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# Thermal environment in indoor spaces with under-floor air distribution systems: 2. Determination of design parameters (1522-RP)

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This paper reports on the continued study of the thermal environment in indoor spaces with under-floor air distribution systems with a focus on the determination of supply airflow rate. Supply airflow rate of Under-floor Air Distribution (UFAD) needs to be carefully determined to achieve thermally comfortable conditions in an occupied space. The design parameters, such as airflow rate, temperature of supply air, and types and number of diffusers need to be properly calculated to ensure an acceptable vertical temperature difference between the head and ankle of occupants. This study introduced an empirical model to predict the vertical temperature difference between the head and ankle of occupants and calculated the supply airflow rate for UFAD design. This investigation developed the model based on a database summarizing vertical temperature distributions that correspond to various airflow and thermal conditions. The model used dimensionless numbers to group design parameters in order to represent the two driving factors of thermal stratification, namely, inertial and buoyance forces. Linear regression analysis was conducted to correlate the empirical equations of stratification for swirl, square, and linear diffusers. With the model, this study developed an airflow calculation method for UFAD as well as a graphical interface for designers.

**Keywords:** Supply airflow rate, thermal stratification, swirl diffuser, square diffuser, linear diffuser, thermal comfort

#### 1. Introduction

UFAD systems create a partially stratified environment. Unlike traditional displacement ventilation and overhead mixing systems, where either buoyancy or momentum dominates the air distribution, both buoyancy and momentum in UFAD systems make a comparable contribution to the vertical temperature gradient. This causes complexity in estimating the temperature difference between the head and ankle of occupants. Computational Fluid Dynamics (CFD) is capable of giving detailed analysis to thermal stratification of UFAD systems. However, it requires substantial computing time and a sophisticated understanding of the boundary conditions at the diffusers. For system design, it would be ideal to have design equations so that

engineers can predict key design parameters, such as air temperature gradient in indoor spaces and supply airflow rate.

Many previous studies have modeled the air stratification in indoor spaces with UFAD systems (Baumann and Daly 2003, Lau and Chen 2007). Ito and Nakahara (1993) introduced a model to calculate the vertical room air temperature profile, where the room space was separated into two regions: a fully-mixed lower region and an upper region with piston-flow based on the direct measurements of air temperature and airflow visualization. Zhang (2000) developed a similar model which integrated a thermal plume model (Morton et al. 1956), a multiple-layer model (Linden and Cooper 1996), and room heat transfer models (Mundt 1990, Li et al. 1992). These models introduced important concepts for modeling the thermal stratification in indoor spaces. For traditional displacement ventilation where the air is discharged into the room horizontally at a very low velocity, models without considering the air momentum from the diffuser supply air are appropriate. As UFAD has significant vertical airflow projection from floor diffusers, the impact from the vertical momentum is critical and should not be neglected. The buoyancy flux in UFAD is another driving factor associated with space heat gain, which normally includes occupants, electric equipment, and solar radiation. Liu (2006) showed a calculation for converting the heat gain into buoyancy flux, which was implemented in the Gamma-Phi equations (CBE 2007). The number of thermal plumes inside the occupied space is required in this method, which may cause difficulties when the equation is being used in nonoffice spaces where the number of thermal plumes is not clearly identified.

Therefore, it is essential to develop simple models that can be used in calculating the temperature stratification and supply airflow rate for designing indoor spaces with UFAD systems. Our investigation has modeled the thermal stratification using the buoyancy and inertial forces of the occupied space. The supply airflow rate calculation requires the reverse usage of the empirical modeling equations. The Newton-Raphson iteration method was used to solve the desired supply airflow rate based on room design temperature and supply air temperature at the diffusers. This paper reports our effort in this investigation.

