This equation was derived in the following manner. Assume an object is subject to a constant acceleration a so we can write,

$$\ddot{x} = \frac{d\dot{x}}{dt} = a, \tag{1.25}$$

where the overdots represent differentiation with respect to time. Integrate this equation twice with respect to time making use of the initial conditions $x(t=0) = x_0$ and $\dot{x}(t=0) = \dot{x}_0$ to get,

$$\frac{d\dot{x}}{dt} = a \implies \int_{\dot{x}=\dot{x}_0}^{\dot{x}=\dot{x}} d\dot{x} = \int_{t=0}^{t=t} a dt \implies \dot{x} - \dot{x}_0 = a\int_{t=0}^{t=t} dt = at \implies \dot{x} = at + \dot{x}_0, \tag{1.26}$$

$$\dot{x} = \frac{dx}{dt} = at + \dot{x}_0 \implies \int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (at + \dot{x}_0)dt \implies x - x_0 = \frac{1}{2}at^2 + \dot{x}_0 t \implies x = \frac{1}{2}at^2 + \dot{x}_0 t + x_0. \tag{1.27}$$

A key step in the derivation to this equation is the assumption that $a =$ constant. When a is a constant, it may be pulled out of the integrals in Eqs. (1.26) and (1.27). Thus, the ballistic equation is only valid when $a =$ constant. It is not valid when a varies with time. If a is a function of time, then it must be evaluated within the integral. For example, if we have,

$$a = ct, \tag{1.28}$$

where c is a constant, then, using the same initial conditions as before, we have,

$$\frac{d\dot{x}}{dt} = a \implies \int_{\dot{x}=\dot{x}_0}^{\dot{x}=\dot{x}} d\dot{x} = \int_{t=0}^{t=t} a dt \implies \dot{x} - \dot{x}_0 = \int_{t=0}^{t=t} (ct)dt = \frac{1}{2}ct^2 \implies \dot{x} = \frac{1}{2}ct^2 + \dot{x}_0, \tag{1.29}$$

$$\dot{x} = \frac{dx}{dt} = \frac{1}{2}ct^2 + \dot{x}_0 \implies \int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (\frac{1}{2}ct^2 + \dot{x}_0)dt \implies x - x_0 = \frac{1}{6}ct^3 + \dot{x}_0 t \implies x = \frac{1}{6}ct^3 + \dot{x}_0 t + x_0. \tag{1.30}$$

Thus, we see that the position in Eq. (1.30) is different than the result given by the ballistic equation.

1.2. Dimensions and Units

A dimension is a qualitative description of the physical nature of some quantity.

Notes:

(1) A basic or primary dimension is one that is not formed from a combination of other dimensions. It is an independent quantity.
(2) A secondary dimension is one that is formed by combining primary dimensions.
(3) Common dimensions include:

$$\begin{aligned} M &= \text{mass} \\ L &= \text{length} \\ T &= \text{time} \\ \theta &= \text{temperature} \\ F &= \text{force} \end{aligned}$$

(4) If M,L, and T are primary dimensions, then $F = ML/T^2$ is a secondary dimension. If F, L, and T are primary dimensions, then $M = FT^2/L$ is a secondary dimension.

A unit is a quantitative description of a dimension. A unit gives "size" to a dimension. Common systems of units in engineering are given in Table 1.1.

Notes:

(1) The mole is the amount of substance that contains the same number of elementary entities as there are atoms in 12 g of carbon 12 ($= 6.022 \times 10^{23}$, known as Avogadro's constant). The elementary entities must be specified, e.g., atoms, molecules, particles, etc. The unit kmol (aka kgmol) is also frequently used, with 1 kmol = 1000 mol = 6.022×10^{26} entities. The unit lbmol is used in the English system of units. Since 1 lbm = 0.453 kg, 1 lbmol = 453.592 mol. The number of kmols of a substance, n, is found by dividing the mass of the substance, m (kg) by the molecular weight of the substance, M (in kg/kmol): $n = m/M$. For example, the atomic weight of carbon 12 is 12

Table 1.1. Common units used in engineering.

primary dimension	SI (Systéme International d' Unités)	BG (British Gravitational	EE (English Engineering)
L, length	meter (m)	foot (ft)	foot (ft)
T, time	second (s)	second (s)	second (s)
θ, temperature	Kelvin (K)	degree Rankine (°R)	degree Rankine (°R)
M, mass	kilogram (kg)	-not primary-	pound mass (lbm or lb)
N, amount of a substance	mole (mol)	mole (mol)	pound mole (lbmol)
F, force	-not primary-	pound force (lbf)	pound force (lbf)

kmol/kg, hence, 1 kg of carbon 12 contains: (12 kg)/(12 kg/kmol) = 1 kmol = 1000 mol (or 12 g of C12 contains 1 mol).

(2) In the SI system, force is a secondary dimension and is given by: $F = ML/T^2$ where $1\,\text{N} = 1\,\text{kgm/s}^2$.

