If a one-time amount of $500 is invested at an annual interest rate of 8% (compounded annually), find its future worth at the end of 30 years.

\[ F = P(1 + i)^n = 500(1.08)^{30} \approx 500 \times 31.53 \]

Table: \( i = 8\% \)

\[ F = P \left( \frac{F}{P}, 8\%, 20 \right) = 500 \times 10.0627 = 5031.35 \]

If you need to have $800 in savings at the end of 4 years and your savings account yields 5% annual interest, how much do you need to deposit today?

\[ P = \frac{F}{(1 + i)^n} = \frac{800}{(1.05)^4} = $658.14 \]

No table @ \( i = 5\% \)
A company borrows $100,000 today at 12% nominal annual interest compounded monthly. Find the monthly payment of a 5-year loan.

\[ P = \$100,000. \]

\[ \text{Annual Payment} = \frac{P \left( \frac{1}{1}, 1\%, 60 \right)}{1} \]

\[ A = P \left( \frac{1}{1}, 1\%, 60 \right) = \$100,000 \left( 0.01222 \right) \]

\[ A = \$2,220 \text{ PER MONTH} \]

\[ A = \frac{P \left( 1 + i \right)^n - 1}{i \left( 1 + i \right)^n - 1} \]

\[ A = \frac{\$100,000 \left( 1.01 \right)^{60} - 1}{0.01 \left( 1.01 \right)^{60} - 1} \]

\[ A = \$2,224 \text{ PER MONTH} \]

A new sander costs $3,600 and has an annual maintenance cost of $400. The salvage value after 7 years is $600. Assuming an annual interest rate of 10%, what is the present worth?

\[ P = \$3,600 \quad i = 10\% \]

\[ A = \$400 \quad n = 7 \text{yr} \]

\[ F = \$600 \]

\[ -P = A \left( \frac{P}{A}, 10\%, 7 \right) = \frac{\$400 \left( 4.8684 \right)}{1} = \$1,947.36 \]

\[ -P = F \left( \frac{P}{F}, 10\%, 7 \right) = \frac{\$600 \left( 0.5132 \right)}{1} = \$310.56 \]

\[ P = \$3,600 - \$1,947.36 + \$410.56 = -\$1,366.80 \]

10% TABLE

\[ \text{Benefit-Cost Analysis} \]

\[ \frac{B}{C} \geq 1 \quad \frac{\$410.56}{\$3,600 + \$1,947.36} = 0.074 \]

\[ \frac{B}{C} < 1 \quad \text{PROJECT IS NOT JUSTIFIED} \]
The annual nominal interest rate on the unpaid portion of a contract is 17%. Find the effective annual interest rate if the interest is compounded quarterly.

\[ r = 0.17 \]

\[ M = \frac{12}{4} = 3 \text{ (Compounded Quarterly)} \]

\[ i_e = \left(1 + \frac{r}{M}\right)^M - 1 = \left(1 + \frac{0.17}{3}\right)^3 - 1 = 0.18 \]

An asset is purchased for $100,000. The estimated life is 7 years and the salvage value is $15,000. Assuming the item is depreciated via straight-line method, find the book value of the asset at the end of 3 years.

\[ D_j = \frac{C - S_n}{n} \]

\[ P3132 \]

\[ D_j = \frac{100,000 - 15,000}{7} = 12,143 \]

\[ BV = \text{Initial Cost} - \sum D_j \]

\[ BV = 100,000 - 3(12,143) = 96,3571 \]
An asset costs $100,000 and has a useful life of 10 years. The salvage value at the end of 10 years is estimated to be $10,000. Using the Modified Accelerated Cost Recovery System (ACRS), find the book value of the asset at the end of year 3.

\[ C = \$100,000 \]
\[ S_{10} = \$10,000 \]
\[ N = 10 \text{ YEARS} \]

**Pg 132 MACKS FACTOR TABLE**

**Find factor % for recovery period of 10 years for the first 3 years**

**Recovery Rate**

<table>
<thead>
<tr>
<th>( N )</th>
<th>FACTOR</th>
<th>( D_j = (\text{FACTOR})C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>$10,000</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
<td>$18,000</td>
</tr>
<tr>
<td>3</td>
<td>0.144</td>
<td>$14,400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sum D_j = $42,400 )</td>
</tr>
</tbody>
</table>

**BV = Initial Cost - \( \sum D_j = \$100,000 - \$42,400 \)**

**BV = \$57,600**
Find EUAC.
period=8 years = n
MARR=12% \( i = 0.12 \)
\( \text{cost} = 55,000 = P \)
\( \text{Salvage} = 4,000 \)
annual maintenance = 3,500
annual returns = 15,000

\( \text{EUAC} = \text{Equivalent Uniform Annual Cost} \)
\( \text{EUAB} = \text{Equivalent Uniform Annual Benefit} \)