## 2. Model development

## 2.1. Model for temperature difference between head and ankle levels

The temperature difference between the head and ankle of occupants needs to be carefully adjusted for thermal comfort design. The vertical temperature difference between the head and ankle,  $\Delta T_{oc.}$  should not exceed 3 K (5.4 °F) according to ASHRAE (2010). In this study,  $\Delta T_{oc.}$  was correlated into a function of the Archimedes number, which is the ratio of buoyancy force over inertial force:

$$Ar = \frac{Gr}{Re^2} \tag{1}$$

where Gr is the Grashof number and Re is the Reynolds number. For a UFAD system, the Grashof and Reynolds numbers can be calculated as follows:

$$Gr = \frac{g\beta(T_r - T_s)L^3}{v^2} \tag{2}$$

$$Re = \frac{u_d L}{v} \tag{3}$$

where g is the gravitational constant,  $\beta$  is the thermal expansion coefficient,  $T_r$  and  $T_s$  are return and diffuser supply air temperatures, respectively, L is the characteristic length, v is the kinematic viscosity, and  $u_d$  is the diffuser air velocity.  $u_d$  can be computed by  $V_d/A_d$  with  $V_d$  as the flow rate per diffuser and  $A_d$  as the effective opening area of the diffuser. Under atmospheric pressure and room temperature, the thermal expansion coefficient

$$\beta = \frac{1}{T_x + 273.15} \tag{4}$$

where  $T_x$  is the average temperature of an occupied zone.

On the other hand, the temperature difference between return and diffuser supply can be determined by the following energy balance equation:

$$T_r - T_s = \frac{Q_{Rm}}{c_p \rho V} \tag{5}$$

 $Q_{Rm}$  is the heat extraction rate of room space,  $c_p$  is the air specific heat, and V is the total supply airflow rate.

Note that room space in this study was defined as the space above the supply plenum and below the return plenum (if any). It is important to differentiate the heat gains to the supply plenum (Bauman et al. 2007), room space, and return plenum of a building with a UFAD system as only the heat gain remaining in the room space contributes to the thermal stratification with buoyancy force. To calculate the heat extraction rate of a room space, the plenum heat gain ratio is a critical assumption to be made for UFAD design. Baumann et al. (2006) developed a simplified first-law heat transfer model to evaluate the heat flux going into the supply and return plenums. His study indicated that the ratio of total heat transfer into the supply plenum varied from 18% to 36.7% without a hung ceiling and 18.3% to 42.5% when a hung ceiling was presented. Schiavon et al. (2010) developed an empirical equation which can easily be used to determine the heat gain ratio of the supply plenum, room space, and return plenum based on the inputs for zone orientation and floor types.

With Eqs. (2) to (5), the Archimedes number can be determined by a few major design parameters related to thermal stratification as:

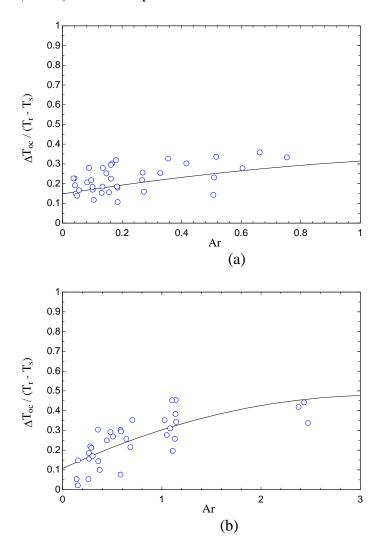
$$Ar = \frac{gQ_{Rm}H}{c_p \rho u_d^2 V(T_x + 273.15)}$$
 (6)

where *H* is the height of the room space.

With the Archimedes number, this study developed empirical equations to calculate the vertical temperature difference between the head and ankle,  $\Delta T_{oc}$ , by using a database that had a broad range of airflow and thermal boundary conditions in various indoor spaces with the UFAD system, including office, classroom, and conference room. The database contained interior and exterior zones for each type of indoor space. Three diffuser types, that is, swirl, square, and

linear diffusers were analyzed in the database. As described in the companion paper (Lee et al. 2012), the database contained a total of 108 cases and described the vertical thermal stratification in the indoor spaces with the UFAD systems by five temperatures, namely, the supply and return temperatures, the air temperatures at the head and ankle level, and the average temperature of the occupied zone.

