3. At a particular temperature, the surface tension of water is 0.073 N/m. Under ideal conditions, the contact angle between glass and water is zero. A student in a laboratory observes water in a glass capillary tube with a diameter of 0.1 mm. What is the theoretical height of the capillary rise?

(A) 0.00020 m
(B) 0.013 m
(C) 0.045 m
(D) 0.30 m

\[ h = \frac{4\sigma\cos \theta}{\gamma d} \]

\[ = \frac{4(0.073)(\cos 0)}{(4\pi)(0.0001)} = 0.298 \text{ m} \]

4. What is the atmospheric pressure on a planet if the absolute pressure is 100 kPa and the gage pressure is 10 kPa?

(A) 10 kPa
(B) 80 kPa
(C) 90 kPa
(D) 100 kPa

\[ P_{\text{obs}} = P_0 + P_{\text{atm}} \]

\[ 100 \text{ kPa} = 10 \text{ kPa} + P_{\text{atm}} \]

\[ P_{\text{atm}} = 90 \text{ kPa} \]

5. 100 g of water are mixed with 150 g of alcohol (\( \rho = 790 \text{ kg/m}^3 \)). What is the specific gravity of the resulting mixture, assuming that the two fluids mix completely?

(A) 0.63
(B) 0.82
(C) 0.86
(D) 0.95

\[ S_G = \frac{\rho}{\rho_w} \]

\[ S_G = \frac{790 \text{ kg/m}^3}{150 \text{ kg/m}^3} = 0.53 \]

\[ S_G_{\text{mix}} = \frac{0.79(150) + (1)(100)}{150 + 100} = 0.87 \]
4. Kinematic viscosity can be expressed in which of the following units?

(A) ft$^2$/sec
(B) sec$^2$/ft
(C) lbm sec$^2$/ft
(D) lbm/sec

5. Which of the following does not affect the rise or fall of liquid in a small-diameter capillary tube?

(A) adhesive forces
(B) cohesive forces
(C) surface tension
(D) viscosity of the fluid

6. The film width in a surface tension experiment is 10 cm. If mercury is the fluid (surface tension = 0.52 N/m), what is the maximum force that can be applied without breaking the membrane? Neglect gravitational force.

\[ F = 2 \sigma L = (2)(0.52 \text{ N/m})(0.1 \text{ m}) \]

(A) 0.1 N
(B) 1.0 N
(C) 2.0 N
(D) 3.4 N
1. What height of mercury column is equivalent to a pressure of 100 psig? The density of mercury is 848 lbm/ft³.

\[
P = \frac{1}{\gamma_m} \left( \frac{12 \text{ in}^2}{1 \text{ ft}^2} \right)^2 \left( \frac{32 \text{ ft}^4}{1 \text{ in}^2} \right) \frac{100 \text{ lb/in}^2}{848 \text{ lbm/ft}^3} \]

\[
P = 14400 = 848(1)h
\]

\[
h = 16.98 \text{ ft}
\]

(A) 2 ft  
(B) 4 ft  
(C) 11 ft  
(D) 17 ft

2. A fluid with a vapor pressure of 0.2 Pa and a specific gravity of 12 is used in a barometer. If the fluid's column height is 1 m, what is the atmospheric pressure?

\[
\rho_v = 0.2 \text{ Pa} \quad \gamma_{\text{Hg}} = 9810 \frac{\text{N}}{\text{m}^3}
\]

\[
SG = 12 = \frac{\gamma}{\gamma_v}
\]

\[
\gamma = (12)(9810 \frac{\text{N}}{\text{m}^3}) = 117720 \frac{\text{N}}{\text{m}^3}
\]

\[
h = 1 \text{ m}
\]

\[
P_{\text{atm}} = P_v + \gamma h = 0.2 \text{ Pa} + (117720 \frac{\text{N}}{\text{m}^3})(1 \text{ m}) = 117.7 \text{ kPa}
\]

(A) 9.80 kPa  
(B) 11.76 kPa  
(C) 101.3 kPa  
(D) 117.7 kPa

3. One leg of a mercury U-tube manometer is connected to a pipe containing water under a gage pressure of 14.2 lbf/in². The mercury in this leg stands 30 in below the water. What is the height of mercury in the other leg, which is open to the air? The specific gravity of mercury is 13.6.

\[
\rho_{\text{Hg}} = 14.7 \text{ psi} = 101.3 \text{ kPa}
\]

\[
P_{\text{atm}} = 14.7 \text{ psi} = 101.3 \text{ kPa}
\]

\[
P_c + P_2 = P_a + P_{\text{atm}}
\]

\[
14.2 \text{ psi} + \delta h_B = \gamma h_A + 14.7 \text{ psi}
\]

\[
(14.2 \frac{14}{1 \text{ in}^2})(12 \text{ in}^2) + (62.4 \frac{1}{2 \text{ in}^2})(2.5 \frac{1}{2} \text{ ft}) =
\]

\[
(818.6 \frac{1}{2 \text{ in}^2})(h_A) + (14.7 \text{ psi})(12 \text{ in}^2)
\]

\[
204.8 + 15.6 = 818.6 h_A + 214.8
\]

\[
2200.8 = 818.6 h_A
\]

\[
h_A = 2.59 \text{ ft}
\]

(A) 0.7 ft  
(B) 1.5 ft  
(C) 2.6 ft  
(D) 3.2 ft
What is the resultant force on one side of a 10 in diameter vertical circular plate standing at the bottom of a 10 ft pool of water?

(A) 326 lbf  
(B) 386 lbf  
(C) 451 lbf  
(D) 643 lbf

\[
F = \gamma h
\]

\[
A = \frac{\pi}{4}(0.5)^2 = 0.5\pi \text{ ft}^2
\]

\[
\gamma = 62.4 \frac{lb}{ft^2}
\]

\[
h = 9.5\text{ ft}
\]

\[
F = 62.4(0.54)(9.5) = 323 \text{ lbf}
\]

5. A special closed tank with the dimensions shown contains water. If the pressure of the air is 100 psig, what is the pressure at point P, which is located halfway up the inclined wall?

\[
P_p = P_{atm} + \gamma gh
\]

\[
P_{atm} = \frac{1}{1216}(12)^2 = 19400 \frac{lb}{ft^2}
\]

\[
P_p = 19400 \frac{lb}{ft^2} + 62.4 \frac{lb}{ft^2} (65 \text{ ft}) = 18456 \frac{lb}{ft^2}
\]

\[
P_p = 18456 \frac{lb}{ft^2} \left( \frac{1}{1216} \right)^{-1} = 128 \frac{lb}{in^2}
\]

\[
P_{ave} = \frac{1}{2} \gamma (h_1 + h_2) = 19400 \frac{lb}{ft^2} + \frac{1}{2} (62.4 \frac{lb}{ft^2})(30+100) \text{ ft} = 18456 \frac{lb}{ft^2} = 128 \frac{lb}{in^2}
\]

\[
F = P_{ave} \cdot A
\]
6. A triangular gate with a horizontal base 4 ft long and an altitude of 6 ft is inclined 45° from the vertical with the vertex pointing upward. The hinged horizontal base of the gate is 9 ft below the water surface. What normal force must be applied at the vertex of the gate to keep it closed?

(A) 1430 lbf
(B) 1570 lbf
(C) 1670 lbf
(D) 1720 lbf

Water flows through a multisectional pipe placed horizontally on the ground. The velocity is 3.0 m/s at the entrance and 2.1 m/s at the exit. What is the pressure difference between these two points? Neglect friction.

(A) 0.2 kPa
(B) 2.3 kPa
(C) 28 kPa
(D) 110 kPa
What is the mass flow rate of a liquid ($\rho = 0.690 \text{ g/cm}^3$) flowing through a 5 cm (inside diameter) pipe at 8.3 m/s?