 Convert given values to a uniform amount (A)

\[
P \times \frac{i(1+i)^n}{(1+i)^n - 1} = P \left( \frac{A}{P}, 12\%, 8 \right) = \$55,000 \left( 0.2013 \right) = \$11071.5 \text{ (cost)}
\]

\[
A = F \frac{i}{(1+i)^n - 1} = F \left( \frac{A}{F}, 12\%, 8 \right) = \$4000 \left( 0.0813 \right) = \$325.2 \text{ (benefit)}
\]

\[
A = \$3500 \text{ already an annual cost} \text{ (cost)}
\]

\[
\text{EUAC} = \$11071.5 + \$3500 - \$325.2 = \$14626.3
\]

\[
\text{EUAB = Annual returns = } \$15,000
\]

Benefit Cost Analysis

\[
\frac{\text{EUAB}}{\text{EUAC}} \geq 1
\]

\[
\frac{\$15,000}{\$14,626.3} = 1.053 \text{ (Project is justified)}
\]

\[
\text{EUAB - EUAC} = \$15,000 - \$14,626 = \$374 \text{ (Profit)}
\]
Engineering Econ Review
Eric Showalter, Ph.D., P.E.

Engineering Econ allows us to compare money spent or received (cash flows) in the future to an amount of money spent today. The most familiar example is an automobile loan. Borrow $20,000 today to buy a car and pay it off in a series of 60 equal payments. How do they decide on the size of my payment? Engineering econ. As engineers we are constantly faced with decisions based on an investment today vs. a benefit in the future.

Basics:
I or i = the interest rate per interest period
i_n = the nominal interest rate, often called r or APR (annual % rate)
i_e = effective interest rate
n = number of compounding periods (often years, but not always)
m = number of compounding periods in one year
EOY = end of year (BOY is beginning of year)
Payback period is the amount of time required to recoup your initial investment.
Return on investment is the interest rate that makes 2 investments equivalent
PV = present value  FV = future value
Annuity (A) is a series of equal payments
Gradient (G) is an increasing series, either arithmetic or geometric
In the tables: n is the number of periods
P is present value, F is future value, A is annuity, G is gradient
The P/F column is used to find (P)resent value given a (F)uture value
The P/A column is used to find (P)resent value given an (A)nnuity, etc.

Keys to success in solving most engineering econ problems:
- Draw a cash flow diagram
- Identify (P)resent values, (F)uture values, (A)nnuities, (i)nterest rate and (n)umber of periods (not all will be given)
- Calculate the present value of each amount, or the annuity amount, or the future amount
- Compare results

Resource: http://www.feexam.ou.edu/ there are reviews of every area on the FE. I only used a few of the example questions here.

Effective interest rate vs. nominal. i_e = (1 + i_n/m)^m For example 9% nominal annual interest compounded monthly is (1+0.09/12)^12-1 = 0.0938 = 9.38% effective.

Another type is: X% annual growth with payments every Y years is an effective rate of (1+X)^Y-1. If costs grow by 3.5% every year, and we re-pave a road every 5 years, then the re-paving cost increases by (1+0.035)^5-1 = 18.77% every 5 years.
Cash flow diagram = a drawing that is used to help understand the size and timing of expenses and revenues. Be sure you are consistent; i.e. all cash flows in to you should be in the same direction.

Single payment now vs. single payment in the future. We expect our money to grow if we lend it, so a payment in the future should be larger than a payment today. It grows by \((1+i)^n\). On the other hand, if we know we will receive money in the future, we would accept less at the present instead. The reduction is the reciprocal of the last equation, \(1/(1+i)^n\).

Borrow $100 today at 4% interest. How much do you owe after 5 years? ($121.67)

\[
F = P(1+i)^n = 100(1.04)^5 = 121.67
\]

You win $1,000, payable one year from now. How much would you take today, instead, if interest is 6%? ($943.40).

\[
P = F(1+i)^{-n} = 1000(1.06)^{-1} = 943.40
\]

One present payment vs. a uniform series of payments. Be sure you understand what the cash flow diagram looks like. What happens when \(n \to \infty\)?

Use capitalized cost formula

\[
P = \frac{A}{i}
\]

You borrow $100,000 today, to be paid back in 10 equal payments at the end of each year, at 6% interest. How much are the payments? ($13,590).

\[
A = P\left(\frac{A}{P}, i, n\right) = 100000\left(0.1359\right) = 13590
\]

One future payment vs. a uniform series of payments. Cash flow diagram (this one does not look "logical").
You invest $10,000 at the end of each year for 5 years. If \( i = 4\% \), what is the value of the account at EOY 5? ($54,163).

\[
F = A \frac{(1+i)^n - 1}{i} = A \frac{F}{i} (4.9\%, 5) = \$10,000 \times (5.4163) = \$54,163
\]

**Arithmetic Gradients.** Payments increase or decrease by the same amount each period. For example, $100 at the end of year one, $200 at the end of year 2, $300 at the end of year 3, etc. Cash flow diagram begins at EOY 2. You may see an arithmetic gradient on top of an annuity.

