3. Conservation of Mass (COM)

In words and in mathematical terms, COM for a system is:

The mass of a system remains constant. $\Rightarrow$ $$\frac{D}{Dt}\Bigg(\underbrace{\int_{V_{\text{system}}} \rho\, dV}_{\text{mass of the system}}\Bigg) = 0 \qquad (20)$$

where *D/Dt* is the Lagrangian derivative (implying that we're using the rate of change as we follow the system), *V* is the volume, and ρ is the density. Using the Reynolds Transport Theorem, Eq. (20) can be converted into an expression for a control volume,

$$\frac{D}{Dt}\left(\int_{V_{\text{system}}} \rho dV\right) = \frac{d}{dt}\left(\int_{\text{CV}} \rho dV\right) + \int_{\text{CS}} \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = 0 \qquad (21)$$

$$\boxed{\underbrace{\frac{d}{dt}\int_{\text{CV}} \rho\, dV}_{\substack{\text{rate of increase of}\\ \text{mass inside the CV}}} + \underbrace{\int_{\text{CS}} \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}_{\substack{\text{net rate at which mass leaves}\\ \text{the CV through the CS}}} = 0} \qquad \textbf{COM for a CV} \qquad (22)$$

Helpful Hints:

1. Carefully draw your control volume. Don't neglect to draw a control volume or draw a control volume and then use a different one.
2. Make sure you understand what each term in conservation of mass represents.
3. Carefully evaluate the dot product in the mass flux term.
4. You must integrate the terms in conservation of mass when the density or velocity are not uniform.
5. Note that the first term in Eq. (22) is the rate of increase of mass in the CV, which can be re-written as,

   $$\frac{d}{dt}\int_{\text{CV}} \rho dV = \frac{dM_{CV}}{dt} \qquad (23)$$

   where M_{CV} is the mass inside the control volume. Similarly, the second term in Eq. (22), which is the net rate at which mass leaves the CV through the CS, may be written as,

   $$\int_{\text{CS}} \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \sum_{\text{all outlets}} \dot{m}_{\text{out}} - \sum_{\text{all inlets}} \dot{m}_{\text{in}} \qquad (24)$$

   where $\dot{m}$ is a mass flow rate. Combining Eqs. (22) - (24) gives:

   $$\boxed{\frac{dM_{CV}}{dt} = \sum_{\text{all inlets}} \dot{m}_{\text{in}} - \sum_{\text{all outlets}} \dot{m}_{\text{out}}} \quad \text{Alternate form of COM} \qquad (25)$$

   Note that the net flow rate of mass term has been moved to the right side of the equation.

Let's consider a few examples to see how COM is applied.