- (A) 11 kg/s
- (B) 69 kg/s
- (C) 140 kg/s
- (D) 340 kg/s

\[
\dot{m} = \rho \cdot V \\
\dot{m} = \frac{(690 \text{ kg/m}^3) \cdot (8.3 \text{ m/s}) \cdot (0.002 \text{ m}^2)}{5} \\
\dot{m} = 11.5 \text{ kg/s}
\]

3. The mean velocity of 100°F water in a 1.76 in (inside diameter) tube is 5 ft/sec. The kinematic viscosity is $\nu = 7.39 \times 10^{-6}$ ft$^2$/sec. What is the Reynolds number?

- (A) $7.9 \times 10^3$
- (B) $8.3 \times 10^3$
- (C) $8.8 \times 10^4$
- (D) $9.9 \times 10^4$

\[
\Re = \frac{V \cdot D}{\nu} = \frac{5 \text{ ft/sec} \cdot 1.76 \text{ in}}{7.39 \times 10^{-6} \text{ ft}^2/\text{sec}} \\
\Re = 9.9 \times 10^4
\]

4. What is the head loss for water flowing through a horizontal pipe if the gage pressure at point 1 is 1.03 kPa, the gage pressure at point 2 downstream is 1.00 kPa, and the velocity is constant?

- (A) $3.1 \times 10^{-3}$ m
- (B) $3.1 \times 10^{-2}$ m
- (C) $2.3 \times 10^{-2}$ m
- (D) 2.3 m

\[
\Delta h = \frac{P_1 - P_2}{\gamma} = \frac{1.03 \text{ kPa} - 1 \text{ kPa}}{9.81 \text{ kJ/} \text{kg}} = 0.003 = 3 \times 10^{-3} \text{ m}
\]

5. The hydraulic radius is

- (A) the mean radius of the pipe.
- (B) the radius of the pipe bend on the line.
- (C) the wetted perimeter of a conduit divided by the area of flow.
- (D) the cross-sectional fluid area divided by the wetted perimeter.

\[
R_h = \frac{A}{P}
\]
For the following problems use the NCEES FE Reference Handbook as your only reference.

6. What horizontal force is required to hold the plate stationary against the water jet? (All of the water leaves parallel to the plate.)

\[
F = 0.05 \frac{M^2}{s} (1000 \text{ kg} \cdot \text{m}^3) (D - 0.7 \text{ m}^2) \\
F = 35.2 \text{ N}
\]

(A) 17.7 N  
(B) 35.4 N  
(C) 42.2 N  
(D) 67.5 N

\[
Q = 0.05 \frac{m^3}{s} \\
D = 30 \text{ cm} \\
A = \frac{\pi}{4} (0.3)^2 = 0.071 \text{ m}^2 \\
V = \frac{Q}{A} = \frac{0.05 \frac{m^3}{s}}{0.071 \text{ m}^2} = 0.704 \frac{m}{s}
\]

7. Water flows with a velocity of 17 ft/sec through 18 ft of cast-iron pipe (specific roughness = 0.00085 ft). The pipe has an inside diameter of 1.7 in. The kinematic viscosity of the water is \(5.94 \times 10^{-6} \text{ ft}^2/\text{sec}\). The loss coefficient for the standard elbow is 0.9. What percentage of the total head loss is caused by the elbow?

\[
Re = \frac{V D}{\nu} = 17 \frac{(\frac{17}{12})}{(\frac{5.94}{10^{-6}})} = 40541.3 \\
Re = 4.1 (10)^5 \\
Moody chart plot Re and \(e/D\) \\
f = 0.031
\]

\[
h_c = f \frac{L}{D} \frac{V^2}{2g} = (0.031)(0.031)(17^2 \frac{12}{2(1.7)}) = 567.8 ft \\
Fitting
\]

\[
h_c = C \frac{V^2}{2g} = (0.9) \frac{12^2}{2(1.7)} = 130 ft \\
\%	ext{head loss due to fitting:} \\
\%	ext{head loss due to fitting} = \frac{130 ft}{(567.8 + 130) ft} \approx 18\%
\]
1. A 70% efficient pump pumps 60°C water from ground level to a height of 5 m. How much power is used if the flow rate is 10 m³/s?

(A) 80 kW
(B) 220 kW
(C) 700 kW
(D) 950 kW

\[ \text{Brake Power} = \frac{\rho y H Q}{\eta} \]

\[ BP = \frac{9642 \cdot 5 \cdot 10^2}{0.7} \]

\[ BP = 688 \text{ kW} \]

2. The acoustic velocity in a specific gas depends only on which of the following variables?

(A) \( c_p \), specific heat at constant pressure
(B) \( k \), ratio of specific heats
(C) \( c_v \), specific heat at constant temperature
(D) \( T \), absolute temperature

\[ C = \sqrt{kRT} \]

\[ k = \frac{c_p}{c_v} \]
4. The velocity of the water in the stream is 1.2 m/s. What is the height of water in the pitot tube?

\[ h = \frac{V^2}{2g} \]

(A) 3.7 cm  
(B) 4.6 cm  
(C) 7.3 cm  
(D) 9.2 cm

5. A venturi meter with a diameter of 6 in at the throat is installed in an 18 in water main. A differential manometer gauge is partly filled with mercury (the remainder of the tube is filled with water) and connected to the meter at the throat and inlet. The mercury column stands 15 in higher in one leg than in the other. Neglecting friction, what is the flow through the meter? The specific gravity of mercury is 13.6.

\[ Q = \frac{C_n A_n}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2g\left(\frac{P_1}{g} + z - \frac{P_2}{g} - z_2\right)} \]

(A) 3.70 ft\(^3\)/sec  
(B) 6.29 ft\(^3\)/sec  
(C) 8.62 ft\(^3\)/sec  
(D) 10.5 ft\(^3\)/sec
6. What is the velocity of water under a 50 ft head discharging through a 1 in diameter round-edged orifice?

\[ V = \sqrt{2gh} = \sqrt{2(\pi)(50)} = 56 \text{ ft/sec} \]

Note
VENA CONTRACTA PS 112 TABLE
\[ V_c = CA = 0.62 \text{(A)} \]

(A) 3.6 ft/sec  
(B) 9.8 ft/sec  
(C) 25 ft/sec  
(D) 56 ft/sec.

7. A 1:1 model of a torpedo is tested in a wind tunnel according to the Reynolds number criterion. At the testing temperature, \( \nu_{\text{air}} = 1.41 \times 10^{-6} \) and \( \nu_{\text{water}} = 1.31 \times 10^{-6} \). If the velocity of the torpedo in water is 7 m/s, what should be the air velocity in the wind tunnel?

(A) 0.6 m/s  
(B) 7.0 m/s  
(C) 18 m/s  
(D) 75 m/s

\[ \frac{V_{\text{air}}}{V} = \left( \frac{\nu_{\text{air}}}{\nu_{\text{water}}} \right)^{1/3} \]

\[ V_{\text{air}} = \frac{V_{\text{water}}}{\left( \frac{\nu_{\text{air}}}{\nu_{\text{water}}} \right)^{1/3}} \]

\[ V_{\text{air}} = 1.41(10)^{5} \left( \frac{7}{5} \right) \left( \frac{1.31(10)^{6}}{1.41(10)^{5}} \right)^{1/3} = 75.3 \frac{m}{s} \]
Problems 8 and 9 refer to the following situation.

A sharp-edged orifice with a 2 in diameter opening is located in the vertical side of a large tank. The coefficient of contraction for the orifice is 0.62, and the coefficient of velocity is 0.98. The orifice discharges under a hydraulic head of 16 ft.