What is the present value if \( i = 6\% \)? ($5,005.85)

<table>
<thead>
<tr>
<th>EOY</th>
<th>Receipt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1,000</td>
</tr>
<tr>
<td>2</td>
<td>$1,100</td>
</tr>
<tr>
<td>3</td>
<td>$1,200</td>
</tr>
<tr>
<td>4</td>
<td>$1,300</td>
</tr>
<tr>
<td>5</td>
<td>$1,400</td>
</tr>
</tbody>
</table>

\[
P = A \left( \frac{P}{A}, 6\%, 5 \right) = \$1000 \times (4.2124) = \$4212.4
\]

\[
P = C \left( \frac{P}{C}, 6\%, 5 \right) = \$100 \times (7.9385) = \$793.85
\]

\[
P = \$4212.4 + \$793.85 = \$5006.25
\]

**Capitalized Costs, \( n \to \infty \).** When \( n \) becomes “very large,” assume that it goes to infinity. Future values generally go to infinity, but in many cases present values tend toward limit. Capitalized costs go to \( A/i \).

Maintenance on traffic lights costs $5,000 per year, and is expected to be required for a very long time. If \( i = 6\% \), what is the capitalized cost?

\[
P = \frac{A}{i} = \frac{\$5000}{0.06} = \$83333
\]

**PTZ\#ETZ.** Understand what the cash flow diagrams look like. If the problem cash flow diagram does not look like the cash flow diagrams of any of your equations, you will have to use more than one step to find the answer.

I just purchased a machine. It has a 2 year warranty. I can purchase an extended warranty until the machine is 10 years old by making payments of $1000 per year, at the beginning of each year starting when the original warranty runs out, or by paying $7,000 today. Use an interest rate of 6% and determine which option is better.

\[
P = A \left( \frac{P}{A}, 6\%, 8 \right) = \$1000 \times (0.2096) = \$209.6 < \$7000
\]

The warranty is the better option.
Depreciation: Depreciation is the decrease in value of an asset. Straight line is the simplest form. The depreciation each year is equal to (Cost – Salvage)/life. Accelerated schemes are available, including MACRS. In this case the depreciation is equal to a factor from the table times Cost.

Book Value: BV is the Cost minus accumulated depreciation.

What is the depreciation each year for a trailer that cost $10,000 new, has a life of 5 years, and a salvage value of $2,000? Use straight line depreciation. What is the book value after 2 years?

\[
D_1 = \frac{C - S}{n} = \frac{10000 - 2000}{5} = 1600
\]

\[
BV = \text{INITIAL COST} - \sum D_i
\]

\[
= 10000 - 2(1600) = 6800
\]

Example Problems
An amount P is invested at interest rate i per compounding period. F is the account balance after n compounding periods. Select the formula that relates F to P.

(A) \( F = P(1+i)^n \)
(B) \( F = P(1+i)^{-n} \)
(C) \( F = P(1+i)^n \)
(D) \( F = P(1+i)^{-n} \)

A solar heating system costs $10,000, has an estimated life of 10 years and a scrap value of $1500. Assuming no inflation and an interest rate of 4%, what uniform annual amount must be invested at the end of each of the 10 years in order to replace the machine?

(A) $708
(B) $850
(C) $1000
(D) $1152

\[
A = P \left( \frac{A}{F}, 4\%, 10 \right) = 10000 \left( 0.1233 \right) = 1233 \text{ (cost)}
\]

\[
A = F \left( \frac{A}{F}, 4\%, 10 \right) = 1500 \left( 0.0833 \right) = 124.95 \text{ (benefit)}
\]

\[
A = -1233 + 124.95 = -1108.05 \text{ NOPE}
\]

\[
A = F \left( \frac{A}{F}, 4\%, 10 \right) = (10000 - 1500) \left( 0.0833 \right) = 708
\]
An investment has infinite life and makes annual payments of $3000 for the first 5 years and $1600 per year thereafter. Using 6% interest per annum, compute the present worth of the annual disbursements.

\[(A) \quad $15,000 \quad (B) \quad $25,000 \quad (C) \quad $32,600 \quad (D) \quad $50,200\]

\[
P = A \left( \frac{P}{A}, 6\%, 5 \right) + \frac{A}{i} = \$1400 (P/A, 12\%, 4) + \frac{\$1600}{0.06} = \$32,564
\]

Interest on a debt is 12% per year compounded monthly. Compute the effective annual interest rate.

\[
\hat{r} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.127
\]

\[
\hat{\hat{r}} = 12.7\%
\]

A more efficient heating system adds $7500 to the cost of your project. Adding this system will save $1200 per year. If the discount rate is 8%, about how long will it take to pay back the initial investment?

\[\rightarrow (A) \quad 6 \text{ years} \quad (B) \quad 7 \text{ years} \quad (C) \quad 8 \text{ years} \quad (D) \quad 9 \text{ years} \quad (E) \quad 10 \text{ years}\]

\[F = A \left( \frac{(1+i)^n-1}{i} \right)
\]

\[
$7500 = $1200 \left( \frac{(1+0.08)^n-1}{0.08} \right)
\]

\[n = 5.268 \text{ yrs}
\]
**Probability:** A culvert is required under a highway. If the highway is overtopped, it will cost $20k to repair. If i=6% and the life of each culvert is 20 years, which of the following is most economical?

