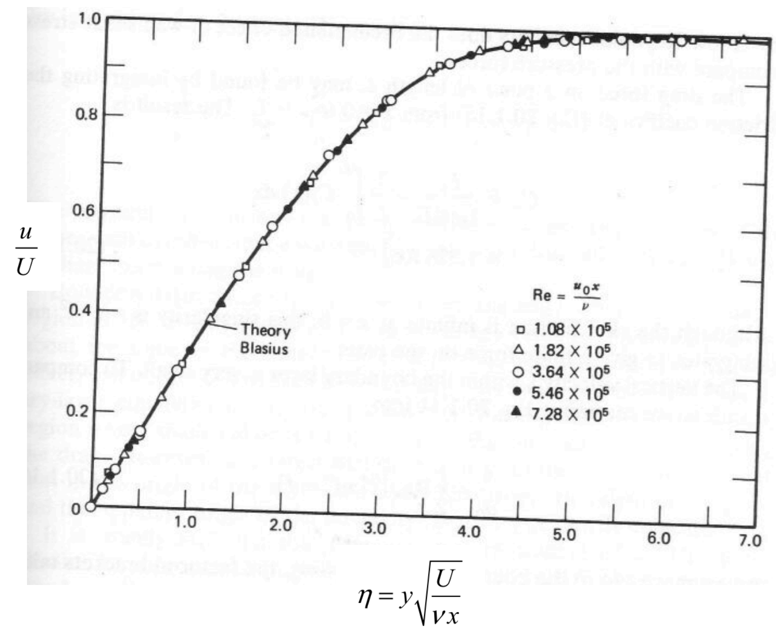
The Blasius solution for a laminar boundary layer with no pressure gradient may be derived, but that derivation is outside the scope of this class. The following information comes from this derivation.



2. The boundary layer thickness, δ, is found from the numerical solution to occur at $\eta=3.5$:

   $$F'(\eta = 3.5) = \frac{u}{U}(\eta = 3.5) = 0.99$$

   The boundary layer thickness is then:

   $$\eta = 3.5 = \delta\sqrt{\frac{U}{2\nu x}} \Rightarrow \quad \frac{\delta}{x} = \frac{5.0}{\mathrm{Re}_x^{1/2}}$$

3. The displacement and momentum thicknesses can also be found numerically using their definitions:

   $$\frac{\delta_D}{x} = \frac{1.72}{\mathrm{Re}_x^{1/2}} \quad \text{and} \quad \frac{\delta_M}{x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

4. The shear stress at the plate surface in dimensionless form is known as the **friction coefficient, c_f**:

   $$c_f \equiv \frac{\sigma_{yx}\big|_{y=0}}{\frac{1}{2}\rho U^2} = \frac{U\mu\left(\frac{d\left(u/U\right)}{d\eta}\frac{\partial\eta}{\partial y}\right)\Big|_{y=0}}{\frac{1}{2}\rho U^2} = \frac{\sqrt{2}F''\big|_{\eta=0}}{\sqrt{\mathrm{Re}_x}} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

   Note that the friction coefficient is a ratio of the shear stress to the dynamic pressure in the flow.

5. The **drag coefficient, c_D**, defined as the dimensionless drag acting on the plate between x=0 and x=L, is found by integrating the shear force over the plate area:

$$c_D \equiv \frac{\int_{x=0}^{x=L} \left(\sigma_{yx}\big|_{y=0} \right) dx}{\frac{1}{2}\rho U^2 L} = \frac{1.328}{\mathrm{Re}_L^{1/2}} \quad \text{(where } c_D \text{ is the drag coefficient per unit depth)}$$

Note that although the boundary layer assumptions break down near the leading edge of the plate (Re_x is not >> 1), the distance over which this is the case is small in comparison to the typical lengths of interest. This discrepancy is generally neglected in engineering applications.

6. This solution is only valid for laminar boundary layers. As a rule of thumb, the transition from laminar flow to turbulent flow occurs at: $\boxed{\mathrm{Re}_x \approx 500{,}000}$.

7. Recall that he boundary layer equations on which the Blasius solution is based are valid only when Re_x >> 1. In practice, it has been found that the Blasius solution is accurate when $\mathrm{Re}_L > 1000$. For $1 \le \mathrm{Re}_L \le 1000$, the following relation developed by Imai (1957) is more appropriate:

$$c_D = \frac{1.328}{\mathrm{Re}_L^{1/2}} + \frac{2.3}{\mathrm{Re}_L}$$

8. In summary, for flow over a flat plate with no pressure gradient, the "exact" Blasius solution gives:

$$\boxed{\begin{array}{lll} \dfrac{\delta}{x} = \dfrac{5.0}{\mathrm{Re}_x^{1/2}} & \dfrac{\delta_D}{x} = \dfrac{1.72}{\mathrm{Re}_x^{1/2}} & \dfrac{\delta_M}{x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}} \\ c_f = \dfrac{0.664}{\mathrm{Re}_x^{1/2}} & c_D = \dfrac{1.328}{\mathrm{Re}_L^{1/2}} & \\ \mathrm{Re}_x < 10^6 & & \end{array}}$$