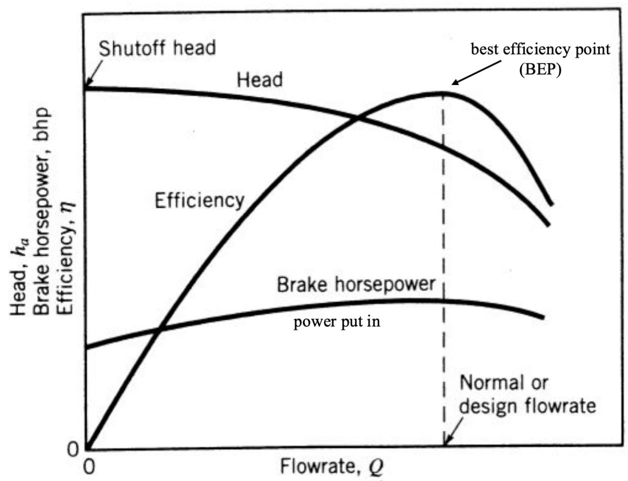
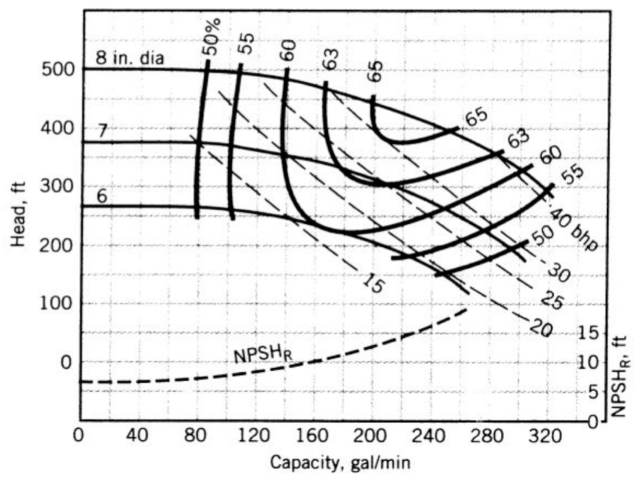


(A) This figure is from Munson, B.R., Young, D.F., and Okiishi, T.H., *Fundamentals of Fluid Mechanics*, 3rd ed., Wiley.



■ FIGURE 12.12 Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameters.

(B) This figure is from Munson, B.R., Young, D.F., and Okiishi, T.H., *Fundamentals of Fluid Mechanics*, 3rd ed., Wiley.

Figure 12.28. Two examples of pump performance plots.

12.5. Net Positive Suction Head (NPSH)

Along the suction side of the impeller blade near the pump inlet are regions of low pressure (Figure 12.29).

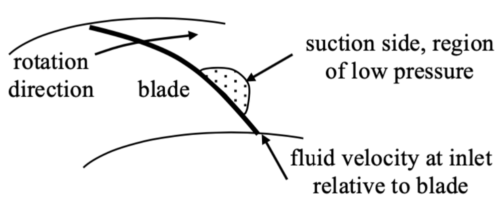


Figure 12.29. A sketch showing the region of low pressure on the suction side of an impeller blade.

If the local pressure is less than the vapor pressure of the liquid, then cavitation will occur: $p \leq p_v \implies$ cavitation. Recall that cavitation is "boiling" (liquid turning to vapor) that occurs when the pressure is less than the liquid's vapor pressure. Cavitation can not only decrease the performance of a pump, but it can also cause pump damage, vibration, and noise. Vapor bubbles caused by cavitation move into regions of higher pressure, collapse violently, and produce localized regions of very high pressure or high speed jets of water that can chip away at surfaces. The result is that the pump material erodes away. One can often hear when cavitation occurs because of the noise generated by the collapsing vapor bubbles.

Let's define a quantity that will aid us in determining when cavitation in a pump will occur,

$$\underline{\text{Net Positive Suction Head}}, \text{NPSH} \coloneqq \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g}\right)_S - \frac{p_v}{\rho g}. \tag{12.18}$$

Notes:

(1) The first term in NPSH is the head at the suction side of the pump near the impeller inlet. This region is where we expect to have the smallest head. Note that we define our reference plane for elevation along the centerline of the pump: $z = 0$.

