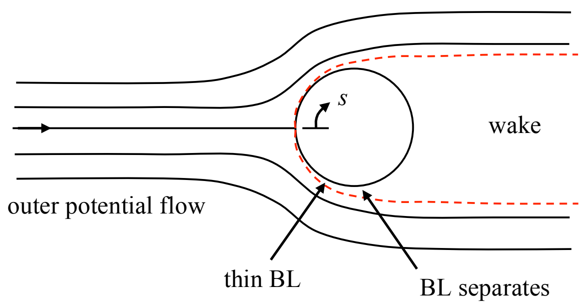
7. Boundary Layer Separation

Consider flow around a cylinder as shown below:



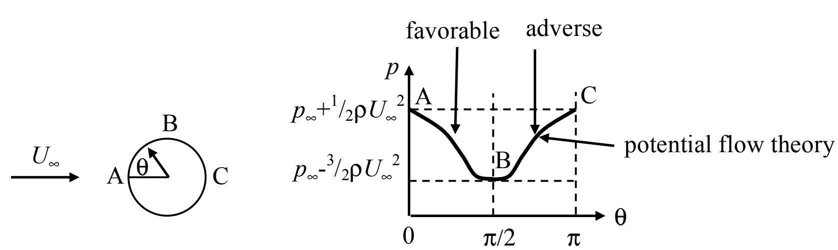
Let s be the distance from the stagnation point along the cylinder surface.

On the front side of the cylinder the boundary layer grows with increasing distance as we might expect. (Note: The pressure near the cylinder surface decreases with distance from the stagnation point on the front half $\Rightarrow dp/ds < 0$.) At a particular location on the cylinder surface, the boundary layer no longer remains "attached" to the surface. This point is termed the boundary layer separation point. The occurrence of boundary layer separation is a result of energy dissipation in the boundary layer due to viscous effects and adverse pressure gradients.

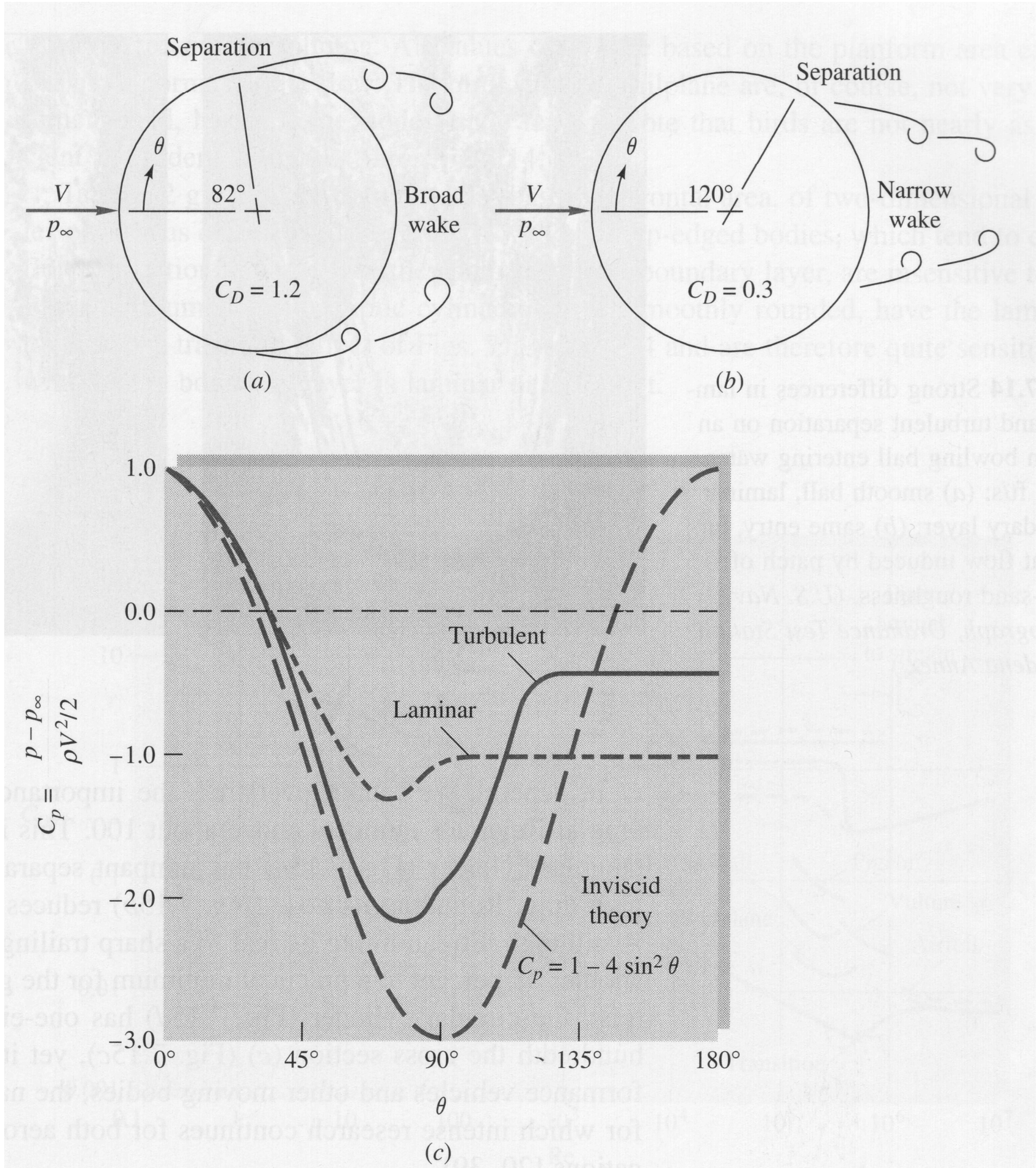
Define:

