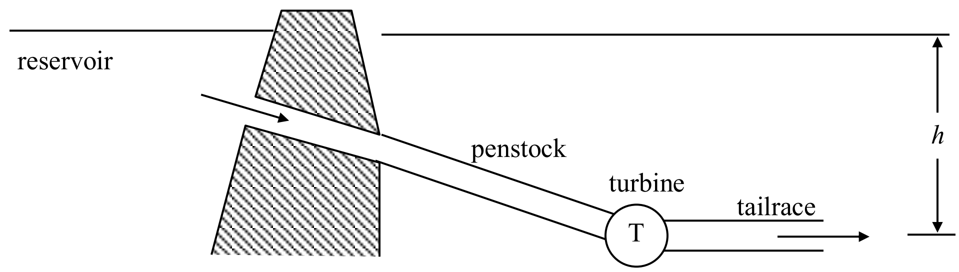
The tailrace (discharge pipe) of a hydro-electric turbine installation is at an elevation, h, below the water level in the reservoir:



The frictional losses in the penstock (the pipe leading to the turbine) and the tailrace are represented by the loss coefficient, k, based on the mean velocity, U, in those pipes (which have the same cross-sectional area, A). The flow discharges to atmospheric pressure at the exit from the tailrace. The water density is denoted by ρ and the acceleration due to gravity by g.

a. What is the drop in total head across the turbine?
b. What is the power developed by the turbine assuming that it is 90% efficient?
c. What is the optimum velocity, U_{opt}, which will produce the maximum power output from the turbine assuming that h, k, A, ρ, and g are constant?

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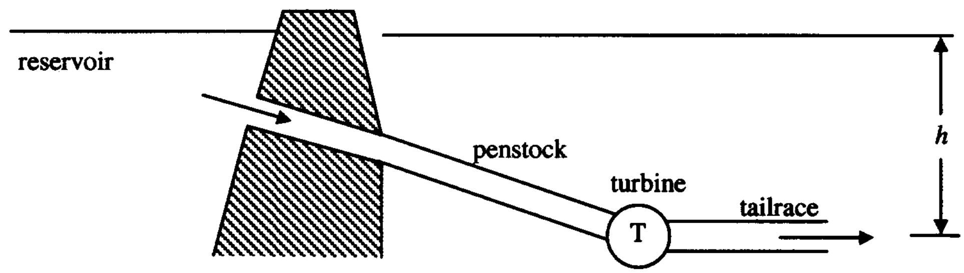

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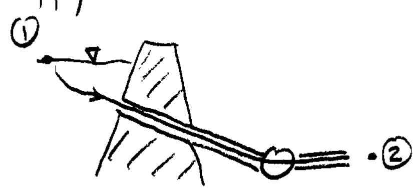
- Apply the Extended Bernoulli Eqn from (1) to (2):



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2 = \left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1 - H_L + H_s$$

where $p_1 = p_2 = p_{atm}$

$\bar{V}_1 \approx 0$

$\bar{V}_2 = U$

$z_1 - z_2 = h$

$H_L = K\frac{U^2}{2g}$

$H_S = \frac{P}{\rho Q g}$

NOTE: Flow is expected to be turbulent $\alpha_2 \approx 1$

$$\Rightarrow \frac{U^2}{2g} = h - K\frac{U^2}{2g} + \frac{P}{\rho Q g}$$

Solution...

$$\frac{U^2}{2g}(1+K) = h + \frac{P}{\rho U A g}$$

a) $$H_s = \frac{P}{\rho U A g} = \boxed{\frac{U^2}{2g}(1+K) - h}$$

the magnitude of the drop in total head across the turbine

b) $$P = \rho U A g \left[\frac{U^2}{2g}(1+K) - h\right]$$

If only 90% efficient, then

$$\boxed{P_{extracted} = -0.9 \rho U A g \left[\frac{U^2}{2g}(1+K) - h\right]}$$

minus sign because we're removing power from the fluid (we're dealing with a turbine)

c) to optimize, find the maximum point:

$$\frac{dP}{dU} = -0.9 \rho A g \left[\frac{3 U_{opt}^2}{2g}(1+K) - h\right] = 0$$

$$\Rightarrow U_{opt}^2 = \frac{2g}{3(1+K)} h$$

$$\boxed{\therefore U_{opt} = \sqrt{\frac{2gh}{3(1+K)}}}$$