Efficient Error Estimating Coding: Feasibility and Applications

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Outline

• Wireless Networking Background
• Motivation: Benefit of BER estimating
• EEC design
  – How to estimate BER with low computational overhead and redundancy?
  – Complexity and Redundancy
• Conclusion
Trends in Wireless Networking

- Application/Network use or relay **correct packet**
- Not correct: Request retransmission
Trends in Wireless Networking

- Many designs to support **partially correct packet** with **Error Correction Coding**
  - Ex) Incremental Redundancy ARQ
- Well-suited to delay-sensitive applications.
Benefits of BER at Relaying Node

- BER-aware packet retransmission
  - Add maximum BER that Error-Correction Code can tolerate
- BER-aware packet scheduling
  - Prioritize the forwarding of packets with lower BER
- BER-aware packet forwarding
  - Decode-and-Forward vs Amplify-and-Forward
Benefits of BER at **Sender**

- **Rate adaptation**
  - Better adapt its rate by a feedback based on BER.

- **BER-aware routing**
  - Instead of optimizing for minimizing the expected number of transmission, we can optimize for maximizing the goodput of end-to-end route.
EEC Design

• How to make structure to estimate BER?
  1. Naive sampling with known pilot

Redundancy
for similar estimation quality
when BER is small

EEC : 2%
Naïve : 40%
EEC Design

• How to make structure to estimate BER?

  2. Sample a group of data bits with a single bit
     - Assume each bit has a probability of error $p$.

  Group size : $g$
  Number of parities : 1
  Number of errors in a group follows Binomial distribution $(g+1,p)$
EEC Design

• How to make structure to estimate BER?
  2. Sample a group of data bits with a single bit

Parity information is sufficient, when p is small enough
EEC Design

• How to make structure to estimate BER?
  2. Sample a group of data bits with a single bit

![Histograms](image)

Sum of odd terms and even terms are comparable,
when \( p \) is not small enough.
EEC Design : Brief Summary

• How to make structure to estimate BER?
  2. Sample a group of data bits with a single bit

  \[ \phi(g + 1, p) : \text{sum of the odd terms} \]

  - when \( p \) is small enough, 
    parity information is sufficient and \( \phi(g + 1, p) \) smaller

  - when \( p \) is not small enough, 
    \( \phi(g + 1, p) \) and sum of even terms are comparable
Single-level EEC

• How to know sum of odd terms $\phi(g + 1, p)$?
  - When $s$ is large, the fraction of #1 parities $\sim \phi(g + 1, p)$

Since parity is sufficient at small $p$

for $\phi(g + 1, p) < c_2$

BER estimation is possible by

$\phi(g + 1, p) / (g+1)$
Multi-level EEC

• How to estimate BER for \([1/n, 1/4]\)?
  
  - Total \(\lfloor \log_2 n \rfloor\) levels each with \(2^i\) group size

\[\phi(2^i, p)\] is monotonically increasing with \(i\) and \(p\)
Multi-level EEC

- How to estimate BER for \([1/n, 1/4]\) ?

  - Find the suitable constants \(c_1\) and \(c_2\) such that there always exists some level \(i\) such that \(\phi(2^i, p)\) falls within \((c_1, c_2)\) for all \(p\) in \([1/n, 1/4]\)

\[
\phi(2, p) < c_2 \text{ for all } p \leq 1/4 \text{ guarantees } \phi(2^i, p) < c_2 \text{ at least at the first level}
\]

\[
\phi(2^{[\log_2 n]}, p) > c_1 \text{ for all } p \geq 1/n \text{ guarantees } \phi(2^i, p) > c_1 \text{ at least at the last level}
\]

\[
\phi(2^{j+1}, p) < c_2, \text{ where } j \text{ is the largest } i \text{ such that } \phi(2^i, p) \leq c_1
\]

\[
\begin{array}{|c|ccccccc|}
\hline
\phi(2^i, p) & i = 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
p = 0.25 & 0.38 & 0.47 & 0.50 & 0.50 & 0.50 & 0.50 \\
p = 0.05 & 0.095 & 0.17 & 0.28 & 0.40 & 0.48 & 0.50 \\
p = 0.01 & 0.020 & 0.039 & 0.075 & 0.14 & 0.24 & 0.36 \\
\hline
\end{array}
\]

\[
c_1 < 0.3, \ c_2 > 0.375, \text{ and } c_2 > 2c_1(1 - c_1)
\]
Multi-level EEC

• How to estimate BER for \([1/n, 1/4]\)?
  
  - Since such sum of odd terms within \((c_1, c_2)\)
    
    \[ \sim \text{expected number of errors per group}, \]

  \[ \hat{p} = \phi(2^i, p) / 2^i \]
EEC Redundancy and Computational Overhead

Redundancy

- $O(\log n)$ levels with $O(1)$ parity bits per level $\sim O(\log n)$
- For BER range $[1/1000, 0.15]$
  - 9 EEC levels, 32 parity bits per level.
  - Relative redundancy to 1500-byte packet $= 2.4\%$

Computation Overhead

**Theorem 2.** The EEC encoding, decoding, and estimating time complexity are all $O(n)$. 
Conclusion

• Benefit of BER estimating in different scenarios
• Design criteria for Error-Estimation code
  – Estimation quality, Low redundancy and computational overhead
• Structure of EEC
  – The property of sum of odd terms with a single parity, when BER is small