an adverse pressure gradient as one in which $dp/ds > 0$, and
a favorable pressure gradient as one in which $dp/ds < 0$

Consider the pressure near the surface of the cylinder determined from potential flow theory:



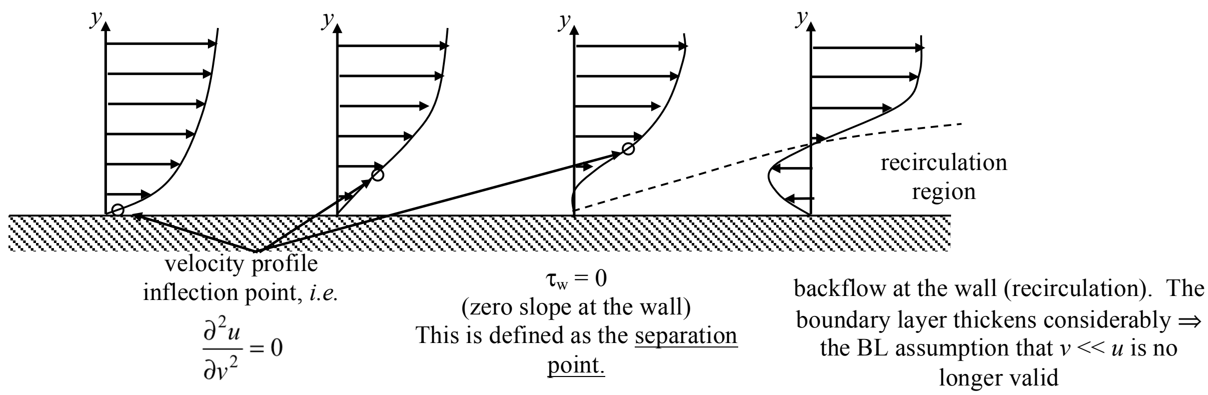
Imagine a fluid particle in the BL. It experiences approximately the same pressure as that in the outer potential flow (refer to the notes discussing the BL equations). As the fluid particle moves in the BL, it loses energy due to viscous effects. As a result, it does not have enough energy to "coast up the pressure hill" from point B to C. The fluid moves up the pressure hill as far as possible and then BL separation occurs.



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)

Inviscid analysis works reasonably well on the upstream side but fails miserably on the downstream side due to boundary layer separation. Notice that the pressure in the wake is relatively uniform and equal to the pressure at the boundary layer separation point. Hence, delaying boundary layer separation helps to recover the pressure in the wake as a well as decreasing the area of the wake. As a result, the drag due to pressure forces (aka form drag) decreases as boundary layer separation is delayed.

Let's look at the velocity profile at different points along a plate for a flow with an adverse pressure gradient ($dp/dx > 0$):



In an adverse pressure gradient flow the boundary layer velocity profile will always have an inflection point. This can be shown by considering the boundary layer momentum equation:

We don't cover this eqn in ME309. You needn't worry about it.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$ (Bernoulli's equation has been used to write: $\frac{1}{\rho}\frac{dp}{dx} = -U\frac{dU}{dx}$.)