<table>
<thead>
<tr>
<th>Type of culvert</th>
<th>Cost new</th>
<th>Probability of overtopping in any one year</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 inch RCP</td>
<td>$20,000</td>
<td>0.20</td>
</tr>
<tr>
<td>2 ft by 4 ft box</td>
<td>$40,000</td>
<td>0.10</td>
</tr>
<tr>
<td>Twin 2 ft by 4 ft box</td>
<td>$60,000</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\[
P_1 = \frac{20000}{A} + (0.2)(20000)\left(\frac{P}{A}, 6\%, 20\right) = \$65579
\]
\[
P_2 = \frac{40000}{A} + (0.1)(40000)\left(\frac{P}{A}, 6\%, 20\right) = \$62939
\]
\[
P_3 = \frac{60000}{A} + (0.05)(60000)\left(\frac{P}{A}, 6\%, 20\right) = \$71470
\]

If the sum of $12,000 is borrowed and the debtor is obligated to pay the creditor $900 for each year the loan is in existence, then, the simple interest is: I would have to assume that the $12,000 is paid back at the end of the loan period.

\[
\text{CAPITAL COST } \ P = \frac{A}{i}
\]
\[
i = \frac{\frac{900}{12000}}{0.075} = 7.5\%
\]

A Sprawl Transportation Authority bond has par value of $5000 and term of 10 years. The bond pays 5% nominal annual interest on par value. Estimate the selling price of the bond if the market interest rate is 6%.

- (A) $4632
- (B) $5000
- (C) $7500
- (D) $8000

Bond value or face value is the money the holder will receive when bond matures.

\[
P = A\left(\frac{P}{A}, 6\%, 10\right) + F\left(\frac{P}{F}, 6\%, 10\right)
\]
\[
P = 5000\left(7.3601\right) + 5000\left(0.5584\right) = \$4633
\]
\[5000\left(0.05\right) = \$250
\]
Two alternative investments have the cash flows indicated in the table. At 6% interest, which alternative should be selected based on future value? (negative is a cost, positive a benefit)

\[
F_A = P\left(\frac{F}{F, 6\%, 2}\right) + A\left(\frac{F}{A, 6\%, 2}\right) \\
= -1830(1.236) + 1000(2.06) = -201
\]

\[
F_B = -2350(1.236) + 1200(2.06) = -432
\]

\[F_A \text{ IS ACCEPTABLE}\]

Consider two alternatives, A and B, having cash flows as shown in the figure. If the MARR is 8%, determine which alternative should be selected using the benefit cost analysis method.

\[
\frac{B}{C} = \frac{P - A(\frac{F}{A, 8\%, 2})}{C} = \frac{500(2.577)}{50} = 51.542
\]

\[\frac{B}{C} = 1.031 \geq 1 \text{ (column A is ACCEPTABLE)}\]

Break even analysis on the FE exam is most likely a simple problem with no discounting (no interest). If there is no interest rate, then it is simply a matter of comparing cost to income, where cost is usually fixed cost + variable cost*units and income is usually income per unit*units.

Corp Inc. produces 4,000 robots per year. Their fixed costs are $1,000,000 per year, and the variable cost per robot is $2,500. The selling price is $10,000 per robot. Find the breakeven point and gross profit at this maximum capacity.

\[\$10,000X - \$1,000,000 - 2500X = 0\]

\[\$7,500X = \$10,000\]

\[X = 1331,3 \text{ ROBOTS TO BREAK EVEN}\]

\[\text{GROSS PROFIT}\]

\[
\$7500X - \$1(10)^6 | \frac{X=1000}{\text{ROBOTS}} = \$29(10)^6
\]
Bill Ding borrows $10,000 today from Dale Ight and another $15,000 at the beginning of year 5. The agreed interest rate is 6%, and the loan is to be repaid in full at the end of year 9. What amount will Bill owe Dale to pay off the loan?

**End of year 9 is the beginning of year 10.**

### 4%

#### Compound Interest Factors

<table>
<thead>
<tr>
<th>n</th>
<th>SINGLE PAYMENT</th>
<th>UNIFORM PAYMENT SERIES</th>
<th>GRADIENT SERIES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Compound Amount Factor</td>
<td>Present Value Factor</td>
<td>Sinking Fund Factor</td>
</tr>
<tr>
<td>1</td>
<td>1.040</td>
<td>.9615</td>
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### 6%

#### Compound Interest Factors

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<th>GRADIENT SERIES</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Compound Amount Factor</td>
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<td>.0476</td>
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<td>.0420</td>
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## ENGINEERING ECONOMICS