Figure 1 shows the vertical temperature difference correlation with the Archimedes number for the swirl, square, and linear diffusers. Square diffusers normally had the widest range of Ar number because of the lower momentum from the discharge with a larger opening area. The higher Ar was related to stronger buoyancy or lower momentum. This normally implied a higher cooling load inside the occupied space or a lower supply airflow rate from the diffuser. Figure 1 illustrates a higher Ar enhancing the room thermal stratification as  $\Delta T_{oc} / (T_r - T_s)$  became higher. When  $\Delta T_{oc} / (T_r - T_s)$  reached the lower end, the UFAD system created a complete mixing in the occupied zone and the stratification became insignificant. Ideally,  $\Delta T_{oc} / (T_r - T_s)$  should drop down to zero when Ar becomes substantially small.



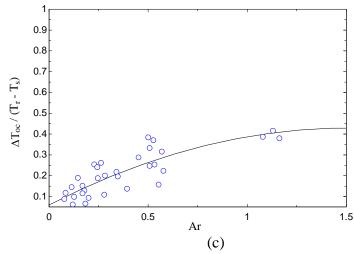


Figure 1. Correlations between  $\Delta T_{oc} / (T_r - T_s)$  and Ar for (a) swirl diffuser, (b) square diffuser, and (c) linear diffuser, developed from the database

By using regression for the data shown in Figure 1, the correlations above can be described as a quadratic function:

$$\frac{\Delta T_{oc}}{T_r - T_s} = aAr^2 + bAr + c \tag{7}$$

Table 1 summarizes the coefficients in Eq. (7).

Table 1. Coefficients in Eq. (7)

	a	b	С
Swirl diffuser	-0.0720	0.2385	0.1480
Square diffuser	-0.0362	0.2316	0.1076
Linear diffuser	-0.1623	0.4902	0.0594

Mathematically, when Ar becomes larger, the  $\Delta T_{oc}$  /  $(T_r - T_s)$  increases before reaching the peak, then decreases afterwards. However, for a real UFAD system, the corresponding Ar is unlikely to go so large as to reach the boundary of -b/2a which is the axis of symmetry of the quadratic function. The computed peak values of Eq. (8) for swirl, square, and linear diffusers were 0.3, 0.5, and 0.4, respectively. This agrees with the findings on displacement ventilation (Chen and Glicksman 2003) where about 30% of the space heat again was attributed to the space between the head and ankle. The R-square values for the three correlation equations above are 0.28, 0.63 and 0.75 for swirl, square and linear diffusers respectively. The fact of low R-square value of swirl diffusers suggests that, except Archimedes number, other variables could also have an impact on the equations. Since designers like to have as few parameters as possible, Archimedes number is used as a main design parameter in our model development. The models

would not produce highly fidelity results but they would be acceptable for building ventilation design. The accuracy will be discussed late in this paper.

### 2.2. Model for determining air temperature at the ankle level

The temperature at the ankle level (0.1 m above the floor),  $T_{0.1}$ , is another important parameter for thermal comfort in UFAD system design. The Mundt model (1992) is well-known in estimating  $T_{0.1}$  for traditional displacement ventilation. It was derived based on the energy balance of the near-floor air and the floor surface. The model assumed that the radiant heat received from the ceiling was balanced by the convective heat leaving the near-floor air above the floor surface as:

$$h_r A_f (T_{suf,c} - T_{sur,f}) = h_c A_f (T_{sur,f} - T_{0.1})$$
(8)

