### 12.6. Pump Similarity

Most pump performance data ($H$-$Q$ curves) are given only for one value of the pump rotational speed and one pump impeller diameter. Is there some way to determine the pump performance data for other speeds and diameters without requiring additional testing? There is...using dimensional analysis! Recall that we typically are interested in knowing the head rise across a pump, $H$ ($= h/g$ where $h$ is the specific energy rise across the pump), power required to operate the pump, $\dot{W}$ (bhp), and pump efficiency, $\eta$, as a function of the volumetric flow rate through the pump, $Q$:

$$h, \dot{W}, \eta = fcns(Q, \rho, \mu, D, \omega), \tag{12.19}$$

where $\rho$ and $\mu$ are the fluid density and dynamic viscosity, $D$ is the pump impeller diameter, and $\omega$ is the pump rotational speed.

Performing a dimensional analysis we find the following,

$$\Psi, \Pi, \eta = fcns(\Phi, \mathrm{Re}), \tag{12.20}$$

where,

$$\Psi := \text{dimensionless head coefficient} = \frac{gH}{\omega^2 D^2}, \tag{12.21}$$

$$\Pi := \text{dimensionless power coefficient} = \frac{\dot{W}}{\rho \omega^3 D^5}, \tag{12.22}$$

$$\eta := \text{efficiency} = \frac{\rho Q g H}{\dot{W}}, \tag{12.23}$$

$$\Phi := \text{dimensionless flow coefficient} = \frac{Q}{\omega D^3}, \tag{12.24}$$

$$\mathrm{Re} := \text{Reynolds number} = \frac{\rho \omega D^2}{\mu}. \tag{12.25}$$

*Notes:*

(1) In most pump flows, Re is very large $\Longrightarrow$ the variations in viscous effects from one flow to another are small $\Longrightarrow$ Re similarity can be neglected.
  (a) If Re is considerably different from one flow to another, e.g., pump water vs. pumping molasses, then Re effects cannot be ignored. The flow physics for large Re, where viscous forces $\ll$ inertial forces, are different than for small Re where viscous forces $\gg$ inertial forces.
  (b) Thus, for large Re:

$$\Psi, \Pi, \eta = fcns(\Phi). \tag{12.26}$$

(2)

$$\eta = \frac{\rho Q g H}{\dot{W}} = \frac{\Psi \Phi}{\Pi}. \tag{12.27}$$

(3) For similarity between geometrically similar flows (assuming large Re), we have the following pump scaling laws:

$$\Phi_1 = \Phi_2 \implies \left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2, \tag{12.28}$$

$$\Psi_1 = \Psi_2 \implies \left(\frac{gH}{\omega^2 D^2}\right)_1 = \left(\frac{gH}{\omega^2 D^2}\right)_2, \tag{12.29}$$

$$\Pi_1 = \Pi_2 \implies \left(\frac{\dot{W}}{\rho \omega^3 D^5}\right)_1 = \left(\frac{\dot{W}}{\rho \omega^3 D^5}\right)_2, \tag{12.30}$$

$$\eta_1 = \eta_2 \quad \text{(since } \Phi_1 = \Phi_2,\ \Psi_1 = \Psi_2,\ \Pi_1 = \Pi_2 \text{ and } \eta = \Psi\Phi/\Pi\text{)}. \tag{12.31}$$

(a) For a given pump ($D =$ constant) using the same fluid ($\rho, \mu =$constants) and the same gravity ($g =$constant),

$$\left(\frac{Q}{\omega}\right)_1 = \left(\frac{Q}{\omega}\right)_2, \tag{12.32}$$

$$\left(\frac{H}{\omega^2}\right)_1 = \left(\frac{H}{\omega^2}\right)_2, \tag{12.33}$$

$$\left(\frac{\dot{W}}{\omega^3}\right)_1 = \left(\frac{\dot{W}}{\omega^3}\right)_2. \tag{12.34}$$

The efficiency remains relatively constant when only changing the pump rotational speed (as given at the start of this note).

(b) For a given pump speed ($\omega =$constant), but varying diameters (assuming a geometrically similar family of pumps), and using the same fluid ($\rho, \mu =$constants) and the same gravity ($g =$constant),

$$\left(\frac{Q}{D^3}\right)_1 = \left(\frac{Q}{D^3}\right)_2, \tag{12.35}$$

$$\left(\frac{H}{D^2}\right)_1 = \left(\frac{H}{D^2}\right)_2, \tag{12.36}$$

$$\left(\frac{\dot{W}}{D^5}\right)_1 = \left(\frac{\dot{W}}{D^5}\right)_2. \tag{12.37}$$

Note that we are assuming that all length scales within the pump are scaled in the same way to maintain geometric similarity. This is not true in practice since pump impellers with different diameters are often put in the same pump casing. Also, surface roughness isn't scaled proportionally. The result is that the pump scaling laws are only approximations.