<table>
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<tr>
<th>Factor Name</th>
<th>Converts</th>
<th>Symbol</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Payment</td>
<td>to ( F ) given ( P )</td>
<td>( (F/P, i^%, n) )</td>
<td>( (1 + i)^{n} )</td>
</tr>
<tr>
<td>Compound Amount</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Payment</td>
<td>to ( P ) given ( F )</td>
<td>( (P/F, i^%, n) )</td>
<td>( (1 + i)^{-n} )</td>
</tr>
<tr>
<td>Present Worth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Series</td>
<td>to ( A ) given ( F )</td>
<td>( (A/F, i^%, n) )</td>
<td>( \frac{i}{(1 + i)^{n} - 1} )</td>
</tr>
<tr>
<td>Sinking Fund</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Recovery</td>
<td>to ( A ) given ( P )</td>
<td>( (A/P, i^%, n) )</td>
<td>( \frac{i[(1 + i)^{n} - 1]}{i[(1 + i)^{n} - 1]} )</td>
</tr>
<tr>
<td>Uniform Series</td>
<td>to ( F ) given ( A )</td>
<td>( (F/A, i^%, n) )</td>
<td>( \frac{1}{i} )</td>
</tr>
<tr>
<td>Compound Amount</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Series</td>
<td>to ( P ) given ( A )</td>
<td>( (P/A, i^%, n) )</td>
<td>( \frac{1 + i}{i[(1 + i)^{n} - 1]} )</td>
</tr>
<tr>
<td>Present Worth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Gradient</td>
<td>to ( P ) given ( G )</td>
<td>( (P/G, i^%, n) )</td>
<td>( \frac{1 + i}{i[(1 + i)^{n} - 1]} )</td>
</tr>
<tr>
<td>Present Worth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Gradient</td>
<td>to ( F ) given ( G )</td>
<td>( (F/G, i^%, n) )</td>
<td>( \frac{1 + i}{i[(1 + i)^{n} - 1]} )</td>
</tr>
<tr>
<td>Future Worth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Gradient</td>
<td>to ( A ) given ( G )</td>
<td>( (A/G, i^%, n) )</td>
<td>( \frac{1 + i}{i[(1 + i)^{n} - 1]} )</td>
</tr>
<tr>
<td>Uniform Series</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## NOMENCLATURE AND DEFINITIONS

- \( A \)........ Uniform amount per interest period
- \( B \)........ Benefit
- \( BV \)....... Book value
- \( C \)......... Cost
- \( d \)......... Combined interest rate per interest period
- \( D_j \)...... Depreciation in year \( j \)
- \( F \)......... Future worth, value, or amount
- \( f \)......... General inflation rate per interest period
- \( G \)......... Uniform gradient amount per interest period
- \( i \)......... Interest rate per interest period
- \( i_e \)...... Annual effective rate
- \( m \)......... Number of compounding periods per year
- \( n \)......... Number of compounding periods; or the expected life of an asset
- \( P \)......... Present worth, value, or amount
- \( r \)......... Nominal annual interest rate
- \( S_p \)...... Expected salvage value in year \( n \)

## Subscripts

- \( j \)......... at time \( j \)
- \( n \)......... at time \( n \)
- \( \dagger \)...... \( F/G = (F/A \times n/i) \times (A/G) \)

## NON-ANNUAL COMPOUNDING

\[ i = \left(1 + \frac{L}{m}\right)^m - 1 \]

## BREAK-EVEN ANALYSIS

By altering the value of one of the variables in a situation, holding all of the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the break-even point.

Break-even analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income will just cover expenses.

The payback period is the period of time required for the profit or other benefits of an investment to equal the cost of the investment.

## INFLATION

To account for inflation, the dollars are deflated by the general inflation rate per interest period \( f \), and then they are shifted over the time scale using the interest rate per interest period \( i \). Use a combined interest rate per interest period \( i \) for computing present worth values \( P \) and Net \( P \). The formula for \( j \) is \( j = i - f \times (i \times f) \).
1. Permanent mineral rights on a parcel of land are purchased for an initial lump-sum payment of $100,000. Profits from mining activities are $12,000 each year, and these profits are expected to continue indefinitely. The interest rate earned on the initial investment is most nearly

(A) 8.3%
(B) 9.0%
(C) 10%
(D) 12%

\[ i = \frac{A}{P} = \frac{12000}{100000} = 0.12 = 12\% \]
2. $1000 is deposited in a savings account that pays 6% annual interest, and no money is withdrawn for three years. The account balance after three years is most nearly

