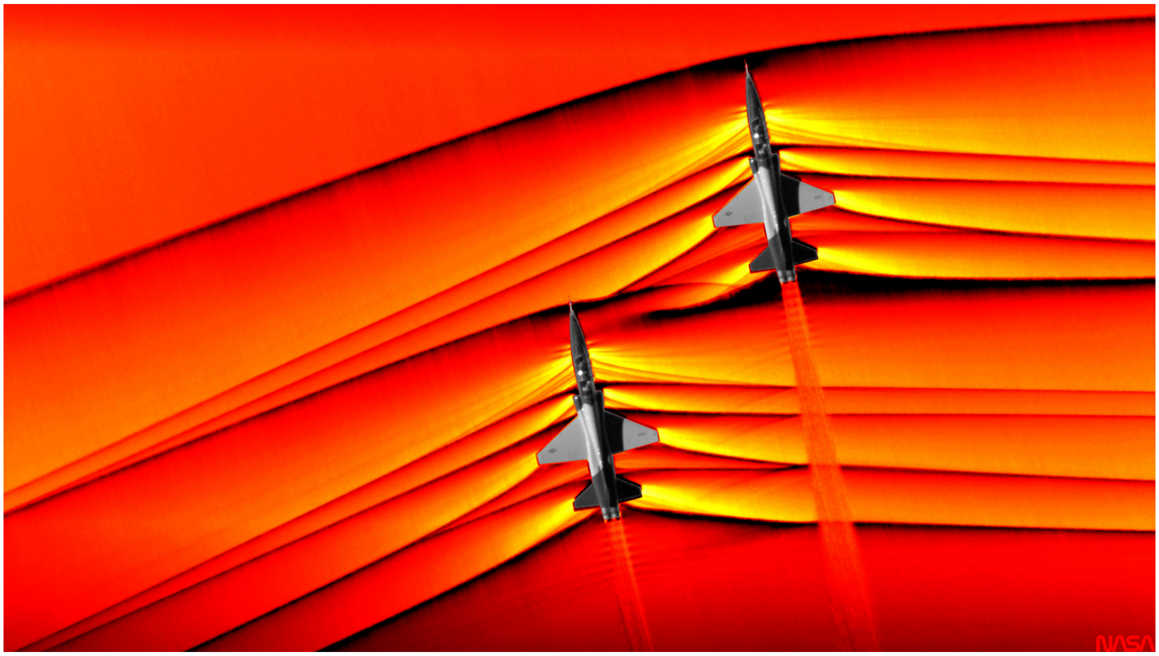
# Compressible Flow – Introduction



https://imgur.com/r/wallpapers/ufMF4X2

# Compressible Flow – Introduction

## Thermodynamics Review

*Some Definitions*

$u$ := specific internal energy
$h$ := specific enthalpy = $u + p/\rho$
$c_v$ := specific heat at constant volume
$c_p$ := specific heat at constant pressure
$k$ := specific heat ratio = $c_p/c_v$ ($k$ = 1.4 for air)
$R$ := gas constant ($R_{air}$ = 287 J/(kg.K) = 53.3 (ft.lbf)/(lbm.degR))
adiabatic := no heat transfer
reversible := system and surroundings can be restored exactly to their initial states
isentropic := no change in entropy, i.e., $\Delta s = 0$
adiabatic and reversible => isentropic
$T$-$s$ diagrams are frequently used to illustrate processes

*Ideal Gas Relations*

$p = \rho R T$ (use absolute pressure and temperature; $R$ is the gas constant, not the universal gas constant))

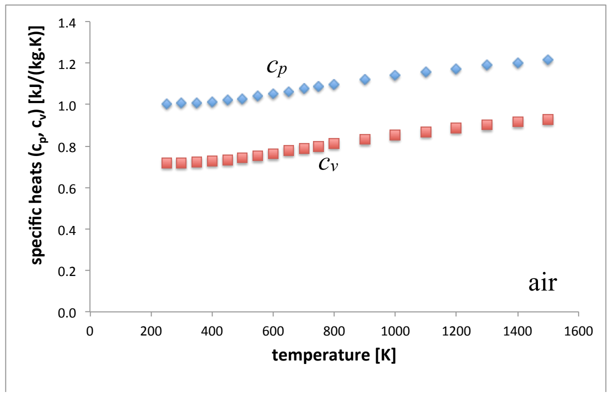
$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T)\,dT$$

$$h_2 - h_1 = \int_{T_1}^{T_2} c_p(T)\,dT$$

$$c_p = c_v + R$$

$$c_p = \frac{kR}{k-1} \quad \text{and} \quad c_v = \frac{R}{k-1}$$

$$ds = c_v(T)\frac{dT}{T} - R\frac{d\rho}{\rho} = c_p(T)\frac{dT}{T} - R\frac{dp}{p}$$




*Perfect Gas Relations*

perfect gas = ideal gas with constant specific heats

$$u_2 - u_1 = c_v(T_2 - T_1) \quad (c_{v,air} = 718 \text{ J/(kg.K)} = 133 \text{ (ft.lbf)/(lbm.degR)})$$

$$h_2 - h_1 = c_p(T_2 - T_1) \quad (c_{p,air} = 1005 \text{ J/(kg.K)} = 187 \text{ (ft.lbf)/(lbm.degR)})$$

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} - R\frac{\rho_2}{\rho_1} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

isentropic process involving a perfect gas: $\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{k-1}$, $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$, $\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{k}$

# Compressible Flow – Introduction

*1st Law for steady flow of a gas with one inlet and one outlet and uniform inlet and outlet properties*

$$\underbrace{\frac{d}{dt}\int_{CV} e\rho\, dV}_{=0\ (\text{steady})} + \int_{CS}\left( h + \tfrac{1}{2}V^2 + \underbrace{gz}_{\substack{\text{negligible compared}\\ \text{to other terms since}\\ \text{its a gas}}} \right)\underbrace{\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}_{=d\dot{m}} = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{on CV}}$$

From COM: $\dot{m} = \dot{m}_{\text{in}} = \dot{m}_{\text{out}}$. Combining with 1st Law:

$$\Delta\left(h + \tfrac{1}{2}V^2\right) = q_{\text{into CV}} + w_{\text{on CV}}$$

For an adiabatic ($q_{\text{into}} = 0$) flow with no work other than pressure work ($w_{\text{on}} = 0$):

$$\Delta\left(h + \tfrac{1}{2}V^2\right) = 0 \quad \text{or} \quad h + \tfrac{1}{2}V^2 = \text{constant}$$

For a perfect gas ($\Delta h = c_p \Delta T$):

$$c_p T + \tfrac{1}{2}V^2 = \text{constant}$$