(2) The second term in NPSH is the vapor pressure head. This head is when the liquid turns to vapor. The vapor pressure is typically given in terms of an absolute pressure so the suction pressure should also be an absolute pressure. Note that vapor pressure increases as temperature increases.

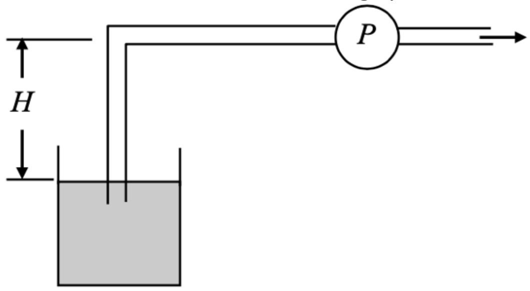
(3) NPSHR $\coloneqq$ Net Positive Suction Head Required to avoid cavitation. This quantity is a *pump property* and is determined experimentally.

(4) NPSHA $\coloneqq$ Net Positive Suction Head Available to the pump. This quantity is a *system property* and can be determined via analysis or experiments. NPSHA is related to the total head available to the pump at the pump inlet (minus the vapor head).

(5) We must have NPSHA $>$ NPSHR to avoid cavitation. Regulations typically recommend at least a 10% margin for safety. For critical applications such as for power generation or flood control, a 100% margin is often used.

(6) NPSHR increases with increasing flow rate since the pressure at the suction side of the pump blade near the pump inlet will decrease (consider Bernoulli's equation). Similarly, the pressure will decrease with increasing blade rotation speed resulting in an increased NPSHR.

Determine the NPSHA for the following system.



SOLUTION:
Choose point 1 to be on the surface of the tank and point 2 to be just upstream of the pump. Apply the EBE from 1 to 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \quad (1)$$

where,

$p_2 = p_s$,
$p_1 = p_{atm}$,
$\bar{V}_2=\bar{V}_s$ (assume turbulent flow => $\alpha_1 \approx 1$),
$\bar{V}_2=0$,
$z_2=z_1+H$,
$H_{L,12}=H_{L,12}$,
$H_{S,12}=0$.

Substitute and simplify,

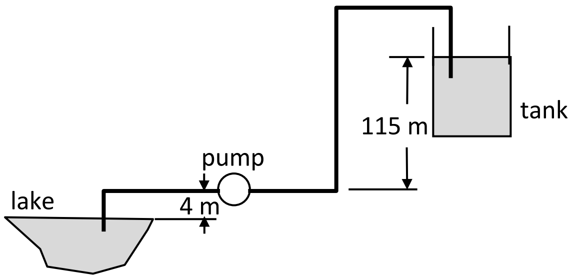
$$\left(\frac{p}{\rho g}+\frac{\bar{V}^2}{2g}\right)_s=\frac{p_{atm}}{\rho g}+\underbrace{(z_1-z_2)}_{=-H}-H_{L,12}. \quad (2)$$

From the definition of NPSH we have,

$$\boxed{NPSHA=\left(\frac{p}{\rho g}+\frac{\bar{V}^2}{2g}\right)_s-\frac{p_v}{\rho g}=\frac{p_{atm}-p_v}{\rho g}-H-H_{L,12}}. \quad (3)$$

Notes:
1. Increasing H, H_{L12}, or p_v decreases NPSHA and increases the likelihood of cavitation since the difference between NPSHA and NPSHR is reduced.
2. Increasing p_{atm} increases NPSHA and decreases the likelihood of cavitation.
3. The vapor pressure p_v varies with temperature.
4. The vapor pressure p_v is usually given in terms of absolute pressure and, thus, the atmospheric pressure should also be an absolute pressure.