(A) $1120
(B) $1190
(C) $1210
(D) $1280

\[ F = P(1+i)^n = 1000(1+0.06)^3 = 1190.49 \]
3. An oil company is planning to install a new 80 mm pipeline to connect storage tanks to a processing plant 1500 m away. The connection will be needed for the foreseeable future. An annual interest rate of 8% is assumed, and annual maintenance and pumping costs are considered to be paid in their entireties at the end of the years in which their costs are incurred.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>initial cost</td>
<td>$1500</td>
</tr>
<tr>
<td>service life</td>
<td>12 yr</td>
</tr>
<tr>
<td>salvage value</td>
<td>$200</td>
</tr>
<tr>
<td>annual maintenance</td>
<td>$400</td>
</tr>
<tr>
<td>pump cost/hour</td>
<td>$2.50</td>
</tr>
<tr>
<td>pump operation</td>
<td>600 hr/yr</td>
</tr>
</tbody>
</table>

\[
\frac{\$2.50}{hr} \times \frac{600 \text{ hr}}{yr} = \frac{\$1500}{yr}
\]

The capitalized cost of running and maintaining the 80 mm pipeline is most nearly

(A) $15,000

(B) $20,000

(C) $24,000

(D) $27,000

\[
P = \frac{A}{i} = \frac{\$1900}{0.08} = \$23750
\]
4. New 200 mm diameter pipeline is installed over a distance of 1000 m. Annual maintenance and pumping costs are considered to be paid in their entireties at the end of the years in which their costs are incurred. The pipe has the following costs and properties.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$1350</td>
</tr>
<tr>
<td>Annual interest rate</td>
<td>6%</td>
</tr>
<tr>
<td>Service life</td>
<td>6 yr</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$120</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>$500</td>
</tr>
<tr>
<td>Pump cost/hour</td>
<td>$2.75</td>
</tr>
<tr>
<td>Pump operation</td>
<td>2000 hr/yr</td>
</tr>
</tbody>
</table>

\[
\frac{52.75 \times 2000 \text{ hr}}{\text{yr}} = \$ 55_00 = A
\]

What is most nearly the equivalent uniform annual cost (EUAC) of the pipe?

(A) $5700
(B) $5900
(C) $6100

\[
A = P \left(\frac{A}{P}, 6\%, 6\right) = 1350 \times 0.2034 = \$ 274.59 \text{ (cost)}
\]

\[
A = F \left(\frac{A}{F}, 6\%, 6\right) = 120 \times 0.1434 = \$ 17.2 \text{ (benefit)}
\]

\[
A = \$ 500 \text{ (cost)}
\]

\[
A = \$ 5500 \text{ (cost)}
\]

\[
EUAC = \Sigma A = - \$274.59 + \$ 17.2 - \$ 500 - \$ 5500 = - \$ 6257
\]
5. New 120 mm diameter pipeline is installed over a distance of 5000 m. Annual maintenance and pumping costs are considered to be paid in their entirety at the end of the years in which their costs are incurred. The pipe has the following costs and properties.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$2500</td>
</tr>
<tr>
<td>Annual interest rate</td>
<td>10%</td>
</tr>
<tr>
<td>Service life</td>
<td>12 yr</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$300</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>$300</td>
</tr>
<tr>
<td>Pump cost/hour</td>
<td>$1.40</td>
</tr>
<tr>
<td>Pump operation</td>
<td>600 hr/yr</td>
</tr>
</tbody>
</table>

\[
\sum \frac{1.40}{hr} \cdot \frac{600 hr}{yr} = \frac{840}{yr} = A
\]

What is most nearly the equivalent uniform annual cost (EUAC) of the pipe?

- (A) $1200
- (B) $1300
- (C) $1400
- (D) $1500

\[
A = \frac{A}{F(1.10, 12)} = \frac{2500(0.1438)}{1.432} = \frac{347}{(cost)}
\]

\[
A = \frac{300}{F(1.10, 12)} = \frac{300(0.64968)}{1.432} = \frac{14.04}{(benefit)}
\]

\[
A = \frac{300}{(cost)}
\]

\[
A = \frac{840}{(cost)}
\]

\[
A = \sum A = -\frac{367}{14} - \frac{300}{14} - \frac{840}{14} = -\frac{1492}{14}
\]
6. A construction company purchases 100 m of 40 mm diameter steel cable with an initial cost of $4500. The annual interest rate is 4%, and annual maintenance costs are considered to be paid in their entirety at the end of the years in which their costs are incurred. The annual maintenance cost of the cable is $200/yr over a service life of nine years. Using Modified Accelerated Cost Recovery System (MACRS) depreciation and assuming a seven year recovery period, the depreciation allowance for the cable in the first year of operation is most nearly

(A) $640
(B) $670
(C) $720
(D) $860

\[
P = \frac{4500}{A} + P \left( A, 4\% \right) = 4500 + 200 \left( 0.1345 \right)
\]

\[
P = 4526.9
\]

DEPRECIATION IN THE FIRST YEAR

PS132

\[D_j = (\text{factor}) \times \text{MACRS FACTOR} = 14.29\%\]

FROM FACTOR TABLE PS132

SINCE THE DEPRECIATION ALLOWANCE IS FOR THE FIRST YEAR ONLY, THE ANNUAL MAINTENANCE CANNOT BE INCLUDED SINCE IT BEGINS AT YEAR ONE

\[D_j = (\text{factor}) \times \text{cost} = 0.1429 \times 4500 = 643\]
7. A piece of equipment has an initial cost of $5000 in year 1. The maintenance cost is $300/yr for the total lifetime of seven years. During years 1–3, the rate of inflation is 5%, and the effective annual rate of interest is 9%. The uninflated present worth of the equipment during year 1 is most nearly

(A) $3200

(B) $3300

(C) $3400

(D) $3500

\[ P = $5000 \]
\[ A = $300 \]
\[ f = 5\% \]
\[ i_e = 9\% \]

\[ d = i + f + i_f = 0.09 + 0.05 + 0.09(0.05) = 0.145 \]

**Inflation Adjustment**

Will remove the effects of inflation.