Therefore, the air temperature of the near-floor layer is associated with the convective heat entering this layer. Then  $T_{0,1}$  can be calculated as:

$$T_{0.1} = h_c A_f (T_{sur,f} - T_{0.1}) / \rho c_p \dot{V} + T_s$$
(9)

where  $h_r$  is the radiant heat transfer coefficient between ceiling and floor,  $h_c$  is the convective heat transfer coefficient on top of the raised-floor,  $A_f$  is the floor area, and  $T_{sur,c}$  and  $T_{sur,f}$  are the surface temperatures of the ceiling and floor, respectively. Eqs. (9) and (10) can be combined together to eliminate  $T_{sur,f}$ . By assuming  $T_{sur,c} = T_r$ ,  $T_{0,I}$  can be obtained by

$$T_{0.1} = \frac{T_r - T_s}{\frac{\rho c_p \dot{V}}{A_f} (\frac{1}{h_c} + \frac{1}{h_r}) + 1}$$
(10)

Though Eq. (10) performs well in displacement ventilation, it may not be able to work in UFAD as the assumptions of Eqs. (8) and (9) are not valid when heat can be conducted to the supply plenum from the raised-floor and mixing occurs between the near-floor layer and the space above. In order to test the applicability of the Mundt model for UFAD systems, Figure 2 compares the predicted  $T_{0.1}$  from Eq. (10) with those in the database for swirl, square, and linear diffusers.

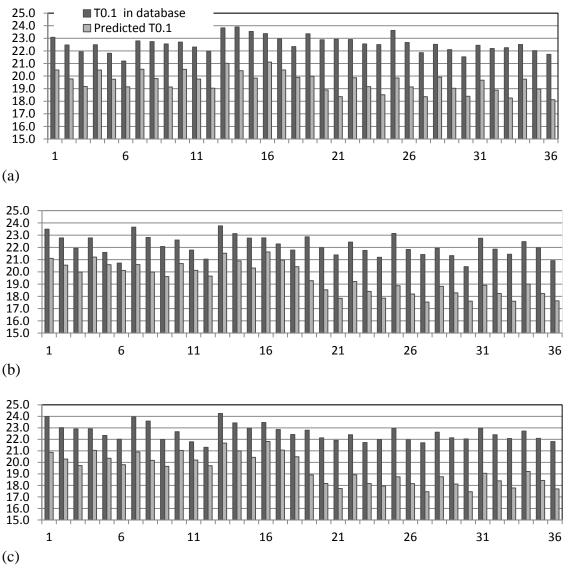
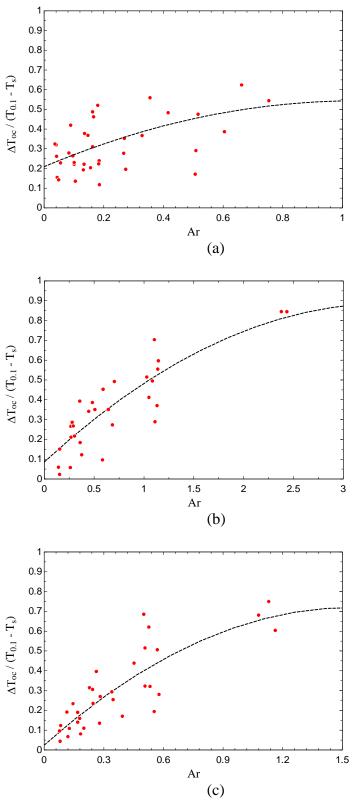


Figure 2. Comparison between the  $T_{0.1}$  predicted using the Mundt model and those in the database for (a) swirl diffusers, (b) square diffusers, and (c) linear diffusers

The comparison showed that the Mundt model consistently underestimated the  $T_{0.I}$  in the UFAD systems for swirl, square, and linear diffusers. UFAD systems normally had a higher near-floor temperature than traditional displacement ventilation systems because heat entered the near-floor layer from the occupied zone due to mixing. Thus, it is necessary to develop new equations for determining  $T_{0.I}$  in a UFAD system. By using the database, Figure 3 shows the correlation between  $\Delta T_{oc}$  /  $(T_{0.I}$  -  $T_s$ ) and Ar for the swirl, square, and linear diffusers. The data for the swirl diffuser had the highest deviation compared to the data for the square and linear diffusers. This is due to the fact that the boundary conditions from the swirl diffusers were sophisticated and the results were sensitive to the boundary conditions.



