Dimensional Analysis – Modeling and Similarity

Blue Oyster Cult song with Godzilla movie clips: https://www.youtube.com/watch?v=muUZjovOFRg

A related paper:

Tretter, T.R., 2005, "Godzilla versus scaling laws of physics", *The Physics Teacher*, Vol. 43, pp. 530 – 532.

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$$y = f_1(x_1, x_2, \dots)$$

$$\Pi_1 = f_2(\Pi_2, \Pi_3, \dots)$$

Example (continued from "Dimensional Analysis – Buckingham-Pi Theorem and Method of Repeating Variables" lecture video)

The ballistic equation:

$$y = -\frac{1}{2}gt^2 + \dot{y}_0 t + y_0$$

$$y = f_1(t, g, \dot{y}_0, y_0) \quad => \quad \frac{y}{y_0} = f_2\left(t\sqrt{\frac{g}{y_0}}, \frac{\dot{y}_0}{\sqrt{g y_0}}\right)$$

Perform an experiment on Earth (g = 9.81 m/s^2) where we measure the ball vertical position y as a function of time t for a fixed y_0 = 1 m and three different initial speeds $\dot{y}_0$ = 10 m/s, 20 m/s, and 30 m/s. Following is the measurement data we would obtain.

g_{Earth} [m/s^2] =	9.81						
y_0 [m] =	1.00						
$ydot_0$ [m/s] =	10.0		$ydot_0$ [m/s] =	20.0		$ydot_0$ [m/s] =	30.0
t [s]	y [m]		t [s]	y [m]		t [s]	y [m]
0.00	1.00		0.00	1.00		0.00	1.00
1.00	6.10		1.00	16.10		1.00	26.10
1.50	4.96		1.50	19.96		1.50	34.96
2.00	1.38		2.00	21.38		2.00	41.38

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Put this data in dimensionless form using the Pi terms previously derived:

$$\frac{y}{y_0} = f_2\left(t\sqrt{\frac{g}{y_0}}, \frac{\dot{y}_0}{\sqrt{g y_0}}\right)$$

For example,

$\dot{y}_0 = 10.0\ \mathrm{m/s}, g = 9.81\ \mathrm{m/s^2}, y_0 = 1\ \mathrm{m} \Rightarrow \frac{\dot{y}_0}{\sqrt{g y_0}} = 3.19,$

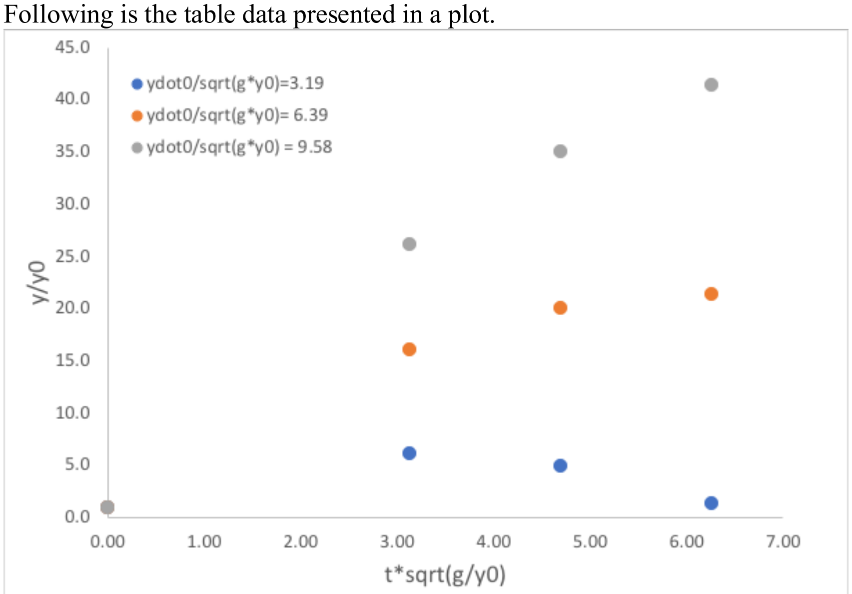
$t = 1.50\ \mathrm{s}, g = 9.81\ \mathrm{m/s^2}, y_0 = 1\ \mathrm{m} \Rightarrow t\sqrt{\frac{g}{y_0}} = 4.70,$

$y = 4.96\ \mathrm{m}, y_0 = 1\ \mathrm{m} \Rightarrow \frac{y}{y_0} = 4.96.$

Completing the rest of the table gives the following.

ydot$_0$/sqrt(g*y$_0$) =	3.19		ydot$_0$/sqrt(g*y$_0$) =	6.39		ydot$_0$/sqrt(g*y$_0$) =	9.58
t*sqrt(g/y$_0$)	y/y$_0$		t*sqrt(g/y$_0$)	y/y$_0$		t*sqrt(g/y$_0$)	y/y$_0$
0.00	1.00		0.00	1.00		0.00	1.00
3.13	6.10		3.13	16.1		3.13	26.1
4.70	4.96		4.70	20.0		4.70	35.0
6.26	1.38		6.26	21.4		6.26	41.4

Following is the table data presented in a plot.



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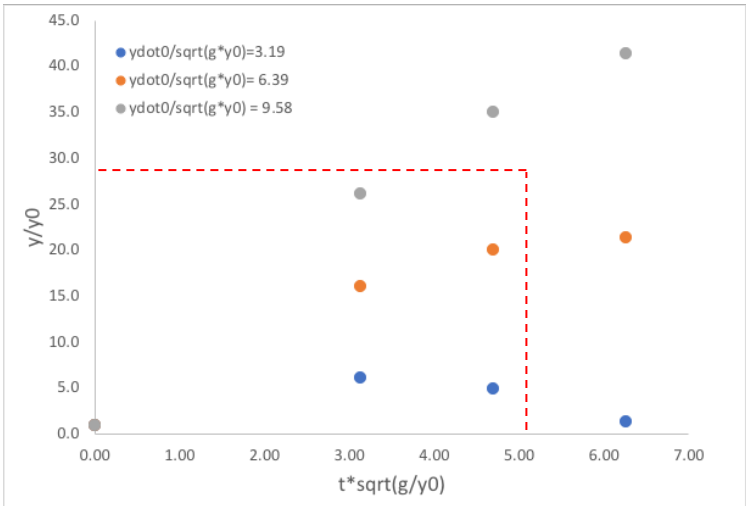
Now let's evaluate what would happen on the Moon where $g_{Moon} = 1.63\ \mathrm{m/s^2}$. In particular, let's arbitrarily say that on the Moon, we're interested in the following conditions,

$$g = 1.63\ \mathrm{m/s^2}, t = 4.00\ \mathrm{s}, \dot{y}_0 = 10.0\ \mathrm{m/s}, y_0 = 1\ \mathrm{m} \Rightarrow t\sqrt{\frac{g}{y_0}} = 5.11, \frac{\dot{y}_0}{\sqrt{g y_0}} = 7.83.$$



Estimating from the plot (or by linear interpolation, which is what I did),

$$y/y_0 = 27.7 \Rightarrow y = 27.7\ \mathrm{m} \quad (\text{since } y_0 = 1\ \mathrm{m}).$$

Note that there is some error in obtaining y/y_0 due to using linear interpolation or by estimating from the plot.

From the ballistic equation for the same lunar conditions,

$$y = -\frac{1}{2}gt^2 + \dot{y}_0 t + y_0,$$

$$\Rightarrow y = 28.0\ \mathrm{m}.$$

Thus, our scaling works (taking into account the error estimating from the plot). We have predicted what would happen on the Moon by using dimensional analysis and running experiments on the Earth!