**Present Worth of Equipment Only**

\[ P = F(1 + d)^{-n} = $5000(1.145)^{-3} = $3330.8 \]
8. A company is considering buying a computer with the following costs and interest rate.

<table>
<thead>
<tr>
<th>Cost Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$3900</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$1800</td>
</tr>
<tr>
<td>Useful life</td>
<td>10 years</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>$390</td>
</tr>
<tr>
<td>Interest rate</td>
<td>6%</td>
</tr>
</tbody>
</table>

The equivalent uniform annual cost (EUAC) of the computer is most nearly

- (A) $740
- (B) $780
- (C) $820
- (D) $850

\[ A = P \left( \frac{A}{F}, 6\%, 10 \right) = \$3900 \cdot 0.1359 = \$530 \quad \text{(cost)} \]

\[ A = F \left( \frac{A}{F}, 6\%, 10 \right) = \$1800 \cdot 0.0759 = \$136.62 \quad \text{(benefit)} \]

\[ A = \$390 \quad \text{(cost)} \]

\[ \text{EUAC} = -A = - \$530 + \$136.62 - \$390 = \$78.62 \]
9. A computer with a useful life of 13 years has the following costs and interest rate.

<table>
<thead>
<tr>
<th>Cost Component</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$5500</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$3100</td>
</tr>
<tr>
<td>Annual maintenance:</td>
<td></td>
</tr>
<tr>
<td>Years 1–8 (y)</td>
<td>$275</td>
</tr>
<tr>
<td>Years 9–13 (y)</td>
<td>$425</td>
</tr>
<tr>
<td>Interest rate</td>
<td>6%</td>
</tr>
</tbody>
</table>

\[ A = \frac{F}{P} \left( \frac{F}{A}, 6\%, 13 \right) = \frac{5500}{0.1130} = 621.5 \quad \text{(Cost)} \]
\[ A_s = F \left( \frac{F}{A}, 6\%, 13 \right) = 3100 \cdot 0.0533 = 164.3 \quad \text{(Benefit)} \]
\[ A_n = A \left( \frac{F}{A}, 6\%, 13 \right) \left( \frac{F}{A}, 6\%, 13 \right) + A \left( \frac{F}{A}, 6\%, 5 \right) \left( \frac{F}{A}, 6\%, 13 \right) \]
\[ = 275(18.821)(0.053) + 150(5.6371)(0.053) = 320 \quad \text{(Cost)} \]
\[ \text{EUAC} = -621.5 + 164.3 - 320 = -777 \]

**Note:** For an accurate calculation, call the future value of each annual maintenance cost and add them together. Now use the combined future value to determine the annual cost distributed over 13 years. The weighted average calculation performed at the top of the page may be good enough. Suggest using this method first due to ease.
10. A computer with an initial cost of $1500 and an annual maintenance cost of $500/yr is purchased and kept indefinitely without any change in its annual maintenance costs. The interest rate is 4\%. The present worth of all expenditures is most nearly

(A) $12,000

(B) $13,000

(C) $14,000

(D) $15,000

\[ P = \frac{A}{i} \]

\[ P = $1500 + \frac{A}{i} = $1500 + \frac{$500}{0.04} \]

\[ P = $14000 \]
A computer with a useful life of 12 years has an initial cost of $2300 and a salvage value of $350. The interest rate is 6%. Using the straight line method, the total depreciation of the computer for the first five years is most nearly

(A) $760  
(B) $810  
(C) $830  
(D) $920

\[ P = $2300 \quad F = $350 \quad i = 6\% \quad n = 12 \]

\[ D_j = \frac{C - S_n}{n} = \frac{$2300 - $350}{12} = $162.5 \text{ per year} \]

Depreciation for the first five years

\[ D_{1.5} = 5(162.5) = $812.5 \]
12. A computer with a useful life of 12 years has an initial cost of $3200 and a salvage value of $100. The interest rate is 10%. Using the Modified Accelerated Cost Recovery System (MACRS) method of depreciation and a 10 year recovery period, what is most nearly the book value of the computer after the second year?