Note that at the wall boundary ($y = 0$), $u = v = 0$ so that:

$$\frac{dp}{dx} = \mu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}$$

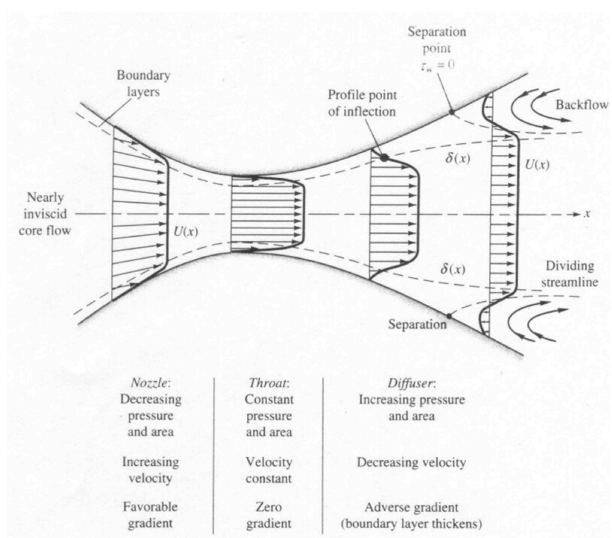
Thus, at the wall boundary in an adverse pressure gradient ($dp/dx > 0$), we must have:

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} > 0$$

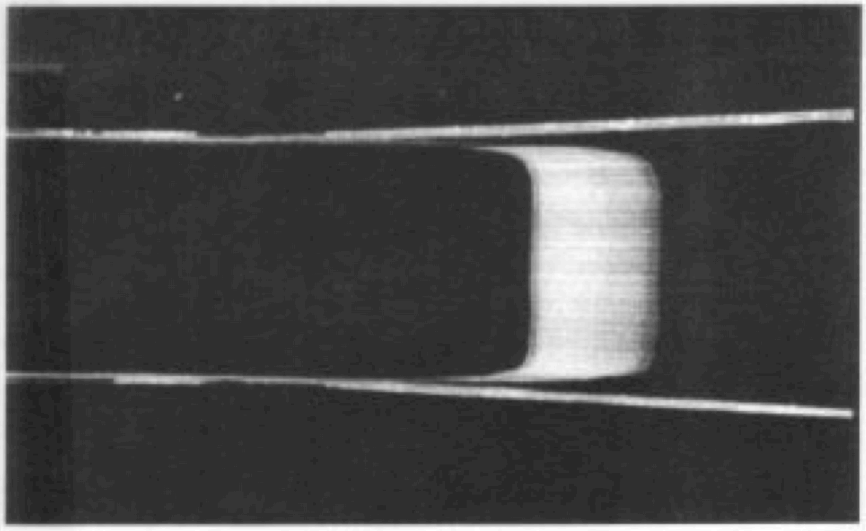
However, at the free stream interface ($y = \delta$) we have:

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=\delta} < 0$$

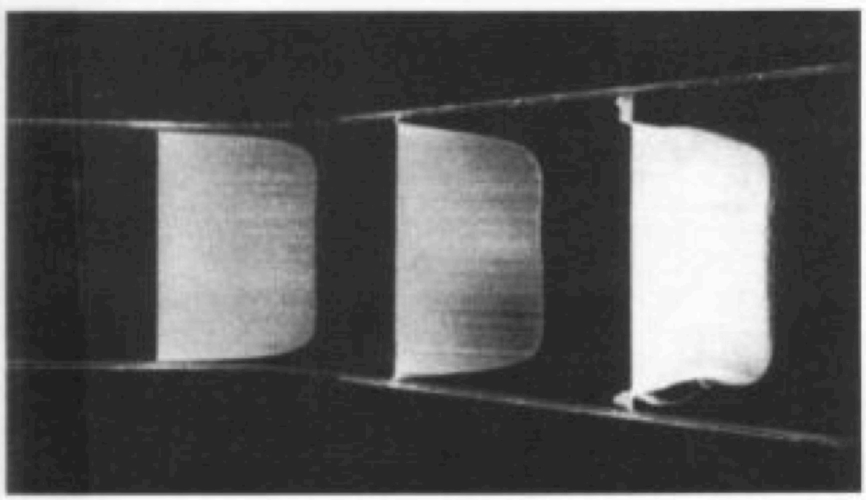
in order for the boundary layer profile to merge smoothly with the outer flow velocity profile. The change in sign of the boundary layer curvature indicates that somewhere within the boundary layer there must be a point of inflection. The inflection point moves toward the outer flow boundary as the flow moves downstream in an adverse pressure gradient flow.



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)



(b)

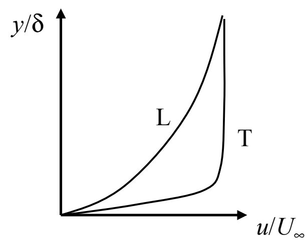


(c)