Since geometric scaling isn't completely satisfied, researchers have proposed the following empirical scaling rules that produce more accurate predictions than the ones given previously,

$$\left(\frac{Q}{D^2}\right)_1 = \left(\frac{Q}{D^2}\right)_2, \tag{12.38}$$

$$\left(\frac{H}{D^2}\right)_1 = \left(\frac{H}{D^2}\right)_2, \tag{12.39}$$

$$\left(\frac{\dot{W}}{D^4}\right)_1 = \left(\frac{\dot{W}}{D^4}\right)_2. \tag{12.40}$$

Since the scaling isn't perfect, the efficiency does not remain the same when scaling impeller size. As was done with the other dimensionless quantities, an empirical relationship can be used to scale the pump efficiencies, such as the one proposed by Moody,

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{1/5}. \tag{12.41}$$

A centrifugal pump with a 12 in. diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60 ft head. The impeller is changed to one with a 10 in. diameter. Determine the expected flowrate, head, and input power if the pump speed remains the same.

SOLUTION:
Since the pump speed remains the same and assuming geometrically similar pumps, the pump scaling laws are,

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^3 \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{D_1}{D_2}\right)^5$$

Using the given parameters,

$Q_1$ = 3200 gpm,
$D_1$ = 12 in,
$D_2$ = 10 in,
$H_1$ = 60 ft,
$\dot{W}_1$ = 60 hp,

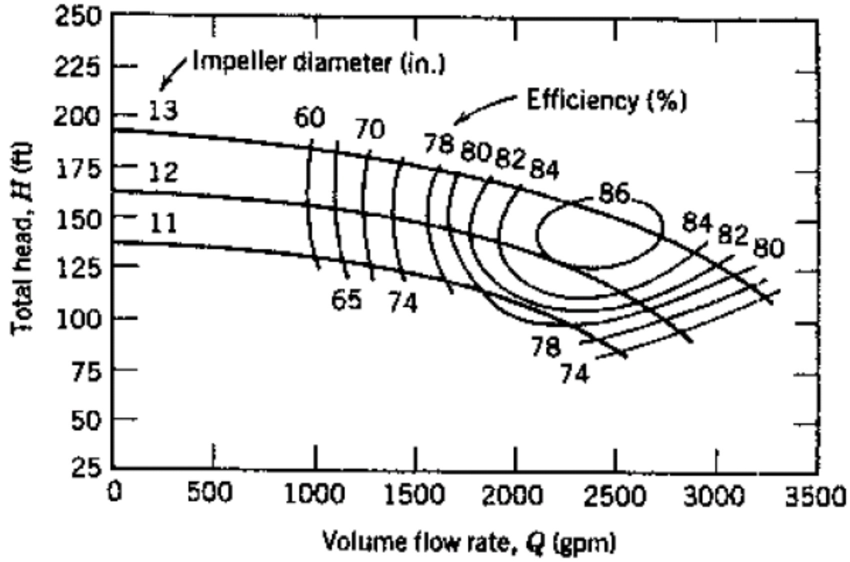
| | | |
|---|---|---|
| $Q_2$ | = | 1850 gpm |
| $H_2$ | = | 41.7 ft |
| $\dot{W}_2$ | = | 24.1 hp |

If the empirical (and more accurate) scaling laws are used,

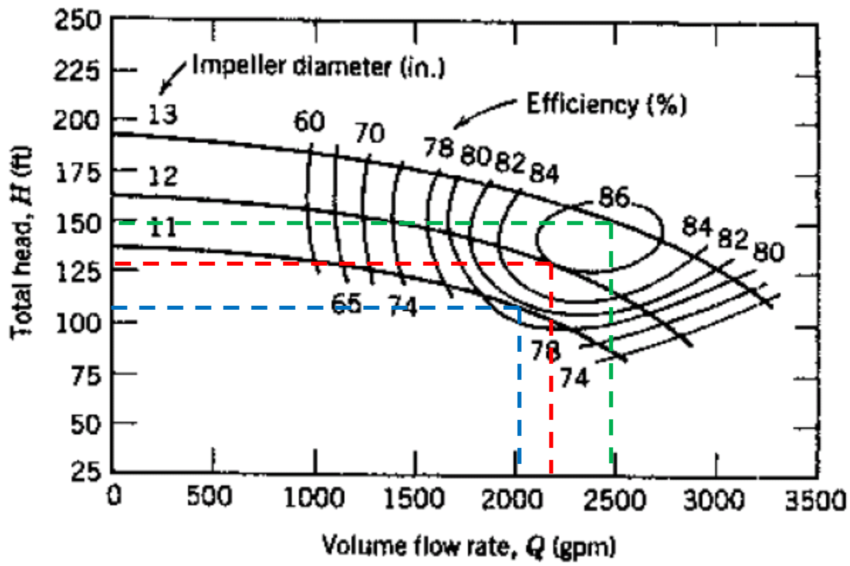
$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{\dot{W}_1}{\dot{W}_2} = \left(\frac{D_1}{D_2}\right)^4$$

| | | |
|---|---|---|
| $Q_2$ | = | 2220 gpm |
| $H_2$ | = | 41.7 ft |
| $\dot{W}_2$ | = | 28.9 hp |

Typical performance curves for a centrifugal pump, tested with three different impeller diameters in a single casing, are shown in the figure below. Specify the flow rate and head produced by the pump at its best efficiency point with a 12 in. diameter impeller. Scale these data to predict the performance of this pump when tested with 11 in. and 13 in. impellers. Comment on the accuracy of the scaling procedure.




SOLUTION:




From the pump performance diagram,

$Q_{12\text{ in.},BEP} = 2200$ gpm

$H_{12\text{ in.},BEP} = 130$ ft

$\eta_{12\text{ in.},BEP} = 86\%$

Using the geometric scaling rule,

$$Q_2 = Q_1 \left( \frac{D_2}{D_1} \right)^3 \tag{1}$$

For $D_2$ = 11 in. and $D_1$ = 12 in., $Q_{12\text{ in.}}$ = 2200 gpm, $\underline{Q_{11\text{ in.}} = 1690\text{ gpm}}$.
For $D_2$ = 13 in. and $D_1$ = 12 in., $Q_{12\text{ in.}}$ = 2200 gpm, $\underline{Q_{13\text{ in.}} = 2800\text{ gpm}}$.

Using the alternate scaling rule that takes into account imperfect geometric similarity,

$$Q_2 = Q_1 \left( \frac{D_2}{D_1} \right)^2, \tag{2}$$

$\underline{Q_{11\text{ in.}} = 1850\text{ gpm}}$
$\underline{Q_{13\text{ in.}} = 2580\text{ gpm}}$

From the pump performance diagram, $Q_{11\text{ in.}} \approx 2000$ gpm.
From the pump performance diagram, $Q_{13\text{ in.}} \approx 2500$ gpm.

The alternate scaling predicts the volumetric flow rate much better than the geometric scaling rule. Indeed, the alternate scaling rule predictions are off by approximately 8% (11 in.) and 3% (13 in.) while the geometric scaling rule is off by 16% (11 in.) and 12% (13 in.).

From the pump scaling rules,

$$H_2 = H_1 \left( \frac{D_2}{D_1} \right)^2 \tag{3}$$

For $D_2$ = 11 in. and $D_1$ = 12 in., $H_1$ = 130 ft, $\underline{H_{11\text{ in.}} = 109\text{ ft}}$.
For $D_2$ = 13 in. and $D_1$ = 12 in., $H_1$ = 130 ft, $\underline{H_{13\text{ in.}} = 153\text{ ft}}$.

From the pump performance diagram, $H_{11\text{ in.}} \approx 110$ ft.
From the pump performance diagram, $H_{13\text{ in.}} \approx 150$ ft.

The geometric scaling rule predictions are excellent with errors of only 1% (11 in.) and 2% (13 in.).

The best efficiencies may be found via the Moody empirical formula,

$$\frac{1-\eta_2}{1-\eta_1} = \left( \frac{D_1}{D_2} \right)^{1/5}, \tag{4}$$

For $D_2$ = 11 in. and $D_1$ = 12 in., $\eta_{1,BEP}$ = 86%, $\underline{\eta_{11\text{ in.},BEP} = 86\%}$.
For $D_2$ = 13 in. and $D_1$ = 12 in., $\eta_{1,BEP}$ = 86%, $\underline{\eta_{13\text{ in.},BEP} = 86\%}$.

From the pump performance diagram, $\eta_{11\text{ in.},BEP} \approx 82\%$ and $\eta_{11\text{ in.},BEP} \approx 87\%$. The Moody formula does a good job of predicting the best efficiency values for both impeller (5% and 1% relative errors for the 11 in. and 13 in. impellers, respectively).