The Continuity Equation

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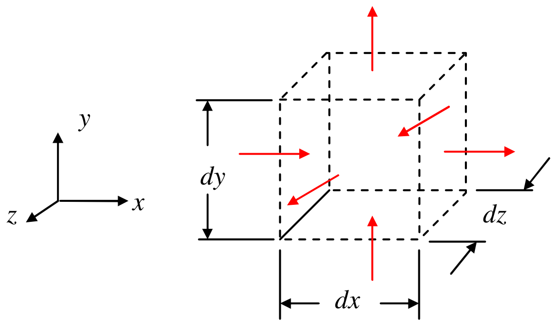

$$\dot{m}_{\text{in through left}} = \dot{m}_{x,\text{center}} + \frac{\partial \dot{m}_{x,\text{center}}}{\partial x}\left(-\tfrac{1}{2}dx\right)$$

$$= \left(\rho u_x dydz\right) + \frac{\partial}{\partial x}\left(\rho u_x dydz\right)\left(-\tfrac{1}{2}dx\right)$$

$$= \left[\rho u_x + \frac{\partial}{\partial x}\left(\rho u_x\right)\left(-\tfrac{1}{2}dx\right)\right]\left(dydz\right)$$

(where $\dot{m}_{x,\text{center}}$ is the mass flux in the x-direction at the center of the control volume

$$\dot{m}_{\text{out through right}} = \left[\rho u_x + \frac{\partial}{\partial x}\left(\rho u_x\right)\left(\tfrac{1}{2}dx\right)\right]\left(dydz\right)$$

$$\dot{m}_{\text{in through bottom}} = \left[\rho u_y + \frac{\partial}{\partial y}\left(\rho u_y\right)\left(-\tfrac{1}{2}dy\right)\right]\left(dxdz\right)$$

$$\dot{m}_{\text{out through top}} = \left[\rho u_y + \frac{\partial}{\partial y}\left(\rho u_y\right)\left(\tfrac{1}{2}dy\right)\right]\left(dxdz\right)$$

$$\dot{m}_{\text{in through back}} = \left[\rho u_z + \frac{\partial}{\partial z}\left(\rho u_z\right)\left(-\tfrac{1}{2}dz\right)\right]\left(dxdy\right)$$

$$\dot{m}_{\text{out through front}} = \left[\rho u_z + \frac{\partial}{\partial z}\left(\rho u_z\right)\left(\tfrac{1}{2}dz\right)\right]\left(dxdy\right)$$

$$\dot{m}_{\text{net,into CV}} = -\left[\frac{\partial}{\partial x}\left(\rho u_x\right) + \frac{\partial}{\partial y}\left(\rho u_y\right) + \frac{\partial}{\partial z}\left(\rho u_z\right)\right]\left(dxdydz\right)$$

$$\left.\frac{\partial m}{\partial t}\right|_{\text{within CV}} = \frac{\partial}{\partial t}\left(\rho dxdydz\right)$$

$$\frac{\partial \rho}{\partial t}\left(dxdydz\right) = -\left[\frac{\partial}{\partial x}\left(\rho u_x\right) + \frac{\partial}{\partial y}\left(\rho u_y\right) + \frac{\partial}{\partial z}\left(\rho u_z\right)\right]\left(dxdydz\right)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho u_x\right) + \frac{\partial}{\partial y}\left(\rho u_y\right) + \frac{\partial}{\partial z}\left(\rho u_z\right) = 0}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho \mathbf{u}\right) = 0}$$