3. Pump Similarity

Most pump performance data (H-Q curves) are given only for one value of the pump rotational speed and one pump impeller diameter. Is there some way to determine the pump performance data for other speeds and diameters without requiring additional testing? There is – using dimensional analysis!

Recall that we typically are interested in knowing the head rise across a pump, H ($=h/g$ where h is the specific energy rise across the pump), power required to operate the pump, $\dot{W}$ (bhp), and pump efficiency, η, as a function of the volumetric flowrate through the pump, Q:

$$h, \dot{W}, \eta = fcns(Q, \rho, \mu, D, \omega) \tag{14}$$

where ρ and μ are the fluid density and dynamic viscosity, D is the pump impeller diameter, and ω is the pump rotational speed.

Performing a dimensional analysis we find the following:

$$\Psi, \Pi, \eta = fcns(\Phi, \text{Re}) \tag{15}$$

where

$$\Psi \equiv \text{dimensionless head coefficient} = \frac{gH}{\omega^2 D^2} \tag{16}$$

$$\Pi \equiv \text{dimensionless power coefficient} = \frac{\dot{W}}{\rho \omega^3 D^5} \tag{17}$$

$$\eta \equiv \text{efficiency} = \frac{\rho Q g H}{\dot{W}} \tag{18}$$

$$\Phi \equiv \text{dimensionless flow coefficient} = \frac{Q}{\omega D^3} \tag{19}$$

$$\text{Re} \equiv \text{Reynolds number} = \frac{\rho \omega D^2}{\mu} \tag{20}$$

Notes:

1. In most pump flows, Re is very large
   ⇒ the variations in viscous effects from one flow to another are small
   ⇒ Re similarity can be neglected

   a. If Re is considerably different from one flow to another, e.g., pump water vs. pumping molasses, then Re effects cannot be ignored. The flow physics for large Re, where viscous forces « inertial forces, are different than for small Re where viscous forces » inertial forces.
   b. Thus, for large Re:

$$\Psi, \Pi, \eta = fcns(\Phi) \tag{21}$$

2. $$\eta = \frac{\rho Q g H}{\dot{W}} = \frac{\Psi \Phi}{\Pi} \tag{22}$$

3. For <u>similarity</u> between <u>geometrically similar flows</u> (assuming large Re), we have the following **pump scaling laws**:

$$\Phi_1 = \Phi_2 \quad \Rightarrow \quad \left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2 \tag{23}$$

$$\Psi_1 = \Psi_2 \quad \Rightarrow \quad \left(\frac{gH}{\omega^2 D^2}\right)_1 = \left(\frac{gH}{\omega^2 D^2}\right)_2 \tag{24}$$

$$\Pi_1 = \Pi_2 \quad \Rightarrow \quad \left(\frac{\dot{W}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}}{\rho\omega^3 D^5}\right)_2 \tag{25}$$

$\Rightarrow \eta_1 = \eta_2$ (since $\Phi_1 = \Phi_2$, $\Psi_1 = \Psi_2$, $\Pi_1 = \Pi_2$ and $\eta = \Psi\Phi/\Pi$)

a. For a given pump (D = constant) using the same fluid (ρ, μ = constant) and the same gravity ($g_1 = g_2$):

$$\left(\frac{Q}{\omega}\right)_1 = \left(\frac{Q}{\omega}\right)_2 \Rightarrow \frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \tag{26}$$

$$\left(\frac{H}{\omega^2}\right)_1 = \left(\frac{H}{\omega^2}\right)_2 \Rightarrow \frac{H_1}{H_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \tag{27}$$

$$\left(\frac{\dot{W}}{\omega^3}\right)_1 = \left(\frac{\dot{W}}{\omega^3}\right)_2 \Rightarrow \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{\omega_1}{\omega_2}\right)^3 \tag{28}$$

The efficiency remains relatively constant when only changing the pump rotation speed (as given at the start of this note).

b. For a given pump speed (ω = constant) but varying diameters (assuming a geometrically similar family of pumps), and using the same fluid (ρ, μ = constant) and the same gravity ($g_1 = g_2$):

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^3 \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{D_1}{D_2}\right)^5 \tag{29}$$

Note that we are assuming that <u>all</u> length scales within the pump are scaled in the same way to maintain geometric similarity. This is not always true in practice since pump impellers with different diameters are often put in the same pump casing. Also, surface roughness isn't scaled proportionally. The result is that the pump scaling laws are only approximations.

Since geometric scaling isn't completely satisfied, researchers have proposed empirical scaling rules that produce more accurate predictions than the ones given above:

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{D_1}{D_2}\right)^4 \tag{30}$$

Since the scaling isn't perfect, the efficiency will not remain the same when scaling impeller size. As was done with the flow rate, head rise, and power, an empirical relationship can be used to scale the pump efficiencies, such as the one proposed by Moody,

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{1/5}. \tag{31}$$