(3) In the EE system, both force and mass are primary dimensions. The two are related via Newton's second law, $g_c F = ma$ where $g_c = 32.2\,\text{lb}_\text{m}\text{ft}/(\text{lb}_\text{f}\text{s}^2)$. It's easiest just to remember that: $1\,\text{lb}_\text{f} = 32.2\,\text{lb}_\text{m}\text{ft/s}^2$.

(4) The slug is the unit of mass in the British Engineering system of units. To convert between a slug and lb_f is: $1\,\text{lb}_\text{f} = 1\,\text{slugft/s}^2$.

(5) The kilogram-force (kgf) is (unfortunately) a not uncommon quantity. The conversion between kgf and Newtons is: 1 kgf = 9.81 N.

(6) It's a good policy to carry units through your calculations. Remember that, unless it's dimensionless, every number has a unit attached to it. For example, if $m = 1\,\text{lb}_\text{m}$ and $g = 32.2\,\text{ft/s}^2$,

$$\text{Poor Practice:} \quad mg = (1)(32.2) = 1\,\text{lb}_\text{f},$$

$$\text{Good Practice:} \quad mg = (1\,\text{lb}_\text{m})(32.2\,\text{ft/s}^2) = \left(32.2\,\text{lb}_\text{m}\text{ft/s}^2\right)\left(\frac{1\,\text{lb}_\text{f}}{32.2\,\text{lb}_\text{m}\text{ft/s}^2}\right) = 1\,\text{lb}_\text{f}$$

Carrying through your units makes it less likely that you'll make unit conversion errors, plus it makes it easier for you and others to follow your work.

Example: What is 70 mph in furlongs per fortnight?
Solution:

$$\left(70\frac{\text{miles}}{\text{hour}}\right)\left(\frac{5280\,\text{feet}}{\text{miles}}\right)\left(\frac{\text{rod}}{16.5\,\text{feet}}\right)\left(\frac{\text{furlong}}{40\,\text{rods}}\right)\left(\frac{24\,\text{hours}}{\text{day}}\right)\left(\frac{7\,\text{days}}{\text{week}}\right)\left(\frac{2\,\text{weeks}}{\text{fortnight}}\right) = 1.88\times10^5\,\frac{\text{furlongs}}{\text{fortnight}} \quad (1.31)$$

Example: What is weight of $1\,\text{lb}_\text{m}$ of water in lb_f?
Solution: Weight is mass multiplied by gravitational acceleration,

$$W = mg = (1\,\text{lb}_\text{m})\left(32.2\,\text{ft/s}^2\right)\left(\frac{1\,\text{lb}_\text{f}}{32.2\,\text{lb}_\text{m}\text{ft/s}^2}\right) = 1\,\text{lb}_\text{f}. \quad (1.32)$$

Dimensional homogeneity is the concept whereby only quantities with similar dimensions can be added (or subtracted). It is essentially the concept of "You can't add apples and oranges." For example, consider the following equation,

$$10\,\text{kg} + 16\,\text{K} = 26\,\text{m}\,\text{s}^{-1}. \quad (1.33)$$

This equation doesn't make sense since it is not dimensionally homogeneous. How can one add mass to temperature and get velocity?!?

Note that dimensional homogeneity is a necessary, but not sufficient, condition for an equation to be correct. In other words, an equation must be dimensionally homogeneous to be correct, but a dimensionally homogeneous equation isn't always correct. For example,

$$10\,\text{kg} + 10\,\text{kg} = 25\,\text{kg}. \quad (1.34)$$

The equation has the right dimensions, but the wrong magnitudes!

Be sure to:

(1) Verify that equations are dimensionally homogeneous.
(2) Carefully evaluate unit conversions. A unit conversion error caused the loss of the $125M Mars Climate Observer spacecraft in 1999!
(3) Always include units with numerical values. An Air Canada Flight in 1983 (now referred to as the "Gimli Glider") ran out of fuel and had to make an emergency landing due, in part, because the fuel load was assumed to be in pounds when in fact it was reported in kilograms.

The Ideal Gas Law is used to find the volume as given in the following formula,

$$V = \frac{mRT}{p},$$

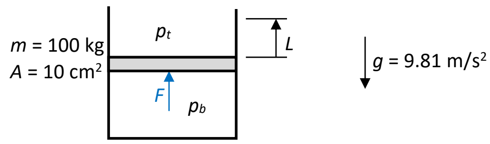
where m = 2 kg, R = 0.189 kJ/(kg.K), T = 300 K, and p = 1 bar (abs). Calculate the volume in m^3. Show all of your calculations and unit conversions.