8. What is the minimum diameter of the jet?
   (A) 1.24 in  
   (B) 1.57 in  
   (C) 2.00 in  
   (D) 2.54 in

![](image)

**AREA VENA CONTRACTA (A_vc)**  
\[ A_{vc} = C_c \cdot A = 0.62 \left( \frac{\pi}{4} \cdot 2^2 \right) = 1.948 \text{ in}^2 \]

\[ A_{vc} = \frac{\pi}{4} \cdot d^2 = 1.948 \text{ in}^2 \]
\[ d = 1.575 \text{ in} \]

---

9. What is the velocity at the vena contracta?
   (A) 5.54 ft/sec  
   (B) 10.8 ft/sec  
   (C) 17.4 ft/sec  
   (D) 31.5 ft/sec

\[ Q = VA \]
\[ V = \frac{Q}{A} \]
\[ Q = C_A \sqrt{2gh} \]
\[ V = C_v \sqrt{2gh} \]
\[ V = 0.98 \sqrt{2(32.2)(16)} \]
\[ V = 31.45 \text{ ft/sec} \]
Problems 10-13 refer to the following situation.

The bottom of a tall tank sits on level ground. The tank is kept filled to a depth of 15 ft, while water discharges at a constant rate through a 0.5 ft diameter hole in the tank side. The center of the hole is 10 ft from the water surface above. The coefficient of velocity for the hole is essentially 1.0.

10. What horizontal distance will the water jet travel before hitting the ground?

(A) 6.5 ft
(B) 7.1 ft
(C) 7.5 ft
(D) 14 ft

\[ V = C_v \sqrt{2gh} = (1) \sqrt{2(32.2)(10)} = 25.4 \text{ ft/sec} \]

\[ x = v \cdot t = 25.4 \frac{\text{ft}}{\text{sec}} \times (0.557 \text{ sec}) \]

\[ x = 14.2 \text{ ft} \]

11. What is the velocity of the water jet?

(A) 21.9 ft/sec
(B) 25.4 ft/sec
(C) 26.9 ft/sec
(D) 30.6 ft/sec

\[ F = \gamma hA = 122.5 \text{ lb} \]
12. If the hole is represented by a sharp-edged orifice with a coefficient of discharge of 0.61, what will be the rate of discharge?

(A) 2.68 ft³/sec
(B) 3.04 ft³/sec
(C) 3.27 ft³/sec
(D) 3.72 ft³/sec

\[ Q = C \cdot \sqrt{A} = (0.61) \left( \frac{\text{ft}}{\text{sec}} \right)^{\frac{3}{2}} \left( \frac{0.5 \text{ ft}}{\text{sec}} \right)^2 = 3.04 \text{ ft}^3/\text{sec} \]

13. Assume the orifice can be moved to any point on the side of the tank. What distance below the water surface should the orifice be located such that the horizontal distance traveled by the jet (before hitting the ground) is the greatest?

(A) 7.5 ft
(B) 8.8 ft
(C) 10 ft
(D) 11 ft
14. A 2 m tall, 0.5 m inside diameter tank is filled with water. A 10 cm hole is opened 0.75 m from the bottom of the tank. What is the velocity of the exiting water? Ignore all orifice losses.

(A) 4.75 m/s  
(B) 4.80 m/s  
(C) 4.85 m/s  
(D) 4.95 m/s

\[ A = 0.01 \text{ m}^2 \]
\[ D = 0.1 \text{ m} \]
\[ V = \sqrt{2gh} \]
\[ V = \sqrt{2 \times 9.81 \times 1.25} \]
\[ V = 4.95 \text{ m/s} \]
1. Reynolds number may be calculated from:

(A) diameter, velocity, and absolute viscosity
(B) diameter, velocity, and surface tension
(C) diameter, density, and kinematic viscosity
(D) diameter, density, and absolute viscosity
(E) characteristic length, mass flow rate per unit area, and absolute viscosity

\[ R_e = \frac{uD}{v} \]

2. Roughening the leading edge of a smooth sphere will reduce its drag coefficient because

(A) the wake width increases
(B) the separation points move to the front of the sphere
(C) the wake eddies increase
(D) the boundary layer becomes turbulent
(E) Stoke's law becomes applicable

3. What is the hydraulic radius of a rectangular flume 2 feet high and 4 feet wide which is running half full?

\[ A = \left( 1 \frac{ft}{sec} \right) \left( \frac{ft}{2} \right) = \frac{1}{2} \text{ sq ft} \]

\[ P = \frac{4}{6} \left( \frac{f}{s} \right) + 1 \left( \frac{f}{s} \right) + 1 \left( \frac{f}{s} \right) = \frac{12}{6} \frac{f}{s} \]

\[ R_h = \frac{A}{P} = \frac{\frac{1}{2} \text{ sq ft}}{\frac{12}{6} \frac{f}{s}} = 0.67 \]

4. Water flows at 10 ft/sec in a 1” inside diameter pipe. What is the velocity if the pipe suddenly increases in diameter to 2”?

\[ V_A = V_A \]

\[ 10 \text{ ft/sec} = V_L (0.00541) \]

\[ V_L = \frac{2.47}{0.00541} \]

5. What pressure differential exists across a perfect venturi with an area reduction ratio of (3:1) if water is flowing through the throat at 40 fps?

\[ \frac{V_L}{V_T} = \frac{V_L}{V_T} \]

\[ V = U \]

\[ (U_A) = (U_L) \]

\[ \frac{V_L}{V_T} = \frac{\frac{1}{3}}{\frac{1}{2}} \]

\[ 40 \text{ fps} (3) = V_L (3) \]

\[ V_L = 13.3 \text{ fps} \]

\[ P_2 - P_1 = \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{6} - \frac{1}{3}}{2(0.00541)} = 22.1 \]
6. If 'L' is defined as the characteristic length, what does the quantity \( \frac{v^2}{Lg} \) represent?

(A) velocity pressure  
(B) Reynolds number  
(C) Froude number  
(D) total pressure  
(E) static pressure

7. Minor losses through valves, fittings, diameter changes, and bends are proportional to

(A) total head  
(B) dynamic head  
(C) static head  
(D) wet head  
(E) velocity

\[ h_f = \frac{v^2}{2g} \]

8. The horsepower of an ideal pump used to move 2 cfs of water into a tank 50 feet above the pump is most nearly

(A) 2  
(B) 11  
(C) 290  
(D) 1213  
(E) 6240

\[ HP = \frac{Q \times H_0}{550} = \frac{(62.4 \frac{ft}{s})(50 \text{ ft})(2 \frac{ft^2}{s})}{550} = 11.3 \] HP

9. A horizontal pipe section 1000 feet long has a total energy loss of 26.2 feet. If the inside pipe diameter is 12 inches and the flow velocity is 10 ft/sec, what is the Darcy-Weisbach friction coefficient?

(A) 0.0170  
(B) 0.0080  
(C) 0.0017  
(D) 0.0002  
(E) 0.0008

\[ h_f = f \frac{L}{D} \frac{v^2}{2g} \]

\[ 26.2 = f \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \frac{10^2}{2(32.2)} \]

\[ f = 0.0148 \]

10. The Reynolds number for a 1-foot diameter sphere moving through a fluid (specific gravity of 1.22, absolute viscosity of 0.00122 lb-sec/ft²) at 10 ft/sec is approximately

(A) 20  
(B) 200  
(C) 2,000  
(D) 20,000  
(E) 200,000

\[ Re = \frac{VD \rho}{\mu} = \frac{10 \frac{ft}{s} (1.2)(76.13)}{0.3920} \]

\[ Re = 19,380 \approx 2000 \]

\[ \rho = 0.00122 \frac{lb-sec}{ft^2} \left( \frac{10 \text{ ft}}{1 \text{ ft}} \right) \]

\[ \mu = 0.3920 \left( \frac{lbm}{in-sec} \right) = 0.3928 \]
11. Water is flowing in a circular pipe between points 1 and 2. The pressure at point 1 is 16.8 psia. The pressure and velocity at point 2 are 17.2 psia and 6.2 ft/sec, respectively. Points 1 and 2 are at the same elevation. Neglecting friction, what is the velocity at point 1?