(c) Figure 3. Correlations between  $\Delta T_{oc}$  / ( $T_{0.1}$  -  $T_s$ ) and Ar for (a) swirl diffusers, (b) square diffusers, and (c) linear diffusers

Similarly, by using regression of the data, the following quadratic function can be obtained with the coefficients shown in Table 2.

$$\frac{\Delta T_{oc}}{T_{0.1} - T_{c}} = kAr^{2} + pAr + q \tag{11}$$

Table 2. The coefficients in Eq. (11)

	k	p	q
Swirl diffuser	-0.3052	0.6382	0.2094
Square diffuser	-0.0673	0.4645	0.0860
Linear diffuser	-0.2888	0.8963	0.0247

The R-square values of these correlation equations are 0.28, 0.80 and 0.76 for swirl, square and linear diffusers respectively. Again, a low R-value of swirl diffusers is observed in this case for the same reason discussed in Eq. (7).

Eq. (11) can be rearranged to calculate  $T_{0.1}$  explicitly:

$$T_{0.1} = T_s + \frac{\Delta T_{oc}}{kAr^2 + pAr + q} \tag{12}$$

# 2.3. Model for calculating supply airflow rate

Eq. (7) and Eq. (12) need to be solved simultaneously to obtain the supply airflow rate. Eq. (7) can be rearranged into:

$$\Delta T_{oc} = \alpha V^{-3} + \beta V^{-2} + \gamma V^{-1} \tag{13}$$

where

$$\alpha = \frac{ag^2 Q_{Rm}^3 H^2}{c_n^3 \rho^3 u_d^4 (T_r + 273.15)^2}$$
(14)

$$\beta = \frac{bgQ_{Rm}^2 H}{c_p^2 \rho^2 u_d^2 (T_x + 273.15)}$$
 (15)

$$\gamma = \frac{cQ_{Rm}}{c_p \rho} \tag{16}$$

On the other hand, Eq. (12) can be rearranged into:

$$\Delta T_{oc} = \frac{T_x - T_s}{\frac{1}{2} + \frac{1}{HV^{-2} + MV^{-1} + q}}$$
(17)

where

$$H = k \left[ \frac{gQ_{Rm}H}{c_p \rho u_d^2 (T_x + 273.15)} \right]^2$$
 (18)

$$M = p \cdot \frac{gQ_{Rm}H}{c_n \rho u_d^2 (T_x + 273.15)}$$
 (19)

by using  $T_{0.1} = T_x - \Delta T_{oc}/2$  with the assumption that  $T_x$  is at the mid-point between the temperatures at the head and ankle levels.

Subtract Eq. (13) from (17), there is

$$F(V) = \frac{T_x - T_s}{\frac{1}{2} + \frac{1}{HV^{-2} + MV^{-1} + q}} - (\alpha V^{-3} + \beta V^{-2} + \gamma V^{-1}) = 0$$
(20)

The Newton-Raphson iteration method can be used to solve the airflow rate in Eq. (20). In each step of the iteration, the solution is updated by

$$V_{n+1} = V_n - \frac{F(V_n)}{F(V_n)} \tag{21}$$

until convergence is reached when

$$V_{n+1} - V_n \le \varepsilon \tag{22}$$

where  $\varepsilon$  is the convergence criterion. The supply airflow rate obtained from Eq. (20) is based on the given room design temperature and supply air temperature at the diffuser.