(A) $1900
(B) $2100
(C) $2300
(D) $2400

\[ P = P_{3200} - F \left( \frac{P}{A, 10\%, 12} \right) \]
\[ = 3200 - 100 \times (0.8186) = 3168.14 \]

**Solution:**

<table>
<thead>
<tr>
<th>Year</th>
<th>Factor</th>
<th>DJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>316.8</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
<td>576.27</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td>887</td>
</tr>
</tbody>
</table>

\[ BV = \text{Initial Cost} - \sum DJ = 3200 - 887 = 2313 \]
13. A computer with a useful life of five years has an initial cost of $6000. The salvage value is $2300, and the annual maintenance is $210/yr. The interest rate is 8%. What is most nearly the present worth of the costs for the computer?

(A) $5200
(B) $5300
(C) $5600
(D) $5700

\[ P = \frac{F}{(1 + i)^n} = \frac{2300}{(1 + 0.08)^5} = \$1565.38 \quad \text{(Benefit)} \]
\[ P = \frac{A}{(1 + i)^n} = \frac{210}{(1 + 0.08)^5} = \$838.47 \quad \text{(Cost)} \]
\[ P = -6000 + 1565.38 - 838.47 = -5273.09 \]
14. A company must purchase a machine that will be used over the next eight years. The purchase price is $10,000, and the salvage value after eight years is $1000. The annual insurance cost is 2% of the purchase price, the electricity cost is $300 per year, and maintenance and replacement parts cost $100 per year. The effective annual interest rate is 6%. Neglect taxes. The effective uniform annual cost (EUAC) of ownership is most nearly

(A) $1200

(B) $2100

(C) $2200

(D) $2300

\[ P = \$10,000 \]
\[ F = \$1000 \]
\[ A = 0.02 \times (\$10,000) = \$200 \]
\[ i = 6\% \]
\[ N = 8 \text{ yrs} \]

\[ A = P \left( \frac{A}{P}, 6\%, 8 \right) = \$10,000 \left( 0.1610 \right) = \$1610 \quad \text{(Cost)} \]

\[ A = F \left( \frac{A}{F}, 6\%, 8 \right) = \$1000 \left( 0.1010 \right) = \$101 \quad \text{(Benefit)} \]

\[ A = 0.02 \times (\$10,000) = \$200 \quad \text{(Cost)} \]

\[ A = \$300 \quad \text{(Cost)} \]

\[ A = \$100 \quad \text{(Cost)} \]

\[ \text{EUAC} = \sum A = \$1610 - \$101 + \$200 + \$300 + \$100 \]

\[ \text{EUAC} = \$2109 \]
15. A company purchases a piece of equipment for $15,000. After nine years, the salvage value is $900. The annual insurance cost is 5% of the purchase price, the electricity cost is $600/yr, and the maintenance and replacement parts cost is $120/yr. The effective annual interest rate is 10%. Neglecting taxes, what is most nearly the present worth of the equipment if it is expected to save the company $4500 per year?

(A) $2300
(B) $2800
(C) $3200
(D) $3500

\[ P = \$15000 \]  
\[ F = \$4500 \]  
\[ A = 0.05 \times \$15000 \]  
\[ A = \$600 \]  
\[ A = \$120 \]  
\[ i = 10\% \]  
\[ n = 9 \text{ yrs} \]

\[ P = \$15000 \quad \text{(cost)} \]

\[ P = F \left( \frac{P}{F}, 10\%, 9 \right) = \$900 (6.4241) = \$3781.69 \quad \text{(benefit)} \]

\[ P = A \left( \frac{P}{A}, 10\%, 9 \right) = 0.05 \times \$15000 (5.7590) = \$4319.25 \quad \text{(cost)} \]

\[ P = A \left( \frac{P}{A}, 10\%, 9 \right) = \$600 (5.7590) = \$3455.40 \quad \text{(cost)} \]

\[ P = A \left( \frac{P}{A}, 10\%, 9 \right) = \$120 (5.7590) = \$691 \quad \text{(cost)} \]

\[ P = A \left( \frac{P}{A}, 10\%, 9 \right) = \$4500 (5.7590) = \$25915 \quad \text{(benefit)} \]

\[ EUAC = \sum P = \$2832 \]
ANNUAL COST  EUAC

1. DEFINE BENEFIT & COST AND PERIOD OF INVESTMENT.
2. DETERMINE DESIRED RANGE OF RETURN.
3. USE FORMULAS

1. CONVERT ALL COSTS TO PRESENT VALUE & THEN SPREAD OUT AS EUAC
2. CONVERT ALL RECEIPTS TO EUAB
3. COMPARE EUAB > EUAC

EX.

PERIOD = 8 yr
MARK = 12%

COST = $55,000
SALVAG = $4,000
MAINTANANCE = $3,500 ANNUALLY
REVENUES = $15,000 ANNUALLY

DETERMINE EUAC:

USE: \( A = P \times (A/P, i, n) \)

\[ A = F \times (A/F, i, n) \]

\[ EUAC = P \times (A/P, i, n) + A \times (A/F, i, n) \times (1 + i)^n \]

\[ = 50,000 \times (A/P, 12\%, 8) + 3,500 \times (A/F, 12\%, 8) \times (1.12^8) \]

\[ = 142,460 \]