(A) 97.8 ft/sec
(B) 9.9
(C) 21.0
(D) 1.52
(E) 4.58

12. The critical depth in a rectangular channel 8 feet wide flowing at a critical velocity of 2 ft/sec is approximately

(A) 0.12 feet
(B) 0.00
(C) 4.00
(D) 0.06
(E) 0.08

13. At a certain section of pipe, water is flowing at a pressure of 80 psi and with a linear velocity of 9 ft/sec. What is the total flow work for 1.5 cubic feet of water which pass that section?

(A) 18,000 ft-lb
(B) 36,000
(C) 120
(D) 12,000
(E) 0

\[ P = 80 \left( \frac{11}{16} \right)^2 = 11520 \left( \frac{11}{32} \right)^2 \]

\[ V = 9 \frac{ft}{sec} \]

\[ H = 1.5 \frac{ft}{sec} \]

\[ W = P \cdot V \cdot H = 11520 \left( \frac{11}{32} \right)^2 \left( 1.5 \frac{ft}{sec} \right) = 17280 \text{ ft-lb} \]
FLUIDS-17

Find the pressure in the tank from the manometer readings shown.

\[ P_{\text{atm}} + (\rho g h)_1 = (\rho g h)_2 + (\rho g h)_3 + P_{\text{Tank}} \]

\[ 100 \text{ kPa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m}) = (750 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m}) + (500 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) + P_T \]

\[ 100000 + 9810 = 22072.5 + 490.5 + P_T \]

\[ P_T = 107112.5 \text{ Pa} \]

\[ p_2 - p_1 = \rho g (z_1 - z_2) \]
\[ p_3 - p_2 = \rho B g (z_2 - z_3) \]
\[ p_4 - p_3 = \rho A g (z_3 - z_4) \]
\[ p_4 - p_1 = (p_4 - p_3) + (p_3 - p_2) + (p_2 - p_1) \]
\[ p_4 = p_1 + g (\rho C (z_1 - z_2) + \rho B (z_2 - z_3) + \rho A (z_3 - z_4)) \]
\[ = 100000 \text{ Pa} + (9.81 \text{ m/s}^2) [(1000 \text{ kg/m}^3)(1 \text{ m}) \]
\[ + (750 \text{ kg/m}^3)(-0.3 \text{ m}) + (500 \text{ kg/m}^3)(0.1 \text{ m})] \]
\[ = 108100 \text{ Pa} \text{ (108 kPa)} \]

The answer is (B).
A jet aircraft is flying at a speed of 1700 km/h. The air temperature is 20°C. The molecular weight of air is 29 g/mol. What is the Mach number of the aircraft?

\[
C = \sqrt{\frac{PR}{\mu T}} = \sqrt{\frac{(1.4) \left( \frac{8314}{\text{mol} \cdot \text{K}} \right) (20 + 273) \text{K} \left( \frac{\text{J} \cdot \text{K}^{-1}}{\text{mol} \cdot \text{K}} \right) 29 \text{ g/mol} \left( \frac{\text{g}}{1000 \text{ g}} \right)}{\frac{117600 \text{ m}^2}{\text{s}^2}}}
\]

\[
C = 343 \frac{\text{m}}{\text{s}}
\]

\[
M_a = \frac{V}{C} = \frac{1700 \text{ km/h} \left( \frac{1000 \text{ m}}{1 \text{ km}} \right)}{343 \frac{\text{m}}{\text{s}}} = 1.38
\]

What is the Mach number of a jet of oxygen gas at standard conditions (1 atm and 25°C) with a speed of 450 m/s? (Assume that for oxygen \( k = 1.40 \) and \( R = 0.260 \text{ kJ/kg \cdot K} \).)

\[
C = \sqrt{\frac{PR}{\mu T}} = \sqrt{\frac{(1.4) \left( 0.26 \frac{\text{kJ}}{\text{kg \cdot K}} \right) (25 + 273) \text{K} \left( \frac{1000 \text{ m} \cdot \text{K}}{\text{kJ}} \right) \left( \frac{1\text{ kg \cdot m} \cdot \text{s}^{-2}}{\text{m} \cdot \text{K}} \right)}{\frac{108472 \text{ m}^2}{\text{s}^2}}}
\]

\[
C = \sqrt{108472 \frac{\text{m}^2}{\text{s}^2}} = 329.351 \frac{\text{m}}{\text{s}}
\]

\[
M_a = \frac{V}{C} = \frac{450 \frac{\text{m}}{\text{s}}}{329.351 \frac{\text{m}}{\text{s}}} = 1.366
\]
A 2 mm (inside diameter) glass tube is placed in a container of mercury. An angle of 40° is measured as illustrated. The density and surface tension of mercury are 13550 kg/m³ and 37.5 x 10⁻² N/m, respectively. How high will the mercury rise or be depressed in the tube as a result of capillary action?

\[ h = \frac{4 \sigma \cos \beta}{\gamma \Delta} \]

\[ \sigma = 37.5 \times 10^{-2} \text{ N/m} \]

\[ \gamma = 13550 \text{ kg/m}^3 \]

\[ \Delta = 2 \text{ mm} \]

\[ \beta = 180° - 40° = 140° \]

\[ h = \frac{4 \times 37.5 \times 10^{-2} \times \cos 140°}{13550 \times 2 \times 9.81 \times 10^{-3}} \approx -0.004 \text{ m} = -0.4 \text{ mm} \]
A sliding-plate viscometer is used to measure the viscosity of a Newtonian fluid. A force of 25 N is required to keep the top plate moving at a constant velocity of 5 m/s. What is the viscosity of the fluid?

\[ \frac{dV}{dy} = \frac{V}{b} = \frac{5 \text{ m/s}}{0.001 \text{ m}} \]

\[ \tau = \frac{F}{A} \quad \Rightarrow \quad \tau = \mu \frac{dV}{dy} = \frac{V}{b} \]

\[ \tau = \frac{F}{A} = \frac{V}{b} \]

\[ \frac{5 \text{ N}}{(25 \text{ cm})(50 \text{ cm}) \left( \frac{m}{100 \text{ cm}} \right)} = \mu \frac{5 \text{ m/s}}{0.001 \text{ m}} \]

\[ \mu = 4 \left( \frac{10}{1} \right)^4 \frac{N \cdot \text{s}}{m^2} \]
Example: What is the pressure 10 feet below the surface of a swimming pool?

\[ P = \gamma h = 62.9 \frac{lb}{ft^3} \cdot 10 \text{ ft} \]

\[ P = 624 \frac{lb}{ft^2} \]
Example: The tank of water has a 3-m column of gasoline (S.G. = 0.73) above it. Atmospheric pressure is 101 kPa. Compute the pressure on the bottom of the tank.

\[
P_b = P_{atm} + \rho \cdot g \cdot h_g + \rho_\text{water} \cdot g \cdot h_w
\]

\[
P_b = 101 \text{ kPa} + 0.73 \left(9810 \text{ N/m}^3\right)(3 \text{ m}) + 9810 \frac{\text{N}}{\text{m}^3} (2 \text{ m})
\]

\[
P_b = 101(10^3) \text{ Pa} + 21483.9 \text{ Pa} + 19620 \text{ Pa}
\]

\[
P_b = 142103.9 \text{ Pa}
\]
Example: Use the manometer measurements to compute the pressure in the pipe.

\[ P_i + (\delta h)_{\text{w}} = (\delta h)_{\text{m}} + P_{\text{atm}} \]

\[ P_i = \gamma h_{\text{m}} + P_{\text{atm}} - \delta h \omega = 13.6 \left( \frac{62.4 \text{ lb}}{ft^3} \right) (2 \text{ ft}) \]

\[ P_i = 13.6 \left( \frac{62.4 \text{ lb}}{ft^3} \right) (2 \text{ ft}) + 14.1 \left( \frac{14 \text{ lb}}{in^2} \right) \left( \frac{12 \text{ in}}{ft} \right)^2 - 62.4 \text{ lb} \left( \frac{1.5 \text{ ft}}{ft^2} \right) \]

\[ = 1697.28 \frac{\text{ lb}}{ft^2} + 2116.8 \frac{\text{ lb}}{ft^2} - 93.6 \frac{\text{ lb}}{ft^2} = 3907.6 \frac{\text{ lb}}{ft^2} \]
Example: Compute the magnitude and location of the resultant force.

