Dimensional Analysis – Buckingham-Pi Theorem and Method of Repeating Variables

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Basic Dimensions

A dimension is a qualitative description of the physical nature of some quantity. A basic dimension is one that is not formed from a combination of other dimensions, i.e., it is an independent quantity. Examples: mass (M), length (L), time (T), temperature (θ)

Buckingham Pi Theorem

(# of Π terms) = (# of variables) – (# of reference dimensions)

- Π terms are dimensionless terms.
- Reference dimensions are the dimensions required to describe the variables in the equation. These are *usually* the same as basic dimensions, but in some cases can be different. For example, let A, B, and C be the variables in an equation with the following dimensions,

  $[A] = M/L^3$
  $[B] = M/(L^3T^2)$
  $[C] = MT/L^3$
  basic dimensions = M, L, T (3 total)
  reference dimensions = (M/L^3), T (2 total)

The Method of Repeating Variables

1. *List all of the independent variables involved in the problem.*
   - e.g., $y = \text{fcn}_1(x, z, t, \ldots)$
   - This is the hardest step in a dimensional analysis.

2. *Express each variable in terms of basic dimensions.*

3. *Determine the number of* Π *terms using the Buckingham-Pi Theorem.*
   (# of Π terms) = (# of variables) – (# of reference dimensions)
   - This is a unique number! Everyone should get this same number.

4. *Select repeating variables where the number of repeating variables is equal to the number of reference dimensions.*
   - Choose from the list of independent variables (the right hand side of the equation in Step 1).
   - All of the reference dimensions must be represented in the repeating variables.
   - This step is not necessarily unique. Different people may select different repeating variables.

5. *Form a* Π *term by multiplying one of the non-repeating variables by the product of the repeating variables, each raised to an exponent that will make the combination dimensionless.*
   - Since the repeating variables are not necessarily unique, the Π terms are not necessarily unique. Different people may have different Π terms. The number of Π terms will be the same, however, because of the Buckingham-Pi theorem.

6. *Double-check that all* Π *terms are indeed dimensionless.*
   - This step can catch errors made in Step 5.

7. *Express the final form of the dimensional analysis as a relationship among the* Π *terms.*
   - e.g., $\Pi_1 = \text{fcn}_2(\Pi_2, \Pi_3, \ldots)$
   - Note that fcn_1 in Step 1 and fcn_2 in Step 7 are different functions, in general. Dimensional analysis won't tell us what these functions are. Other analyses or experiments are required for that.
   - The equation in Step 7 in terms of Π terms contains all of the same information as the equation in Step 1 in terms of dimensional variables.

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Example:
Consider the ballistic equation,

$$y = -\frac{1}{2}gt^2 + \dot{y}_0 t + y_0 .$$

a. How many dimensionless variables are required to describe this equation?
b. What dimensionless terms can be used to describe this equation?

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