Boundary Layers – Pressure Gradient Effects

https://www.youtube.com/watch?v=VUiGhyHC-1A

no dimples: 26+ mpg
with dimples: 29+ mpg

A bit more on the topic:
Let's look at the velocity profile at different points along a plate for a flow with an adverse pressure gradient ($\frac{\partial p}{\partial x} > 0$):

In an adverse pressure gradient flow the boundary layer velocity profile will always have an inflection point. This can be shown by considering the boundary layer momentum equation:

$$\frac{\partial^2 u}{\partial y^2} = 0$$

Note that at the wall boundary ($y = 0$), $u = v = 0$ so that:

$$\frac{\partial u}{\partial y} = 0$$

Thus, at the wall boundary in an adverse pressure gradient ($\frac{\partial p}{\partial x} > 0$), we must have:

$$\frac{\partial u}{\partial y} = 0$$

However, at the free stream interface ($y = G$) we have:

$$\frac{\partial u}{\partial y} = 0$$

in order for the boundary layer profile to merge smoothly with the outer flow velocity profile. The change in sign of the boundary layer curvature indicates that somewhere within the boundary layer there must be a point of inflection. The inflection point moves toward the outer flow boundary as the flow moves downstream in an adverse pressure gradient flow.

This is defined as the separation point, i.e. the BL assumption that $v << u$ is no longer valid.

Notes:

1. Turbulent BLs have more momentum than laminar BLs (consider the velocity profiles)
   - Fluid in a turbulent BL will travel further into an adverse pressure gradient than fluid in a laminar BL
   - BL separation occurs later for a turbulent BL than for a laminar BL

\[
\frac{u}{U} \quad \frac{y}{L} \quad T
\]

(From Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press.)

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Bernoulli’s Eqn:
The component of the force acting in the direction parallel to the incoming flow is known as the lift force. The force acting on an object immersed in a fluid flow is comprised of the force due to pressure variations and the component perpendicular to the incoming flow is known as the drag force. Two contributions to the total drag force:

- Pressure force component: \[ F_p = \int \rho U^2 dA \]
- Shear stress drag component: \[ F_{\tau} = \int \tau dA \]

Forces on Objects Immersed in a Fluid Flow

Lift and drag are often expressed in dimensionless form as a lift and drag coefficient, \( C_L \) and \( C_D \), respectively.

Bluff body (use the frontal projected area)

Streamlined body (use the planform area)
Inviscid analysis works reasonably well on the upstream side but fails miserably on the downstream side due to boundary layer separation. Notice that the pressure in the wake is relatively uniform and equal to the pressure at the boundary layer separation point. Hence, delaying boundary layer separation helps to recover the pressure in the wake as well as decreasing the area of the wake. As a result, the drag due to pressure forces (aka form drag) decreases as boundary layer separation is delayed.

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Dimpled disc cycle wheels (from Zipp) for bicycle racing.
“…the $Re_{cr}$ is reduced by the presence of the dimples….dimples make the flow turbulent at an earlier point so the more energetic turbulent flow may stay attached to the surface for longer.”

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Flow Around a Cylinder (or Sphere) As a Function of Reynolds Number

Note: \( \text{Re} = \frac{VD}{v} \)

\( \text{Re} \ll 1 \)
(creeping or Stoke's flow)

\( 5 < \text{Re} < 50 \)
(fixed eddies)

\( 60 < \text{Re} < 5000 \)
(Karman Vortex Street, periodic shedding of vortices)

\( 5000 < \text{Re} < 200,000 \)

\( \text{Re} \approx 200,000 \)
(drag crisis)

\( \text{Re} > 200,000 \)
Figure 4.12.1. Streamlines of steady flow (from left to right) past a circular cylinder of radius $a$; $Re = 2aU/v$. The photograph at $Re = 0.25$ (from Frandt and Tietjens 1934) shows the movement of solid particles at a free surface, and all the others (from Taneda 1955b) show particles illuminated over an interior plane normal to the cylinder axis.

(From Batchelor, G.K., *An Introduction to Fluid Dynamics*, Cambridge University Press.)
96. Kármán vortex street behind a circular cylinder at $Re=10^5$. The initially spreading wake shown opposite develops into the two parallel rows of staggered vortices that von Kármán's inviscid theory shows to be stable when the ratio of width to streamwise spacing is 0.24. Streaklines are shown by electrolytic precipitation in water. Photograph by Sadaharu Taneda.

97. Smoke at various levels in a vortex street. A smoke filament in air shows, at a Reynolds number of 100, both shear layers (top photograph), only one shear layer (middle), and the incipient flow below the wake (bottom). Zdralewicz 1969

98. Kármán vortices in absolute motion. Here the camera moves with the vortices rather than the cylinder. The streamline pattern closely resembles the inviscid one calculated by von Kármán. The flow is visualized by particles floating on water. Photograph by R. Willis, from Wills 1979. Reproduced, with permission, from the Annual Review of Fluid Mechanics, Volume 5, © 1973 by Annual Reviews Inc.

(From Van Dyke, M., An Album of Fluid Motion, Parabolic Press.)
Commonly used curve fits to the curve shown above are:

- \( \text{Re}_D < 1 \): \( C_D = \frac{24}{\text{Re}_D} \) (Stokes’ drag law)
- \( \text{Re}_D < 5 \): \( C_D = \frac{24}{\text{Re}_D (1 + 3/16 \text{Re}_D)} \) (Oseen’s approximation)
- \( 0 \leq \text{Re}_D \leq 2 \times 10^5 \): \( C_D = \frac{24}{\text{Re}_D} + 6(1 + 81 \text{Re}_D^{2/3}) \) + 0.4
- \( \text{Re}_D < 2 \times 10^5 \): \( C_D = 0.44 \) (Newton’s Law)

Fig. 8.32 Drag coefficient of a sphere as a function of Reynolds number (Ref. 13).

Fig. 8.34 Drag coefficient for circular cylinders as a function of Reynolds number (Ref. 13).