A pump station is used to fill a tank on a hill using water from a lake. The flow rate is 10.5 L/s and atmospheric pressure is 101 kPa (abs). The pump is located 4 m above the lake, and the tank surface level is 115 m above the pump. The suction and discharge lines are 10.2 cm diameter commercial steel pipe. The equivalent length of the inlet line between the lake and the pump is 100 m. The total equivalent length between the lake and the tank is 2300 m, including all fittings, bends, screens, and valves. The overall efficiency of the pump and motor set is 70%.



water density = 1000 kg/m^3
water dynamic viscosity = 1*10^{-3} Pa.s
water vapor pressure = 1820 Pa (abs)

What is the net positive suction head available for this pump?

SOLUTION:

Apply the Extended Bernoulli Equation between the lake surface (1) and the pump inlet (2).

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (1)$$

where

ρ = 1000 kg/m^3
μ = 1*10^{-3} Pa.s
p_V = 1820 Pa (abs)
g = 9.81 m/s^2
p_1 = p_{atm} = 101 kPa (abs)
V_1 ≈ 0
$z_2 - z_1$ = 4 m
H_S = 0 (Point 2 is located upstream of the pump.)
D = 0.102 m
Q = 10.5 L/s = 0.0105 m^3/s
V_2 = $Q/(\pi/4D^2)$ = 1.28 m/s (2)
Re$_D$ = $\rho V_2 D/\mu$ = 131,000 (3)
α_2 ≈ 1 (turbulent flow)
ε = 0.045*10^{-3} m (commercial steel)
H_L = $f\left(\frac{L_e}{D}\right)\frac{\bar{V}_2^2}{2g}$ = 1.61 m (4)

where Re$_D$ = 131,000 and ε/D = 0.0004 $\Rightarrow$ f = 0.0195 (from the Moody chart) (5)
and L_e = 100 m

Re-arrange Eqn. (1) to solve for the NPSHA:

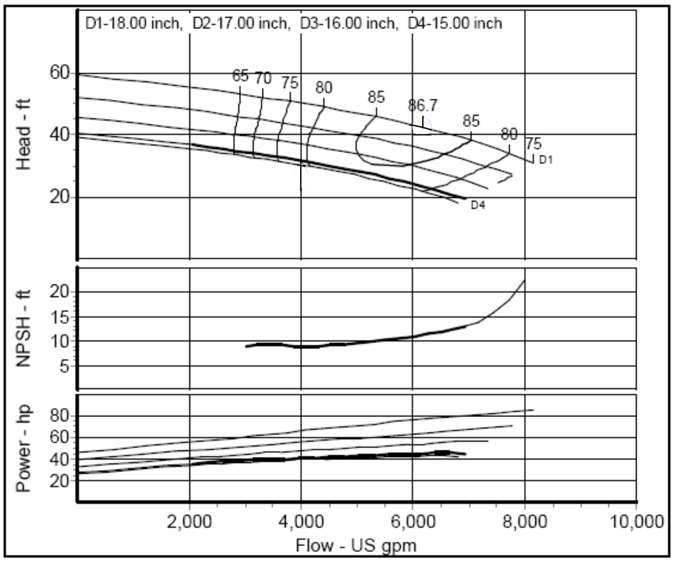
$$NPSHA=\left(\frac{p}{\rho g}+\frac{\bar{V}^2}{2g}\right)_S-\frac{p_V}{\rho g}=\frac{p_{atm}-p_V}{\rho g}+z_1-z_2-H_L \quad (6)$$

$\therefore$ NPSHA = 4.5 m

A Peerless Model 16A 18B pump is proposed as the supply unit for the Purdue Engineering Mall fountain. The following requirements have been provided by the architectural firm:

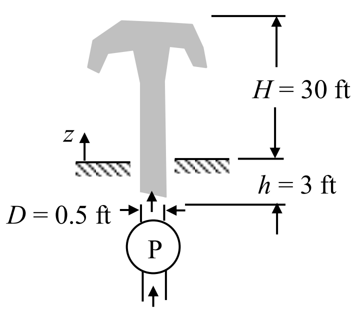
- The pump outlet is to be located 3 feet below ground level.
- The water flow is to reach a peak height of 30 feet above ground level.
- The discharge from the pump is 6 inches in diameter.

