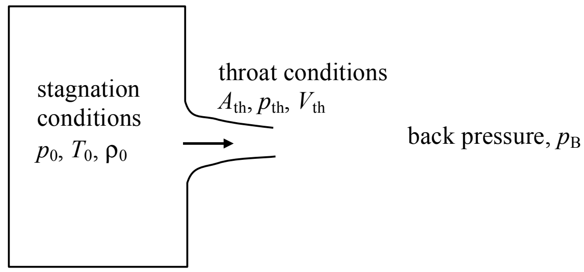
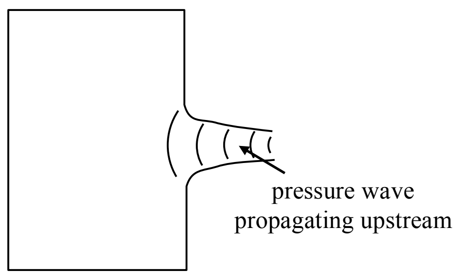
Choked Flow

Consider the flow of a compressible fluid from a large reservoir into the surroundings. Let the pressure of the surroundings, called the **back pressure, p_B,** be controllable:



When $p_B = p_0$ there will be no flow from the reservoir since there is no driving pressure gradient. When the back pressure, p_B, is decreased, a pressure wave propagates through the fluid in the nozzle and into the tank. Thus, the fluid in the tank "is informed" that the pressure outside has been lowered and a pressure gradient is established resulting in fluid being pushed out of the tank.



Thus, when $p_B < p_0$, the fluid will begin to flow out of the reservoir. Furthermore, as $p_B/p_0 \downarrow$, $V_{th} \uparrow$, and the mass flow rate $\uparrow$. Note that the flow through the nozzle will be subsonic (Ma<1) since the fluid starts off from stagnation conditions and does not pass through a minimum until reaching the throat. Additionally, since the flow is subsonic, the pressure at the throat will be the same as the back pressure, i.e., $p_{th} = p_B$. That this is so can be seen by noting that if $p_{th} > p_B$, then the flow would expand upon leaving the nozzle and as a result, the jet velocity would decrease, and the pressure would increase. Thus, the jet pressure would diverge from the surrounding pressure. But the jet must eventually reach the surrounding pressure so the assumption that $p_{th} > p_B$ must be incorrect. A similar argument can be made for $p_{th} < p_B$.

As we continue to decrease p_B/p_0, we'll eventually reach a state where the velocity at the throat will reach Ma=1 ($V_{th} = V^* = c^*$). The pressure ratio at the throat will then be:

$$\frac{p_{th}}{p_0} = \frac{p_B}{p_0} = \frac{p^*}{p_0} = \left(1 + \frac{\gamma - 1}{2}\right)^{\gamma/(1-\gamma)} \tag{52}$$

and the fluid velocity will be:

$$V_{th} = V^* = c^* = \sqrt{\gamma R T^*} \tag{53}$$

Any further decrease in p_B has no effect on the velocity at the throat since the pressure information can no longer propagate upstream into the reservoir. The fluid velocity out of the tank is the same as the speed of the pressure wave into the tank so that the pressure information can't propagate upstream of the throat. Thus, all flow conditions upstream of the throat will remain unchanged. As a result, we can no longer increase the mass flow rate from the tank by changing the back pressure. This condition is referred to as **choked flow conditions**. The maximum, or choked, mass flow rate will be the same as the mass flow rate at the throat where sonic conditions occur:

$$\dot{m}_{\text{choked}} = \rho^* V^* A^* = \frac{p^*}{RT^*} V^* A^*$$
$$= p^* \sqrt{\frac{\gamma}{RT^*}} A^*$$

where Eq. (53) has been utilized. Substituting the following isentropic relations:

$$p^* = \frac{p^*}{p_0} p_0 = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{1-\gamma}} p_0$$

$$T^* = \frac{T^*}{T_0} T_0 = \left(1 + \frac{\gamma - 1}{2}\right)^{-1} T_0$$

and simplifying results in:

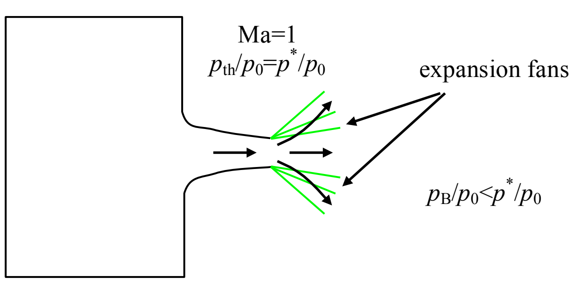
$$\boxed{\dot{m}_{\text{choked}} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(1-\gamma)}} p_0 \sqrt{\frac{\gamma}{RT_0}} A^*} \tag{54}$$

Notes:

1. The choked mass flow rate (Eq. (54)) is the maximum mass flow rate that can be achieved from the reservoir.
2. A quick check to see if the flow will be choked or not is to check if the back pressure is less than or equal to the sonic pressure, i.e.:

$$\text{If } \frac{p_B}{p_0} \le \frac{p^*}{p_0} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{1-\gamma}}, \text{ then the flow will be choked and } \frac{p_{th}}{p_0} = \frac{p^*}{p_0}. \tag{55}$$

3. What happens outside of the nozzle if the back pressure is less than the sonic pressure? The flow must eventually adjust to the surrounding pressure. It does so by expanding in a two-dimensional process known as an **expansion fan**; a topic we'll address later in the course.



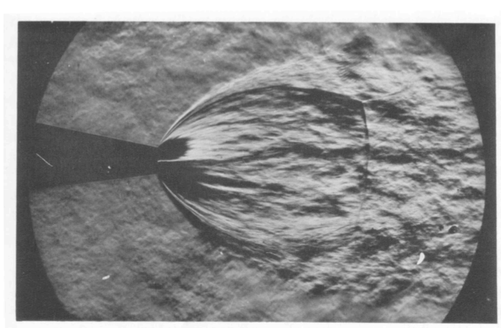


Figure 6.8
External expansion of the jet from a choked converging nozzle, $P_e/P_a \approx 105$. (*Courtesy of E. S. Love, NASA*).

4. The previously described processes are sketched in the following plots: