1.3. Taylor Series Expansion Approximation

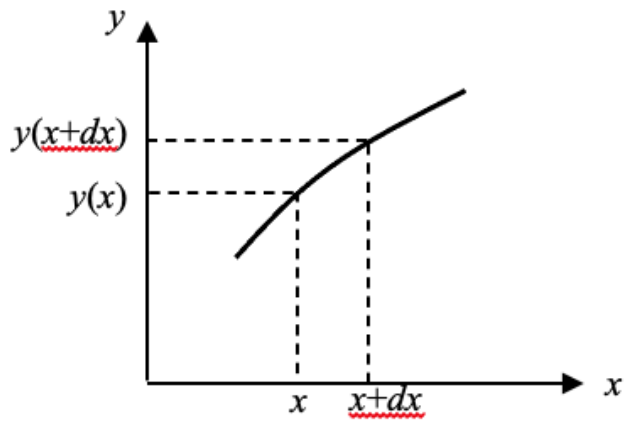

Figure 1.3. A plot used to motivate the use of Taylor Series approximations. If we know the value of $y(x)$, how can we estimate the value of $y(x + dx)$?

If we know the value of some quantity, y, at some location, x, then how can we determine the value of y at a nearby location $x + dx$ (Figure 1.3)? We can use a Taylor series expansion for y about location x,

$$y(x + \delta x) = y(x) + \left.\frac{dy}{dx}\right|_x (\delta x) + \left.\frac{d^2y}{dx^2}\right|_x \frac{(\delta x)^2}{2!} + \ldots \tag{1.35}$$

As δx becomes very small, $\delta x \to dx$, and the higher order terms become negligibly small: $\delta x \gg (\delta x)^2 \gg (\delta x)^3$,

$$\boxed{y(x + dx) \approx y(x) + \left.\frac{dy}{dx}\right|_x (dx).} \tag{1.36}$$

Note that Eq. (1.36) is simply the equation of a line. We can see what's happening more clearly in Figure 1.4. We'll use this approximation often, especially when examining how quantities vary over small distances.

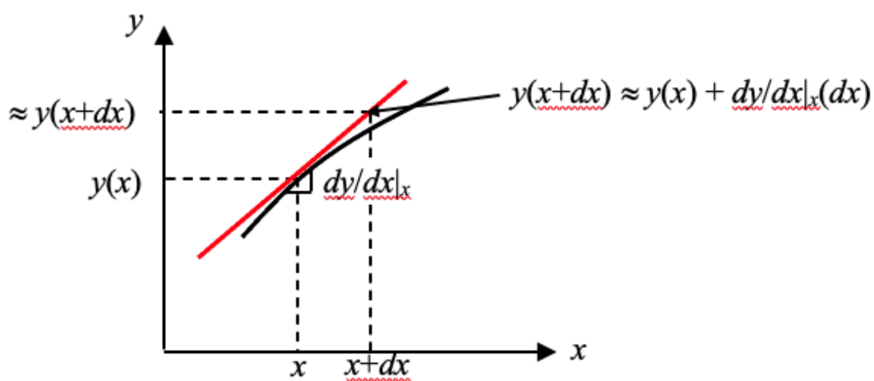

Figure 1.4. A sketch used to illustrate how a truncated Taylor Series can be used to estimate values. The black line is the original function and the red line is the truncated Taylor Series approximation.

Be sure to:

(1) Make sure you understand how this procedure works. It will be used frequently in the remainder of these notes.