\[ F_r = (P_{\text{net}} + sгh_с \sin \theta)A \]

\[
\begin{align*}
A &= \frac{1}{2} (2m)(1.5m) = 1.5 \text{m}^2 \\
\text{water} & \quad 2.0 \text{m} \\
\text{c.g.} & \quad .c.p. \quad t \quad h_c \\
\text{h}_c & \quad 45^\circ \\
& \quad 1.5 \text{m} \\
\end{align*}
\]

\[
\text{moment of inertia } I_c = bh^3/36 = \frac{(2m)(1.5)^3}{36} = 0.188 \text{ m}^4
\]

\[
\sin 45^\circ = \frac{h}{3(1.5)} \quad h = 0.707 \quad h_c = 2.75\text{m} - h = 2.043 \text{m}
\]

\[
Y_c = \frac{h_c}{\sin 45^\circ} = \frac{2.043 \text{m}}{0.707} = 2.889 \text{m}
\]

\[
Y_{cp} = Y_c + \frac{I_c}{Y_c A} = 2.889 \text{m} + \frac{0.188 \text{ m}^4}{(2.889 \text{m})(1.5 \text{m}^2)} = 2.932 \text{m}
\]

\[
F_{r2} = (1.9 Y_c \sin \theta)A = (1.9 \times 2.889 \text{m} \sin 45^\circ)A = 9810 \frac{N}{\text{m}^2} (2.889 \text{m}) \sin 45^\circ (1.5 \text{m}^2)
\]

\[
F_{r2} = 30060 \text{ N} = 30 \text{ kN}
\]

or

\[
F_{r2} = Y_c h_c A = 30 \text{ kN}
\]
Example: Compute the force on the curved corner for a unit width.

\[ F = \gamma \cdot A = \gamma \cdot \frac{1}{4} \pi r^2 = 7.069 \text{ m}^3 \]

\[ \frac{F_H}{h} = \gamma \cdot h \cdot A = (9810 \text{ N/m}^3)(10\text{m} + 1.5\text{m})(3\text{m})(1\text{m}) = 33644.5 \text{ N} \]

\[ F_V = \gamma \cdot V = (9810 \text{ N/m}^3) \left(30 \text{ m}^3 + 7.069 \text{ m}^3 \right) = 363642.8 \text{ N} \]

\[ F = \sqrt{F_H^2 + F_V^2} = 496770 \text{ N} \]
**Example:** Compute the force in the rope.

![Diagram showing a 3 ft segment of a rope with wood 1 ft into the paper, with 2 ft of rope and 1 ft of wood.]

\[ V_{\text{wood}} = (2\, \text{ft})(1\, \text{ft})(1\, \text{ft}) = 2\, \text{ft}^3 \]

**Buoyancy (F_B):**

\[ F_B = \gamma_w V_{\text{wood}} = 62.4 \frac{\text{lb}}{\text{ft}^3} (2\, \text{ft}^3) = 124.8\, \text{lb} \]

**Weight of Wood:**

\[ \omega = 40 \frac{\text{lb}}{\text{ft}^3} (2\, \text{ft}^3) = 80\, \text{lb} \]

**FBD:**

\[ F_B - \omega - F_r = 0 \]

\[ F_r = -\omega + F_B = -80\, \text{lb} + 124.8\, \text{lb} \]

\[ F_r = 44.8\, \text{lb} \]
**Example:** Three kN/s of water flows through the pipeline reducer. Determine the flow rate and velocity in the 300 mm and 200 mm pipes.

\[
\text{Mass Flow Rate (}\dot{m}\text{)}
\]

\[
\dot{m} = \rho Q = \rho VA
\]

\[
\dot{m} = 2 \text{ kN/s} = \frac{2 \text{ kg/s}}{\text{m}^3} = \frac{2}{\text{m}^3}
\]

\[
Q = \frac{\dot{m}}{\rho} = \frac{2 \text{ kg/s}}{9.81 \text{ kg/m}^3} = 0.2058 \text{ m}^3/\text{s}
\]

\[
Q = VA = VA_1
\]

\[
V_1 = \frac{Q}{A_1} = \frac{0.2058 \text{ m}^3/\text{s}}{\frac{\pi (0.3)^2}{4}} = 4.33 \text{ m/s}
\]

\[
V_2 = \frac{Q}{A_2} = \frac{0.2058 \text{ m}^3/\text{s}}{\frac{\pi (0.2)^2}{4}} = 9.73 \text{ m/s}
\]
Example: Water is flowing at 0.884 m$^3$/s through a 15 cm diameter pipe, that has a 90$^\circ$ bend. What is the reaction on the water in the z-direction in the bend?

**IMPULSE MOMENTUM**

\[ \sum F_z = \int \dot{Q} (v_z - v) \]

\[ F_z = (1000 \text{ kg}) (0.884 \text{ m}$^3$/s $\cdot$ (0 - 50)) \]

\[ F_z = -44200 \text{ N} \]

\[ R_z = 44200 \text{ N} \]

\[ Q = 0.884 \text{ m}^3/\text{s} \]

\[ D = 0.15 \text{ m} \]

\[ A = \frac{\pi D^2}{4} (0.15) \approx 0.018 \text{ m}^2 \]

\[ Q = AV \]

\[ V = \frac{0.884 \text{ m}^3/\text{s}}{0.018 \text{ m}^2} = 49.1 \text{ m}$/s
**Example:** The height of water in the pitot tube is measured to be 7.3 cm. What is the velocity at that point in the flow.

\[ z_1 + \frac{P_1}{\gamma} + \frac{v^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{v^2}{2g} \]

\[ \frac{P_1}{\gamma} = \frac{P_2}{\gamma} + \frac{v^2}{2g} \]

\[ \frac{P_1 - P_2}{\gamma} = \frac{v^2}{2g} \]

**(Change in Pressure Head = Velocity Head)**

\[ h_v = \frac{v^2}{2g} \]

\[ v = \sqrt{2gh} = \sqrt{2 \times (9.81 \text{ m/s}^2) \times (0.073 \text{ m})} = 1.2 \text{ m/s} \]
The density of air flowing in a duct is 1.15 kg/m$^3$. A pitot tube is placed in the duct as shown. The static pressure in the duct is measured with a wall tap and pressure gage. Use the gage readings to determine the velocity of the air.

\[ V = \sqrt{\frac{2}{\rho} (p_0 - p_s)} = \sqrt{\frac{2}{1.15 \text{ kg/m}^3} (1 \times 10^3 \text{ Pa} - 6 \times 10^3 \text{ Pa})} = \sqrt{\frac{2}{1.15 \text{ kg/m}^3} (1 \times 10^3 \text{ Pa} \cdot \frac{1}{10^3} \text{ kg} \cdot \frac{1}{10^3} \text{ m}^3)} = 41.7 \text{ m/s} \]
Water flows out of a tank at 12.5 m/s from an orifice located 9m below the surface. The cross-sectional area of the orifice is 0.002 m², and the coefficient of discharge is 0.85. What is the diameter D, at the vena contracta?

\[ A = C_c A \]
\[ A = 0.62 (0.002 \text{ m}^2) = 0.00124 \text{ m}^2 \]
\[ \frac{\pi d^2}{4} = 0.00124 \text{ m}^2 \]
\[ d = 0.02973 \text{ m} \]

\[ C_c = \frac{C_v}{C_u} = \text{COEFF OF DISCHARGE} \]
\[ C_v = \text{COEFF OF VELOCITY} \]
\[ C_u = \text{COEFF OF CONTRACTION} \]

\[ C_v = \sqrt{\frac{V}{V_{\text{local at depth}}} = \sqrt{\frac{12.5^4}{\sqrt{2g h}} = \frac{12.5^4}{\sqrt{2(9.81)(8)}} = 0.941} \]