The pump characteristics are given in the following plot.



a. What head must be supplied by the pump? Report your answer in ft.
b. What flow rate must be supplied by the pump? Report your answer in gal/min (gpm).
c. What pump impeller diameter should be used? (either 15.00, 16.00, 17.00, or 18.00 inch diameter)
d. What is the pump efficiency? Report your answer in terms of a percentage.
e. What power is required to drive the pump? Report your answer in horsepower (hp).
f. What range of NPSH is acceptable at the pump inlet? Report your answer in ft.

SOLUTION:



The head that must be supplied by the pump is:

$$H_S = H + h \Rightarrow \boxed{H_S = 33 \text{ ft}} \quad (1)$$

The flow rate may be found using the pump outlet diameter and the velocity required to achieve the desired height. Apply Bernoulli's Equation from point 1 to point 2.

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1 \quad (2)$$

where

$p_1 = p_2 = p_{atm}$

$V_1 = 0$

$z_1 = -h$

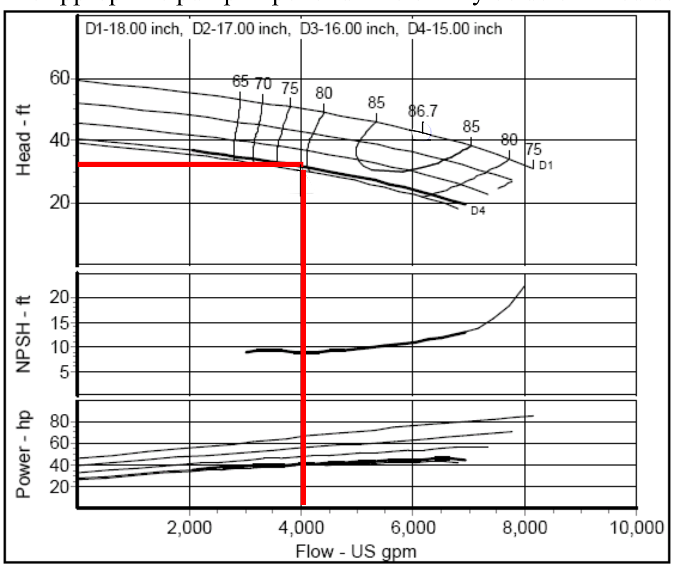
$z_2 = H$

$$\frac{V_1^2}{2g}=z_2-z_1 \Rightarrow V_1=\sqrt{2g\left(z_2-z_1\right)} \quad \text{(using the given data: } V_1 = 46.1\text{ ft/s)} \quad (3)$$

The flow rate is thus:

$$Q=V_1\tfrac{\pi}{4}D^2 \quad \text{(using the given data: } \boxed{Q = 9.05\text{ ft}^3\text{/s} = 4060\text{ gpm}}\text{)} \quad (4)$$

The appropriate pump impeller diameter may be determined using the given pump characteristics plot.



The nearest impeller diameter is the $\boxed{15.00\text{ inch}}$.

The pump efficiency may also be found from the pump characteristic plot and is $\boxed{\sim 80\%}$.

The power required to drive the pump is:

$$\dot{W}_{\substack{\text{input into}\\\text{pump}}}=\frac{\rho Q g H}{\eta}=\frac{1}{0.80}\left(62.4\ \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(9.05\ \frac{\text{ft}^3}{\text{s}}\right)\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)(33\text{ ft})\left(\frac{\text{lb}_\text{f}}{32.2\frac{\text{lb}_\text{m}\cdot\text{ft}}{\text{s}^2}}\right)\left(\frac{\text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}_\text{f}}{\text{s}}}\right) \quad (5)$$

$$\boxed{\therefore \dot{W}_{\substack{\text{input into}\\\text{pump}}}=42.4\text{ hp}}$$

The required NPSH to avoid cavitation at this flow rate is (from the pump plot) ~9 ft so the range of acceptable NPSH is $\boxed{> \sim 9\text{ ft}}$.