Compressible Flow – Flow through a Converging Nozzle

FIG. 3. Free jet Schlieren results under different inlet pressures with different nozzles: (a) conical nozzle: 4 bars, (b) conical nozzle: 7 bars, (c) conical nozzle: 10 bars, (d) MLN nozzle: 4 bars, (e) MLN nozzle: 7 bars, and (f) MLN nozzle: 10 bars.

Compressible Flow – Flow through a Converging Nozzle

1D, steady, adiabatic flow of a perfect gas with no work other than pressure work

\[ \frac{T}{T_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{-\frac{1}{k-1}} \]

\[ \frac{c}{c_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{-\frac{k}{2(k-1)}} \]

1D, steady, isentropic flow of a perfect gas with no work other than pressure work

\[ \frac{p}{p_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{\frac{1}{k-1}} \]

\[ \frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{\frac{k}{2(k-1)}} \]

\[ \frac{A}{A^*} = \frac{1}{\text{Ma}} \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{-\frac{1}{k-1}} \]

Choked Flow

stagnation conditions
\[ p_0, T_0, \rho_0 \]

throat conditions
\[ A_{th}, p_{th}, V_{th} \]

back pressure, \( p_B \)

Combine the mass flow rate at sonic conditions with the isentropic relations at sonic conditions:

\[ \dot{m}_{\text{choked}} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{\frac{1}{k-1}} p_0 \left(\frac{k}{RT_0}\right)^{A^*} \]
Any further decrease in $p_B$ has no effect on the velocity at the throat since the pressure information can no longer propagate upstream into the reservoir. The fluid velocity out of the tank is the same as the speed of the pressure wave into the tank so that the pressure information can’t propagate upstream of the throat. Thus, all flow conditions upstream of the throat will remain unchanged. As a result, we can no longer increase the mass flow rate from the tank by changing the back pressure. This condition is referred to as choked flow conditions. The maximum, or choked, mass flow rate will be the same as the mass flow rate at the throat where sonic conditions occur:

$$m_{\text{choked}}$$

where equation (12.92) has been utilized. Substituting the following isentropic relations:

$$\frac{p}{p_0}, \frac{p^*}{p_0}, \frac{p^*}{p_0}$$

and simplifying results in:

$$\frac{p}{p_0} = \frac{p^*}{p_0}$$

Notes:

1. The choked mass flow rate (equation (12.93)) is the maximum mass flow rate that can be achieved from the reservoir.
2. A quick check to see if the flow will be choked or not is to check if the back pressure is less than or equal to the sonic pressure, i.e.

$$\frac{p_B}{p_0} < \frac{p^*}{p_0}$$

(12.94)

3. What happens outside of the nozzle if the back pressure is less than the sonic pressure? The flow must eventually adjust to the surrounding pressure. It does so by expanding in a two-dimensional process known as an expansion fan; a topic we’ll address later in the course.

$$\frac{p}{p_0} = \frac{p^*}{p_0}$$

Ma=1

$$\frac{p^*}{p_0} = \frac{p_B}{p_0}$$

pressure variations in expansion fan

$\frac{p}{p_0}$

throat

$\frac{p_B}{p_0}$

expansion fans

Figure 12.8

External expansion of the jet from a choked converging nozzle, $p_B/p_0 > 1$. (Courtesy of E. S. Lover, NASA.)