4.8 cm
A nuclear submarine is capable of a top underwater speed of 65 km/h. How fast would a 1/20 scale model of the submarine have to be moved through a testing pool filled with seawater for the forces on the submarine and model to be dimensionally similar?

\[
\frac{[Re]_p}{[Re]_m} = \frac{[V]_p}{[V]_m} = \frac{[L]_p}{[L]_m}
\]

\[
\frac{65 \text{ km/h}}{1000 \text{ m/km}} \cdot \frac{\text{h}}{3600 \text{ s}} = 18.06 \text{ m/s}
\]

\[
V_p = V_m = 361 \text{ m/s}
\]

\[
360 \text{ m/s}
\]
Darcy-Weisbach Example: For a discharge \( Q \) of \( 1.0 \text{ ft}^3/\text{s} \), compute the head loss in 1,500 feet of new 6-inch diameter cast iron pipe \( (\varepsilon = 0.00085 \text{ ft}) \). Assume a water temperature of \( 60^\circ \text{F} \).

\[
Q = 1.0 \frac{\text{ft}^3}{\text{s}}
\]

\[
L = 1500 \text{ ft}
\]

\[
D = 6 \text{ in}
\]

\[
\varepsilon = 0.00085
\]

\[
\frac{\varepsilon}{D} = \frac{0.00085}{0.5} = 0.0017
\]

\[
h_f = f \frac{L}{D} \frac{V^2}{2g}
\]

\[
V = \frac{Q}{A} = \frac{1.0 \frac{\text{ft}^3}{\text{s}}}{\pi (0.5)^2} = \frac{1}{0.196} = 5.1 \frac{\text{ft}}{\text{s}}
\]

\[
V = 1.217(10)^{-5} \frac{\text{ft}}{\text{s}} \quad \text{GET FROM CHART ON PG 114}
\]

\[
Re = \frac{V D}{\varepsilon} = \frac{(5.1 \frac{\text{ft}}{\text{s}})(0.5 \text{ ft})}{1.217(10)^{-5} \frac{\text{ft}^2}{\text{s}}} = 209531.6 = 2.09(10)^5
\]

FROM MOODY CHART (PG 115)

\[
f = 0.023
\]

\[
h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.023)\left(\frac{1500 \text{ ft}}{0.5 \text{ ft}}\right)\left(\frac{(5.1 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})}\right) = 27.8 \text{ ft}
\]
Example: A 6-inch diameter 500 ft long steel pipe \((\varepsilon = 0.00015 \text{ ft})\) conveys flow between two reservoirs which have a difference in water surface elevation of 30 ft. The pipe exit and entrance are square edge. Compute the flow rate.
Example: Assume a pump is added to the previous example and the flow direction is reversed. What pump head is required for a discharge of 2.0 cfs.

\[
D = 0.5 \text{ ft}
\]
\[
A = \frac{\pi}{4} (0.5)^2 = 0.196 \text{ ft}^2
\]
\[
V = \frac{Q}{A} = \frac{2 \frac{ft^3}{s}}{\frac{\text{ft}^2}{s} (0.5)^2} = 10.2 \frac{ft^3}{s}
\]
\[
Z_1 + \frac{P_1}{\rho} + \frac{V_1^2}{2g} + h_P = Z_2 + \frac{P_2}{\rho} + \frac{V_2^2}{2g} + h_L
\]
\[
h_P = Z_2 - Z_1 + 17.5 \frac{V_2^2}{2g}
\]
\[
h_P = 30 \text{ ft} + 17.5 \left(\frac{10.2 \frac{ft^3}{s}}{2g}\right)
\]
\[
h_P = 30 \text{ ft} + 28.2 \text{ ft} = 58.2 \text{ ft}
\]
**Example:** Determine the horsepower required for the pump of the previous problem, assuming the pump efficiency is 75%.

\[ Q = 2.0 \text{ ft}^3/\text{s} \]

\[ h_p = 58.2 \text{ ft} \]

\[ \eta = 0.75 \]

\[
\text{Horse Power (Watts)} = \frac{\text{Horse Power (Pump Brake Power)}}{\text{Watts}}
\]

\[
P = 62.4 \ \frac{\text{lb}}{\text{ft}^2}.
\]

\[
\begin{align*}
\text{BHP} &= \frac{599 H Q}{550 \eta_{\text{pump}}} = \frac{(62.4 \ \frac{\text{lbm}}{\text{ft}^3})(32.2 \ \frac{\text{ft}}{\text{sec}^2})(58.2 \ \text{ft})(2.0 \ \frac{\text{ft}^3}{\text{sec}})}{(550 \cdot 32.2 \ \frac{\text{lbm-ft}}{\text{lbm-sec}^2} \cdot 0.75)} \\
\text{BHP} &= 17.6 \ \text{Horse Power}
\end{align*}
\]

\[
\begin{align*}
\text{BHP} &= \frac{\omega H Q}{550 \eta_{\text{pump}}} = \frac{(62.4 \ \frac{\text{lbm}}{\text{ft}^3})(58.2 \ \text{ft})(2 \ \frac{\text{ft}^3}{\text{sec}^2})}{550 \cdot 0.75} = 17.6 \ \text{HP}
\end{align*}
\]

**Don't forget to use \( \eta \) to avoid it.**
Example: Compute the discharge rate that causes the pressure to drop to vapor pressure. The pipe between the reservoir and pump has a length of 1,000 ft, diameter of 3 feet, and friction factor (f) of 0.02. Neglect minor losses. For a water temperature of 80°F, the vapor pressure ($P_v$) is 0.51 psia.

\[ \begin{align*}
  \text{Example: Compute the discharge rate that causes the pressure to drop to vapor pressure. The pipe between the reservoir and pump has a length of 1,000 ft, diameter of 3 feet, and friction factor (f) of 0.02. Neglect minor losses. For a water temperature of 80°F, the vapor pressure ($P_v$) is 0.51 psia.}
\end{align*} \]
Example: Compute the discharge in each pipe. Neglect minor losses.

\[ Q_1 = Q_L + Q_3 = Q_4 \]

\[ h_{L2} \quad h_{L3} \]

\[ K_2 = 1.663 \]

\[ K_3 = 7.649 \]

\[ K_1 = 0.359 \]

\[ K_4 = 0.120 \]

\[ h_L = KQ^2 \]

\[ K = f \left( \frac{L}{D} \right) \left( \frac{1}{2gA^2} \right) \]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>L(ft)</th>
<th>D(inch)</th>
<th>A(ft^2)</th>
<th>f</th>
<th>K = f(L/D) (1/2gA^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,000</td>
<td>16</td>
<td>1.396</td>
<td>0.020</td>
<td>0.359</td>
</tr>
<tr>
<td>2</td>
<td>3,000</td>
<td>12</td>
<td>0.785</td>
<td>0.022</td>
<td>1.663</td>
</tr>
<tr>
<td>3</td>
<td>2,000</td>
<td>8</td>
<td>0.349</td>
<td>0.020</td>
<td>7.649</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>16</td>
<td>1.396</td>
<td>0.020</td>
<td>0.120</td>
</tr>
</tbody>
</table>
The Darcy friction factor for both of the pipes shown is 0.024. The total flow rate is 300 m³/h. What is the flow rate through the 250 mm pipe?