To achieve acceptable indoor air quality, ASHRAE (2010a) requires the zone outdoor airflow  $V_{oz}$  to be determined as:

$$V_{oz} = V_{bz} / E_z \tag{23}$$

where  $V_{bz}$  is the minimum ventilation rate in the breathing zone and  $E_z$  is the zone air distribution effectiveness.  $V_{bz}$  can be obtained by:

$$V_{bz} = R_p \cdot P_z + R_a \cdot A_z \tag{24}$$

where  $A_Z$  is the net occupied floor area of the ventilation zone,  $P_Z$  is the number of people in the ventilation zone during typical usage,  $R_P$  is the outdoor airflow rate required per person, and  $R_a$  is the outdoor airflow rate required per unit area. ASHRAE (2010a) suggests  $E_Z = 1.2$  for displacement ventilation with low velocity cool air supply and  $E_Z = 1.0$  for a UFAD system with cool air supply. Lee's study (2009) in the stratified air distribution system shows that the  $E_Z$  for indoor spaces with a UFAD system varied from 1.05 to 2.0 depending on different types of indoor spaces with medium height. Empirical equations were also developed in his study to calculate  $E_Z$  based on actual design conditions. Eq. (25) was used to determine  $E_Z$  in Eq. (23) according to his study.

$$E_{z} = 1.9 + 0.93 \frac{VQ_{Rm}}{A_{f}^{2}} + 37.8 \frac{VT_{s}}{A_{f}H} + 103.68 \frac{V^{2}T_{s}}{A_{f}Hn} - 1288.8 \frac{V}{A_{f}H}$$

$$-3240 \frac{V^{2}}{A_{f}Hn} + 0.00591 \frac{Q_{Rm}}{A_{f}}$$
(25)

where n is the number of diffusers. It is important to know that Eq. (25) is an empirical equation written in SI units. The airflow rate in Eq. (25) is measured with  $m^3/s$  and obtained from Eq. (20). The airflow rate solution from Eq. (20) needs to be compared with that of Eq. (23). The design airflow rate  $V_{design}$  should be the greater of V and  $V_{oz}$ .

Finally,  $\Delta T_{oc}$  can be obtained by substituting  $V_{design}$  back into Eq. (17) and the number of diffusers can be calculated by using

$$n = \frac{V_{design}}{V_d} \tag{26}$$

# 3. Computer interface of the design calculation

To help designers using the models developed above, this study has developed a Graphical User Interface (GUI), as shown in Figure 4. The GUI allows its users to specify either diffuser or duct supply temperature. If duct supply temperature is used, the plenum heat gain ratio also needs to be given. When the plenum is shared by multiple rooms, the ratio of plenum flow rate to zonal supply flow rate can be specified to calculate the plenum air temperature increase. For a perimeter zone, solar heat flux needs to be specified. The information on supply airflow rate and effective opening area per swirl, square, and linear diffusers can be obtained from the product manual or the manufacturer. In the output section, users can analyze the thermal stratification by observing the vertical temperature difference between head and ankle. The total supply airflow rate and number of diffusers are computed based on the selected type of diffuser.

It is important to know that this design tool has not incorporated the library of minimum ventilation rate required by ASHRAE (2010a). Therefore, to finalize the total supply airflow rate, the user still needs to manually compare the output to  $V_{oz}$  as obtained in Eq. (24).

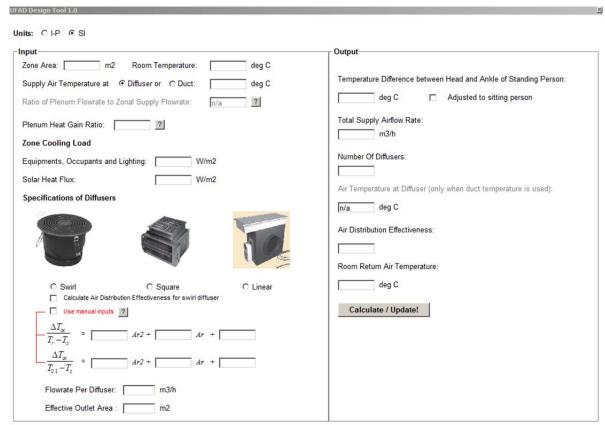


Figure 4. GUI of the design calculation

#### 4. Evaluation of the models

In order to evaluate the performance of the design calculation method, the model was compared to the measured data. Six experimental cases in the database were used for the comparison. The calculated results of the temperature difference between head and ankle, total supply airflow rate, and number of diffusers were compared with the six cases, which covered both interior and perimeter zones and three types of diffusers. Table 3 shows the conditions in the six cases.

Table 3. Specifications of the six cases in the database

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Case #	Diffuser	Floor	Room heat	Room	Supply air	Zone type		
	type	Area	extraction rate	temperature	temperature			
					at diffuser			
	[-]	$[m^2]$	$[W/m^2]$	[°C]	[°C]	[-]		
1	Swirl	20.16	38	22.9	16.3	Interior		
2	Swirl	20.16	46	22.9	16.1	Exterior		
3	Square	20.16	35	24.3	16.8	Interior		
4	Square	20.16	45	23.0	16.6	Exterior		
5	Linear	20.16	31	24.3	19.4	Interior		
6	Linear	20.16	46	22.8	15.3	Exterior		

Figs. 5 to 7 show the comparison between the computed results and the measured data for the six cases in the database. The computed results were in reasonable agreement with the data. The calculation of the number of diffusers would lead to round-off error. The supply airflow rate and temperature difference between head and ankle calculated were within 10% of the data, although the R-values are low.

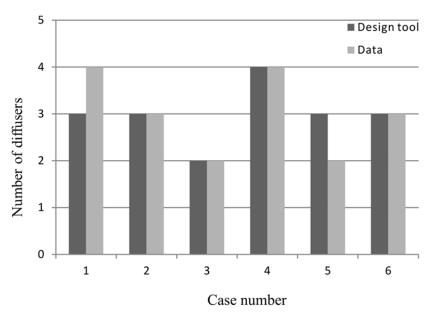


Figure 5. Comparison of number of diffusers obtained by the design tool and the data

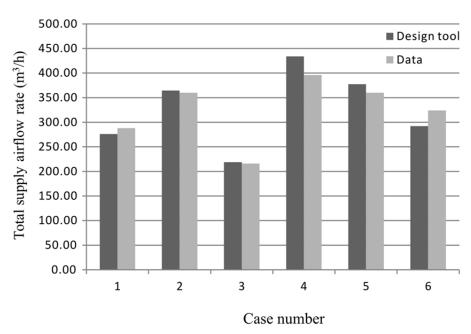


Figure 6. Comparison of total supply airflow rate obtained by the design tool and the data

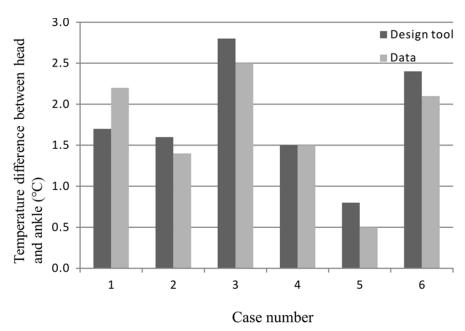


Figure 7. Comparison of temperature differences obtained by the design tool and the data

## 4. Conclusion

This investigation studied further the thermal environment in indoor spaces with the UFAD systems. The study led to the following important findings:

• The thermally stratified environment of the UFAD systems can be effectively modeled by using the Archimedes number which represents the ratio of the buoyancy force from the

- heat sources in the space over the inertial force from the air supply from the diffuser. The temperature difference between the head and ankle levels in the UFAD systems can be correlated with the Archimedes number.
- By using a database of thermal environment information, this study developed a model to
  determine the supply airflow rate from the UFAD systems which can maintain an
  acceptable temperature difference between the head and ankle levels. A graphical user
  interface was also developed to assist designers in using the model. For design purposes,
  the airflow rate obtained needs to be compared with the minimum required airflow rate to
  ensure that the indoor air quality is acceptable.
- The model developed was evaluated by using the experimental data from both exterior and interior zones of the buildings in the database. The model, with the given design conditions, can calculate the correct number of diffusers, total supply airflow rate, and temperature difference between the head and ankle levels of occupants.

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