Dimensions and Units

A dimension is a qualitative description of the physical nature of some quantity.

Notes:
1. A basic or primary dimension is one that is not formed from a combination of other dimensions. It is an independent quantity.
2. A secondary dimension is one that is formed by combining primary dimensions.
3. Common dimensions include:
   [M] = mass
   [L] = length
   [T] = time
   [θ] = temperature
   [F] = force
4. If [M], [L], and [T] are primary dimensions, then [$F = ML/T^2$] is a secondary dimension. If [F], [L], and [T] are primary dimensions, then [$M = FT^2/L$] is a secondary dimension.

A unit is a quantitative description of a dimension. A unit gives "size" to a dimension.

Common systems of units in engineering include:

primary dimension	**SI (Systéme International d' Unités)**	**BG (British Gravitational)**	**EE (English Engineering)**
[L], length	meter (m)	foot (ft)	foot (ft)
[T], time	second (s)	second (s)	second (s)
[θ], temperature	Kelvin (K)	degree Rankine (°R)	degree Rankine (°R)
[M], mass	kilogram (kg)	- not primary -	pound mass (lb_m or lb)
[N], amount of a substance	mole (mol)	mole (mol)	pound mole (lbmol)
electric current	ampere (A)	ampere (A)	ampere (A)
luminous intensity	candela (cd)	candela (cd)	candela (cd)
[F], force	- not primary -	pound force (lb_f)	pound force (lb_f)

Notes:
1. The mole is the amount of substance that contains the same number of elementary entities as there are atoms in 12 g of carbon 12 (= $6.022*10^{23}$, known as Avogadro's constant). The elementary entities must be specified, e.g., atoms, molecules, particles, etc. The unit kmol (aka kgmol) is also frequently used, with 1 kmol = 1000 mol = $6.022*10^{26}$ entities. The unit lbmol is used in the English system of units. Since 1 lb_m = 0.453 kg, 1 lbmol = 453.592 mol. The number of kmols of a substance, *n*, is found by dividing the mass of the substance, *m* (kg) by the molecular weight of the substance, *M* (in kg/kmol): $n = m/M$. For example, the atomic weight of carbon 12 is 12 kmol/kg, hence, 1 kg of carbon 12 contains: (12 kg)/(12 kg/kmol) = 1 kmol = 1000 mol (or 12 g of C^{12} contains 1 mol).
2. In the SI system, force is a secondary dimension and is given by: $F = ML/T^2$ where 1 N = 1 $kg \cdot m/s^2$.
3. In the BG system, mass is a secondary dimension and is given by: $M = FT^2/L$ where 1 slug = 1 $lb_f.s^2/ft$.
4. In the EE system, both force and mass are primary dimensions. The two are related via Newton's second law, $g_cF = ma$ where g_c = 32.2 (lb_m-ft/(lb_f-s^2)). A simple conversion is to remember that: 1 lb_f = 32.2 $lb_m \cdot ft/s^2$.
5. The kilogram-force (kgf) is (unfortunately) a not uncommon quantity. The conversion between kgf and Newtons is: 1 kgf = 9.81 N.
6. It's a good policy to carry units through your calculations. Remember that, unless it's dimensionless, every number has a unit attached to it. For example, if m = 1 lb_m and g = 32.2 ft/s^2,
   Poor Practice: $mg = (1)(32.2) = 1\ lb_f$

Good Practice: $mg = (1\ \text{lb}_m)(32.2\ \text{ft/s}^2) = \left(32.2\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2}\right)\underbrace{\left(\frac{1\ \text{lb}_f}{32.2\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2}}\right)}_{\text{unit conversion}} = 1\ \text{lb}_f$

Carrying through your units makes it less likely that you'll make unit conversion errors, plus it makes it easier for you and others to follow your work.

Example: What is 70 mph in furlongs per fortnight?

SOLUTION: $\left(70\frac{\text{miles}}{\text{hour}}\right)\left(\frac{5280\text{ feet}}{\text{miles}}\right)\left(\frac{\text{rod}}{16.5\text{ feet}}\right)\left(\frac{\text{furlong}}{40\text{ rods}}\right)\left(\frac{24\text{ hours}}{\text{day}}\right)\left(\frac{7\text{ days}}{\text{week}}\right)\left(\frac{2\text{ weeks}}{\text{fortnight}}\right) = 1.88*10^5\ \frac{\text{furlongs}}{\text{fortnight}}$

Dimensional homogeneity is the concept whereby only quantities with similar dimensions can be added (or subtracted). It is essentially the concept of "You can't add apples and oranges."

For example, consider the following equation:

10 kg + 16 °C = 26 m/s

This equation doesn't make sense since it is not dimensionally homogeneous. How can one add mass to temperature and get velocity?!?

Note that dimensional homogeneity is a necessary, but not sufficient, condition for an equation to be correct. In other words, an equation must be dimensionally homogeneous to be correct, but a dimensionally homogeneous equation isn't always correct. For example,

10 kg + 10 kg = 25 kg

The equation has the right dimensions but the wrong answer!

Be Sure To:

1. Verify that equations are dimensionally homogeneous.
2. Carefully evaluate unit conversions. (A unit conversion error caused the loss of the $125M Mars Climate Observer spacecraft in 1999!)