Hydrostatic Pressure Forces and the Center of Pressure

Glen Canyon Dam in AZ (https://www.usbr.gov/uc/rm/crsp/gc/)

Hydrostatic Pressure Forces and the Center of Pressure

$$\mathbf{F}_p = \int_A d\mathbf{F}_p = \int_A -p \, d\mathbf{A}$$

$\downarrow g$

$d\mathbf{F}_p$

x $\hat{\mathbf{n}}$ $d\mathbf{A} = (dxdz)\mathbf{j}$

z

L W

$$\mathbf{F}_{p,\text{bottom}} = \int_{z=0}^{z=W} \int_{x=0}^{x=L} -p \underbrace{\left[dxdz\left(\hat{\mathbf{j}} \right) \right]}_{=d\mathbf{A}} = -\hat{\mathbf{j}} \int_{z=0}^{z=W} \int_{x=0}^{x=L} \underbrace{\rho g H}_{=p_{\text{gage}}} dx\,dz = -\hat{\mathbf{j}} \rho g H W L$$

$$p_{\substack{\text{bottom,} \\ \text{gage}}} = \rho g H$$

$$\mathbf{F}_{p,\text{bottom}} = \underbrace{\int_{z=0}^{z=W} \int_{x=0}^{x=L} -\left(p_{\text{atm}} + \rho g H \right)\left(dxdz\hat{\mathbf{j}} \right)}_{\substack{\text{pressure force due to water} \\ \text{using an absolute pressure}}} + \underbrace{\int_{z=0}^{z=W} \int_{x=0}^{x=L} -\left(p_{\text{atm}} \right)\left[dxdz\left(-\hat{\mathbf{j}} \right) \right]}_{\substack{\text{pressure force on bottom due to atmosphere} \\ \text{using an absolute pressure}}} = -\hat{\mathbf{j}} \rho g H W L$$