CHAPTER 11

Pipe Flows

11.1. Entrance Region

The flow in the entrance region is complex (Figure 11.1) and will not be investigated here. Experiments have shown that the dimensionless length of the entrance region depends on whether the entering flow is laminar or turbulent, with,

$$\text{laminar flow:} \quad \frac{L}{D} \approx 0.06\text{Re}_D, \tag{11.1}$$

$$\text{turbulent flow:} \quad \frac{L}{D} \approx 4.4\text{Re}_D^{1/6}, \tag{11.2}$$

where L is the length of the entrance region and D is the pipe diameter. For many engineering flows, $1 \times 10^4 < \text{Re}_D < 1 \times 10^5 \implies 20 < L/D < 30$. The shorter entrance region length for turbulent flows is due to the fact that turbulent mixing rapidly averages the flow speeds across the pipe cross-section.

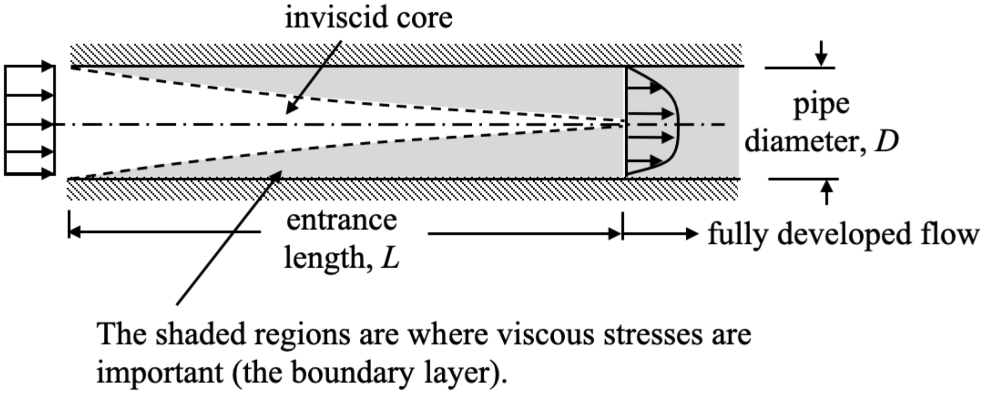

FIGURE 11.1. The structure of a pipe flow entrance region.

11.2. Fully Developed Laminar Circular Pipe Flow (Poiseuille Flow)

The derivation in this section was previously covered in Chapter 8 and is repeated here, in a slightly condensed form, for convenience. Consider the steady flow of an incompressible, constant viscosity, Newtonian fluid within an infinitely long, circular pipe of radius, R (Figure 11.2).

Make the following assumptions,

1. The flow is axi-symmetric and there is no “swirl” velocity. $\implies \frac{\partial}{\partial \theta}(\dots) = 0$ and $u_\theta = 0$
2. The flow is at steady state. $\implies \frac{\partial}{\partial t}(\dots) = 0$
3. The flow is fully-developed in the z-direction. $\implies \frac{\partial u_r}{\partial z} = \frac{\partial u_z}{\partial z} = 0$
4. There are no body forces. $\implies f_r = f_\theta = f_z = 0$