\[ f = 0.024 \]

\[ A_1 = \frac{\pi}{4} (0.15)^2 \]
\[ = 0.018 \text{ m}^2 \]

\[ A_c = \frac{\pi}{4} (0.25)^2 \]
\[ = 0.049 \text{ m}^2 \]

\[ Q_c = 300 \text{ m}^3/\text{h} \]

\[ h_{L1} = h_{L2} \]

\[ \frac{f}{2} \frac{L}{D} \frac{V_1^2}{2g} = \frac{f}{2} \frac{L}{D} \frac{V_2^2}{2g} \]

\[ \frac{L_1 V_1^2}{P_1} = \frac{L_2 V_2^2}{P_2} \]

\[ \frac{V_1}{V_2} = \frac{(650 \text{ m})(0.15 \text{ m})}{(325 \text{ m})(0.25 \text{ m})} \]

\[ V_1 = 1.095 V_2 \]

\[ h = \frac{P_1}{D} \frac{L_1}{D} \frac{V_1^2}{2g} \]

\[ V_2 = 4366.18 \text{ m}^3/\text{h} \]

\[ Q_2 = V_2 A = (4366.18 \text{ m}^3/\text{h}) \left( \frac{\pi}{4} (0.25 \text{ m}^2) \frac{h}{3600 s} \right) = 0.059 \text{ m}^3/\text{s} \]
**EXAMPLE:** Two reservoirs are connected by a 850 feet long 6-inch diameter pipe ($f = 0.020$). A pump with the given characteristic curves is used to lift water from one reservoir to the other. Determine the discharge rate.
What is the hydraulic radius of the trapezoidal irrigation canal shown?

\[ \text{HYDRAULIC RADIUS} \ (H_R) = \frac{\text{CROSS SECTIONAL AREA}}{\text{WETTED PERIMETER}} \]

\[ S = \sqrt{h^2 + b^2} = 5 \text{m} \]

\[ \text{WETTED PERIMETER} = 2(5 \text{m}) + 5 \text{m} = 15 \text{m} \]

\[ \text{AREA} = \frac{(a + b)h}{2} = \frac{(5 \text{m} + 11 \text{m})(4 \text{m})}{2} = 32 \text{m}^2 \]

\[ H_R = \frac{32 \text{m}^2}{15 \text{m}} = 2.13 \text{m} \]
Geometric Elements of Channel Section

example: trapezoidal section

\[
\begin{align*}
\text{top width} & \quad T = 2(12\text{ ft}) + 20 = 44\text{ ft} \\
\text{flow area} & \quad A = \left(\frac{a+b}{2}\right)h = \left(\frac{20\text{ ft} + 8\text{ ft}}{2}\right)6\text{ ft} = 192\text{ ft}^2 \\
\text{wetted perimeter} & \quad = (2)(13.4\text{ ft}) + 20\text{ ft} = 46.8\text{ ft} \\
\text{hydraulic radius} & \quad H_R = \frac{\text{cross area}}{\text{wetted perimeter}} = \frac{192\text{ ft}^2}{46.8\text{ ft}} = 4.1\text{ ft} \\
\text{hydraulic depth} & \quad y_h = \frac{A}{T} = \frac{192\text{ ft}^2}{44\text{ ft}} = 4.36\text{ ft}
\end{align*}
\]
Example: Compute the discharge in a concrete \( (n = 0.015) \) channel with the previous cross-section and slope of 0.10%.

\[ n = 0.015 \quad \text{ROUGHNESS COEF.} \]

\[ S = 0.10\% = 0.001 \]

\[ Q = \frac{V}{n} A R_h^{2/3} S^{1/3} \]

\[ A = 192 \text{ ft}^2 \]

\[ R_h = \frac{A}{P} = 4.1 \text{ ft} \]

\[ V = 1.486 \]

\[ Q = \frac{1.486}{0.015} (192 \text{ ft}^2)(4.1 \text{ ft})^{2/3} (0.001) = 1540.8 \frac{\text{ft}^3}{\text{sec}} \]
Example: Compute the normal depth and velocity.

\[ Q = 400 \text{ cfs} \]
\[ S = 0.0016 \]
\[ n = 0.025 \]

\[ A = \frac{(a+b)}{2} y \]
\[ \frac{y}{1} = \frac{x}{2} \Rightarrow x = 2y \]
\[ v = \sqrt{y^2 + (2y)^2} \]

\[ A = \frac{20 + (20 + (2)(2y))}{2} (y) \]

\[ Q = \frac{V}{n} \times \frac{2}{1} \times S \]

\[ 400 \times \frac{A^3}{6} = 1.486 \left( \frac{20 + (20 + 4y)}{2} \right) (y) \left[ \frac{(20 + (20 + 4y))}{2} \right] \left( \frac{0.0016}{2} \right)^{1/2} \]

\[ y_n = 3.3 \text{ ft} \]

Simplify Equation

\[ A = \frac{20 + (20 + (2)(2y))}{2} \cdot y = \frac{(40 + 4y)}{2} \cdot y = (20 + 2y) \cdot y \]

\[ P = 20 + (2)\sqrt{y^2 + (2y)^2} = 20 + (2)\sqrt{y^2 + 4y^2} = 20 + (2)(2.236) \cdot y = 20 + 4.472 \cdot y \]

\[ R = \frac{A}{P} = \frac{(20 + 2y) \cdot y}{20 + 4.472 \cdot y} \]

\[ AR^{1/3} = \left( \frac{20 + 2y}{20 + 4.472y} \right)^{1/3} = 168.2 \]

\[ y = 3.3 \text{ ft} \]
Example: Compute the normal depth and velocity.

\[ Q = 400 \text{ cfs} \]
\[ S = 0.0016 \]
\[ n = 0.025 \]

\[ Q = \frac{K A R_n^{2/3}}{S^{1/2}} \]
\[ A R_n = \frac{Q n}{K S^{1/2}} = \frac{(400 \text{ cfs}) (0.025)}{(1.486) (0.0016)^{1/2}} = 168.24 \]

\[ T = 20 + 4y \]
\[ P = 20 + 2n = 20 + 2 \sqrt{5} y^2 = 20 + 4.47 y \]

\[ A = \left( \frac{T + B}{2} \right) y = \left( \frac{20 + 4y + 20}{2} \right) y = (20 + 2y) y \]
\[ A^{2/3} = \left( \frac{A}{P} \right)^{2/3} = \left( \frac{20 + 2y}{20 + 4y} \right)^{4/3} \]
\[ y_n = 3.36 \text{ ft} \]

Find \( y_c \) for trapezoidal channels use \( \frac{Q^2}{g} = \frac{A^2}{T} \)

\[ \frac{400^2}{32.2} = \frac{(20 + 2y)^2 y}{20 + 4y} \]
\[ y_c = 2.1 \text{ ft} \]
\[ Q = \frac{400 \text{ ft}^3}{s} \]

\[ S = 0.001 \text{ ft} \]

\[ \nu = 0.025 \]

\[ x = 2y \]

\[ T = 20 + 4y \]

\[ \frac{y}{2} \]

\[ r = \sqrt{y^2 + (2y)^2} \]

\[ r = \frac{y}{2} \]

\[ r = \frac{y}{2} \]

\[ y = \frac{y}{2} \]

\[ L = 20 \text{ ft} \]

\[ A_{\text{cub}} = \frac{Q \cdot u}{k \cdot S^{1/2}} = \frac{(400 \text{ ft}^3)(0.025)}{(1.486)(0.0016)^{1/2}} = \frac{10 \text{ ft}^3}{0.05944} = 168.2 \]

\[ A = \left( \frac{a + b}{2} \right) y = \left( \frac{20 + 2(2y)}{2} \right) y = \left( 10 + 4y \right) y = (20 + 2y) y \]

\[ P = 20 + 2y \]

\[ P = 20 + 2y \sqrt{y^2 + 4y^2} = 20 + 2y \sqrt{1 + 4} y^2 = 20 + 4.972 y \]

\[ A_{\text{cub}} = A \left( \frac{A}{D} \right) = (20 + 2y) y \left( \frac{(20 + 2y) y}{20 + 4.972 y} \right) \]

\[ y = 3.36 \text{ ft} \]

\[ (T H I S \ I S \ T H E \ D E P T H \ N O R M A L \ D E P T H) \]

\[ \text{FIND} \ y_c \text{ PA 167} \]

\[ \frac{Q}{S} = \frac{A^2}{T} \]

\[ A = (20 + 2y) y \]

\[ T = 20 + 4y \]

\[ \frac{400 A^2}{S} = \frac{[(20 + 2y) y]^3}{20 + 4y} \]

\[ y = 3.3 \text{ ft} = y_c \]
Example: Compute the critical depth and velocity for \( Q = 400 \text{ cfs.} \)

\[
A = \frac{(a + b)}{2} h = \frac{(20 + 20 + (2)(2y))}{2} \quad y = \left(\frac{40 + 4y}{2}\right)y = (20 + 4y)y
\]

\[
T = 20 + 2(2y) = 20 + 4y
\]

\[
\frac{Q^2}{g} = \frac{A^2}{T}
\]

\[
\frac{400}{32.2} \frac{ft^3}{sec} = \frac{[20 + 4y]^3}{20 + 4y}
\]

\[
y = 2.1 \quad \text{and} \quad y <
\]
Example: A rectangular concrete \( (n = 0.013) \) channel with a bottom width of 12 meters abruptly changes from a slope of 0.010 to a slope of 0.005. A discharge of 25 \( m^3/s \) is flowing in the channel. Determine whether a hydraulic jump occurs.

