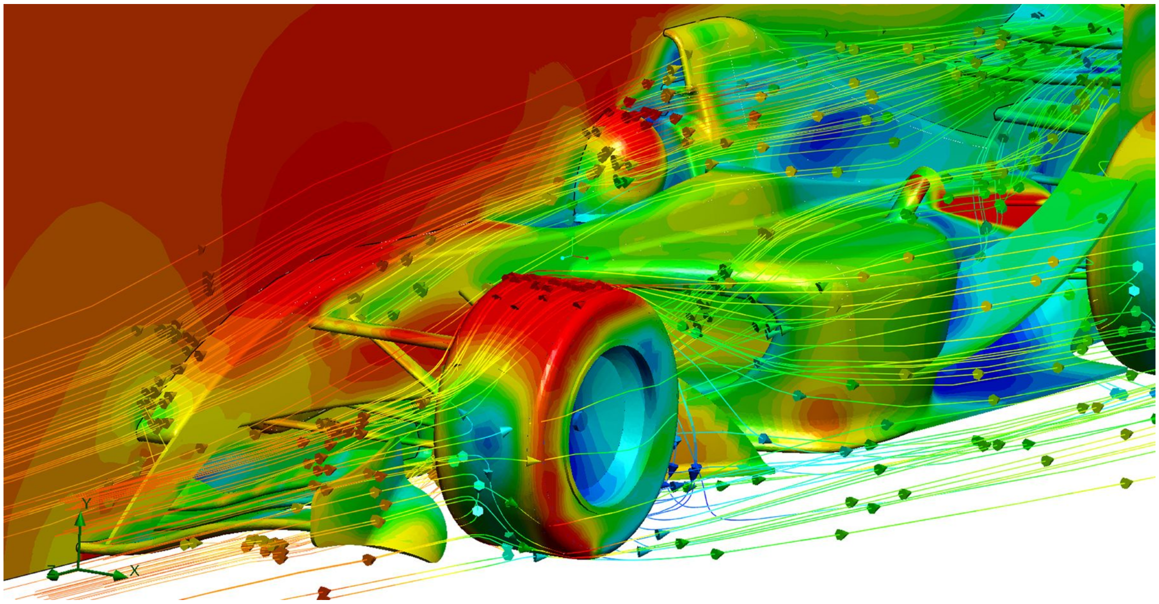
Acceleration of a Fluid Particle



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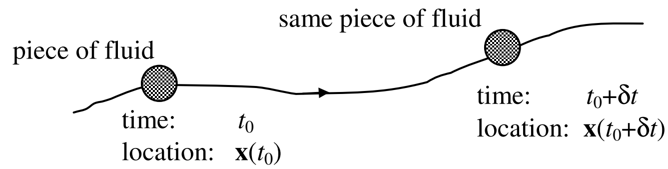

$$T = T\left(t, \mathbf{x}(t)\right)$$

$$\left.\frac{dT}{dt}\right|_{\substack{\text{following a}\\ \text{fluid particle}}} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\underbrace{\frac{dx}{dt}}_{=u_x} + \frac{\partial T}{\partial y}\underbrace{\frac{dy}{dt}}_{=u_y} + \frac{\partial T}{\partial z}\underbrace{\frac{dz}{dt}}_{=u_z}$$

$$\begin{aligned}\frac{DT}{Dt} &= \frac{\partial T}{\partial t} + u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y} + u_z\frac{\partial T}{\partial z}\\ &= \frac{\partial T}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)T\end{aligned}$$

$$\begin{aligned}\underbrace{\frac{D}{Dt}(\cdots)}_{\substack{\text{Lagrangian rate of}\\ \text{change (changes as we}\\ \text{follow a fluid particle)}}} &= \underbrace{\frac{\partial}{\partial t}(\cdots)}_{\substack{\text{local or Eulerian}\\ \text{rate of change (changes}\\ \text{due to unsteady effects)}}} + \underbrace{\left(\mathbf{u}\cdot\nabla\right)(\cdots)}_{\substack{\text{convective rate}\\ \text{of change (changes due to}\\ \text{a change in particle position)}}}\\ &= \frac{\partial}{\partial t}(\cdots) + u_x\frac{\partial}{\partial x}(\cdots) + u_y\frac{\partial}{\partial y}(\cdots) + u_z\frac{\partial}{\partial z}(\cdots)\end{aligned}$$

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)\mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + u_x\frac{\partial \mathbf{u}}{\partial x} + u_y\frac{\partial \mathbf{u}}{\partial y} + u_z\frac{\partial \mathbf{u}}{\partial z}$$