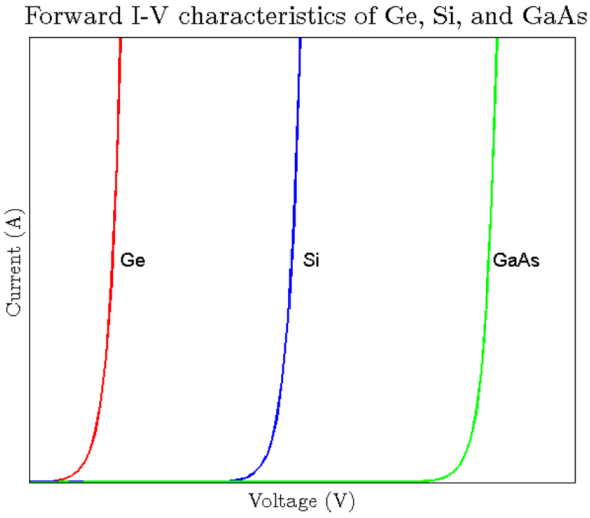


Figure 15: I-V Characteristics

According to the figures of charge density, electric field, and energy band diagram, it's hard to observe the difference between the simulations with higher and lower resolution. However, from the I-V characteristics, it's pretty easy to see the difference between them. Obviously, the turn on voltage obtained from the simulation with lower resolution is much earlier, which means a proper simulation setup is very critical to predict the device characteristics.

Problem 6



A p^+n diode's forward current is based on SDF 6.29 and 6.32a:

$$I = I_0 \left(e^{qV_A/kT} - 1 \right) \tag{24}$$

where

$$I_0 = qA \frac{D_p}{L_p} \frac{n_i^2}{N_D} \tag{25}$$

Assuming the diodes have identical structures and doping concentrations, the only differences in I-V relationships is a result of differences in the D_p/L_p ratio and n_i. The intrinsic carrier concentrations at room temperature for Ge, Si, and GaAs are approximately 10^{13}, 10^{10}, and 2.25×10^6 cm^{-3} respectively (ADF p.117). Clearly, the difference in reverse saturation current will be dominated by the n_i^2 term and not the D_p/L_p term (the mobility differences and effective carrier lifetimes will only vary by roughly an order of magnitude). Therefore, assuming $N_D = 10^{16}$,

$$I_{\text{Ge}} \propto 10^9 \left(e^{qV_A/kT} - 1 \right) \tag{26}$$

$$I_{\text{Si}} \propto 10^3 \left(e^{qV_A/kT} - 1 \right) \tag{27}$$

$$I_{\text{Ge}} \propto 10^{-5} \left(e^{qV_A/kT} - 1 \right) \tag{28}$$

The rough plot of these relations is shown in Figure 16.