\[
Q = 25 \text{ m}^3/\text{s}
\]

\[
S = 0.010
\]

\[
S = 0.005
\]

\[
\frac{Q^2}{g} = \frac{A^3}{T}
\]

\[
A = (12)(\gamma_c)
\]

\[
T = 12 \text{ m}
\]
Example: A 15-ft wide rectangular channel has a roughness coefficient \( n \) of 0.0015 and bottom slope of 0.0015. The channel discharges into a river which may reach a stage of 10-feet above the channel bottom during floods. For a design discharge of 500 cfs, calculate \( y_n \), \( y_c \), and the distance from the channel outlet to the location where normal depth occurs.

**normal depth**

\[
Q = \frac{k}{n} A R^{2/3} S^{1/2} \quad A = (15 ft) y \\
S = 15 ft + 2y
\]

\[
AR^{2/3} = \frac{Q n}{k S^{1/2}} = \frac{500 \frac{ft}{sec} (0.015)}{1.486 \frac{ft}{sec} (0.0015)^{1/2}} = 130.3
\]

\[
(15 ft) y \left[ \frac{(15 ft) y}{15 ft + 2y} \right]^{2/3} = 130.3
\]

\[
y = 4.4 \text{ ft}
\]

**critical depth**

\[
y_c = \left( \frac{q^2}{g} \right)^{1/3} \\
q = \frac{Q}{A} = \frac{500 \frac{ft}{sec}}{15 \text{ ft}} = 33.3 \text{ ft}^2
\]

\[
y_c = \left( \frac{(33.3 \text{ ft}^2)^{1/3}}{22.2 \frac{ft}{sec}} \right) = 3.3 \text{ ft}
\]

*SUBCRIT TO SUPERCRIT FLOW*

\[
y = 4.4 \text{ ft} + 0 \text{ to } 10 \text{ ft}
\]

*NO HYDRO JUMP*
Example: A hydraulic jump occurs in a triangular flume having side slopes of 1:1. The flow rate is 0.45 m³/s and the depth before the jump is 0.30 m. Compute the depth after the jump.

\[ \frac{Q^2}{gA_1} + A_1h_c = \frac{Q^2}{gA_2} + A_2h_c \]

\[ \frac{Q^2}{g} + \frac{1}{2}g \frac{y_1^2}{y_1} = \frac{Q^2}{g} + \frac{1}{2}g \frac{y_c^2}{y_c} \]

\[ \frac{0.45^2}{(9.81)(0.3)^2} + \frac{(0.3)(0.3)}{3} = \frac{0.45^2}{(9.81)y_1^2} + \frac{y_1^2}{3} \]

\[ y = 0.89 \text{ m} \]
Example: Estimate the flow depth in the downstream channel and the energy loss in the hydraulic jump at the foot of the 100-ft wide rectangular broad crested spillway for a discharge of 2,130 cfs.

Compute the depth \( y_1 \) at the toe of the spillway: \( Q = V \cdot A \)

\[
V = \frac{Q}{A} = \frac{2,130 \text{ cfs}}{100 \text{ ft}} = \frac{21.3}{y_1}
\]

\[
z_0 + \frac{y^2}{2g} = z_1 + \frac{V^2}{2g}
\]

\[
z_0 = z_1 + \frac{V^2}{2g} \quad z_1 = y_1
\]

\[
y_1 = 0.4 \sqrt{V} \quad \therefore \quad V_1 = \frac{21.3}{0.4} = 53.25 \frac{A}{s}
\]
Compute the sequent depth: \[ y_s = \frac{y_1}{2} \left( -1 + \sqrt{1 + 8 \frac{F_r^2}{y_1}} \right) \]

\[ v_1 = \frac{z_1.3}{y_1} = \frac{21.3}{0.4} = 53.25 \text{ ft/sec} \]

\[ A = (0.4)(100) = 0.4 \]

\[ T = 100 \]

\[ F_r = \frac{v}{\sqrt{g y_1}} = \frac{53.25}{\sqrt{32.2(0.4)}} = 14.8 \]

\[ y_s = \frac{(0.4)}{2} \left( -1 + \sqrt{1 + 8(14.8)^2} \right) = 8.2 \text{ ft} \]

\[ \frac{Q^2}{g A} + A h_c = \frac{Q^2}{g A} + A h_c \]

\[ h_c = \frac{1}{2} y_1, \quad h_c = \frac{1}{2} y_1 \]

\[ A_1 = (100)(y_1), \quad A_2 = (100)(y_1) \]

Compute the energy loss in the hydraulic jump: \[ E = \frac{\lambda}{2g} + y \]

\[ v_1 = 53.25 \frac{\text{ft}}{\text{sec}} \]

\[ v_2 = \frac{Q}{A_2} = \frac{213.0}{100(8.24)} = 2.6 \frac{\text{ft}}{\text{sec}} \]

\[ E_1 = \frac{\lambda}{2g} + y_1 = (1) \left( \frac{(53.25)^2}{2(32.2)} \right) + 0.4 = 94.4 \text{ ft} \]

\[ E_2 = \frac{\lambda}{2g} + y_2 = (1) \left( \frac{(2.6)^2}{2(32.2)} \right) + 8.2 = 4.3 \text{ ft} \]

\[ E_1 - E_2 = 36.1 \]

\[ E_{\text{loss}} % = \frac{E_{\text{loss}}}{E_{\text{total}}} = \frac{36.1}{94.4} = 38% \]
Example: A 50 acre watershed in Brazos County has a curve number (CN) of 86. Compute the direct runoff volume to result from a design storm with a recurrence interval of 50 years and duration of 24 hours.
**Example:** A hydrograph representative of a certain 60 square mile watershed is given below. The hydrograph resulted from a rainfall with a duration of 2.0 hours. Compute a unit hydrograph for the watershed.

**Given Storm Hydrograph**

<table>
<thead>
<tr>
<th>Time (Hours)</th>
<th>Discharge (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4,000</td>
</tr>
<tr>
<td>4</td>
<td>11,000</td>
</tr>
<tr>
<td>6</td>
<td>16,000</td>
</tr>
<tr>
<td>7</td>
<td>14,000</td>
</tr>
<tr>
<td>10</td>
<td>9,000</td>
</tr>
<tr>
<td>12</td>
<td>5,000</td>
</tr>
<tr>
<td>14</td>
<td>2,000</td>
</tr>
<tr>
<td>16</td>
<td>500</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>
Example: The watershed described in the previous example is located in Brazos County, Texas. A composite curve number of 80 has been estimated for the watershed. Design storm rainfall data are provided below. Compute the runoff hydrograph for the design storm using a computational interval of 2.0 hours.

Design Rainfall Data

50-year recurrence interval
6-hour rainfall duration
Balanced triangular distribution over time