SOLUTION:

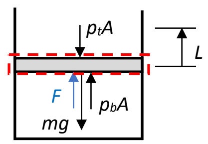
$$V = \frac{(2\ \text{kg})(0.189\ \text{kJ/(kg.K)})(300\ \text{K})}{(1\ \text{bar})} = \left(\frac{2\ \text{kg}}{1}\right)\left(\frac{0.189\ \text{kJ}}{\text{kg.K}}\right)\left(\frac{300\ \text{K}}{1}\right)\left(\frac{1}{1\ \text{bar}}\right)\left(\frac{1000\ \text{J}}{1\ \text{kJ}}\right)\left(\frac{1\ \text{bar}}{10^5\ \text{Pa}}\right)\left(\frac{1\ \text{Pa}}{1\ \text{N/m}^2}\right)\left(\frac{1\ \text{N.m}}{1\ \text{J}}\right) \quad (1)$$

$$\boxed{V = 1.13\ \text{m}^3}.$$

A piston, with a mass of m = 100 kg and cross-sectional area of A = 10 cm^2, is located within a cylinder as shown in the figure. The pressure on the top surface of the piston is a uniform p_t = 200 kPa (abs). The pressure on the bottom of the piston is a uniform p_b = 1 bar (abs). If a force is applied to the piston to move it slowly upwards, i.e., in a quasi-equilibrium process, a distance of L = 1 cm, determine the work done on the piston by the force, in kJ. Show all of your unit conversions.



SOLUTION:



First, determine the force by performing a vertical force balance on the piston, keeping in mind that the process is in quasi-equilibrium so that there is no acceleration of the piston,

$$\sum F = 0 = F + p_b A - mg - p_t A \;\; => \;\; F = mg + (p_t - p_b)A. \quad (1)$$

The work due to the force is,

$$W = \int_{s=0}^{s=L} \boldsymbol{F} \cdot d\boldsymbol{s} = \int_0^L [mg + (p_t - p_b)A]ds = [mg + (p_t - p_b)A]L, \quad (2)$$

$$\boxed{W = mgL + (p_t - p_b)AL}. \quad (3)$$

Note that the first term on the right-hand side is the change in potential energy while the second term is the pressure difference multiplied by the volume traced out by the piston (= AL).

Using the given values, evaluate the terms in Eq. (3). Start with the mgL term,

$$mgL = \underbrace{(100\text{ kg})}_{=m}\underbrace{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}_{=g}\underbrace{(1\text{ cm})}_{=L}\left(\frac{1\text{ N}}{1\ \frac{\text{kg.m}}{\text{s}^2}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ J}}{1\text{ N.m}}\right)\left(\frac{1\text{ kJ}}{1000\text{ J}}\right) = 9.81 * 10^{-3}\text{ kJ}. \quad (4)$$

Now evaluate the $(p_t - p_b)AL$ term,

$$(p_t - p_b)AL = \left[\underbrace{(200\text{ kPa})}_{=p_t}\left(\frac{1000\text{ Pa}}{1\text{ kPa}}\right) - \underbrace{(1\text{ bar})}_{=p_b}\left(\frac{10^5\text{ Pa}}{1\text{ bar}}\right)\right]\underbrace{(10\text{ cm}^2)}_{=A}\underbrace{(1\text{ cm})}_{=L}\left(\frac{1\text{ N/m}^2}{1\text{ Pa}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3\left(\frac{1\text{ J}}{1\text{ N.m}}\right)\left(\frac{1\text{ kJ}}{1000\text{ J}}\right) = 1 * 10^{-3}\text{ kJ}. \quad (5)$$

Combining the numerical values in Eqs. (4) and (5),

$$\boxed{W = 1.08 * 10^{-2}\text{ kJ}}.$$

For the flow of gas in a nozzle,

$$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2),$$

where h_1 and h_2 are the gas's specific enthalpies at the inlet and outlet of the nozzle, respectively, and V_1 and V_2 are the gas speeds at the inlet and outlet, respectively. For the current case, h_1 = 300 kJ/kg, V_1 = 100 m/s, and V_2 = 200 m/s.

Using the given formula, calculate the value for h_2 in kJ/kg.

SOLUTION:
Substitute the given parameter into the equation to solve for h_2, including appropriate unit conversions,

$$h_2 = \left(300\,\frac{\text{kJ}}{\text{kg}}\right) + \frac{1}{2}\left[\left(100\,\frac{\text{m}}{\text{s}}\right)^2 - \left(200\,\frac{\text{m}}{\text{s}}\right)^2\right]\left(\frac{1\text{ kJ}}{1000\text{ N.m}}\right)\left(\frac{1\text{ N.m}}{1\text{ kg.m}^2/\text{s}^2}\right), \tag{1}$$

$$h_2 = \left(300\,\frac{\text{kJ}}{\text{kg}}\right) + \frac{1}{2}\left(-30000\text{ m}^2/\text{s}^2\right)\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right), \tag{2}$$

$$h_2 = \left(300\,\frac{\text{kJ}}{\text{kg}}\right) - \left(15\,\frac{\text{kJ}}{\text{kg}}\right), \tag{3}$$

$$\boxed{h_2 = 285\,\frac{\text{kJ}}{\text{kg}}}. \tag{4}$$