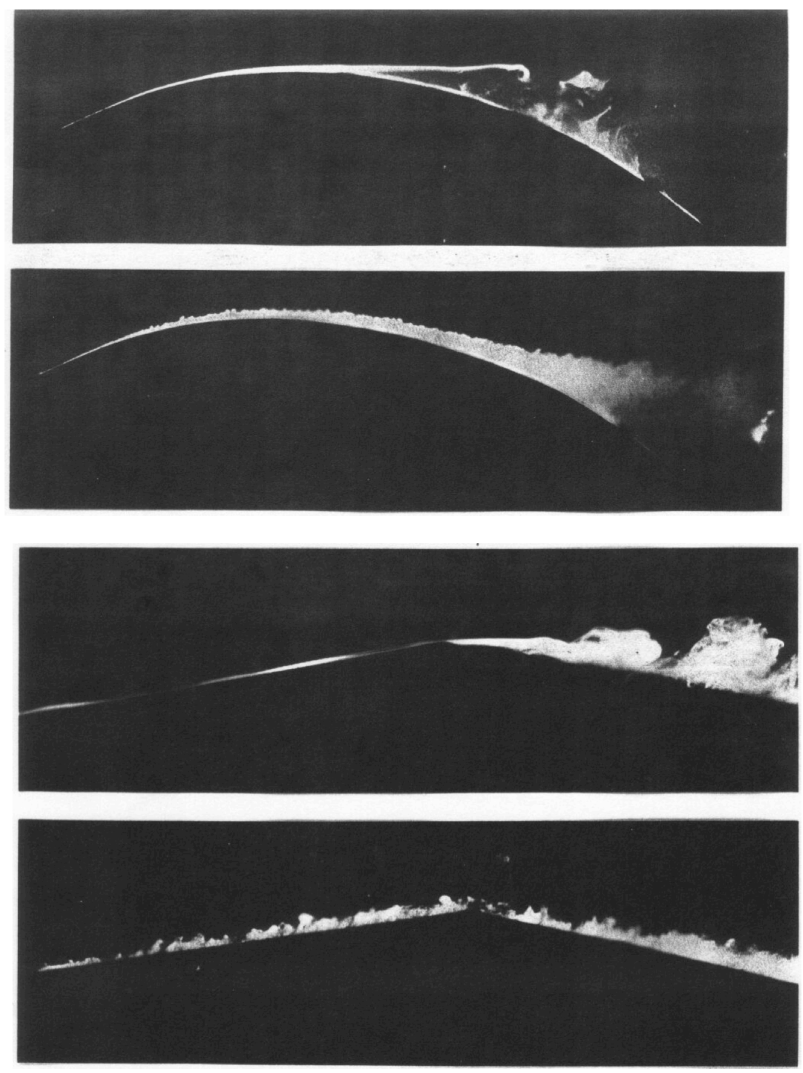
(b) Boundary-layer growth in a small-angle diffuser. (c) Boundary-layer separation in a large-angle diffuser. *[Parts (b) and (c) from the film "Fundamental of Boundary Layers," by the National Committee for Fluid Mechanics Films and the Educational Development Center.]*

Notes:

1. Turbulent BLs have more momentum than laminar BLs (consider the velocity profiles)
   - ⇒ fluid in a turbulent BL will travel further into an adverse pressure gradient than fluid in a laminar BL
   - ⇒ BL separation occurs later for a turbulent BL than for a laminar BL

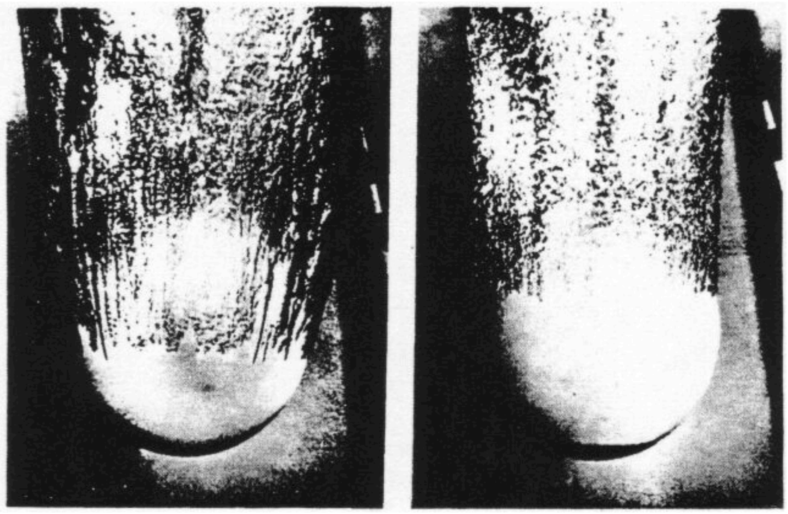


Note that $\tau_{w,\text{ laminar}} < \tau_{w,\text{ turbulent}}$ since the velocity gradient for laminar flow is less than the velocity gradient for turbulent flow.



(From Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press.)

Fig. 7.14 Strong differences in laminar and turbulent separation on an 8.5-in bowling ball entering water at 25 ft/s; (*a*) smooth ball, laminar boundary layer; (*b*) same entry, turbulent flow induced by patch of nose-sand roughness. (*U.S. Navy photograph, Ordnance Test Station, Pasadena Annex.*)



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)