Compressible Flow – Normal Shock Waves

0.30-06 caliber rifle

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For air:

$k = 1.4$, $R = 287$ J/(kg.K) = 53.3 (ft.lb$_f$)/(lb$_m$.degR) = 1716 ft^2/(s^2.degR)

1D, steady, <u>adiabatic</u> flow of a perfect gas with no work other than pressure work

$$T\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right) = \text{constant}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1} \quad \text{and} \quad \frac{T^*}{T_0} = \left(1+\frac{k-1}{2}\right)^{-1}$$

$$\frac{c}{c_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-\frac{1}{2}} \quad \text{and} \quad \frac{c^*}{c_0} = \left(1+\frac{k-1}{2}\right)^{-\frac{1}{2}}$$

1D, steady, <u>isentropic</u> flow of a perfect gas with no work other than pressure work

$$\frac{p}{p_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{1-k}} \quad \text{and} \quad \frac{p^*}{p_0} = \left(1+\frac{k-1}{2}\right)^{\frac{k}{1-k}} \quad \text{(for air } (k_{\text{air}} = 1.4),\ p^*/p_0 = 0.5283)$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{1}{1-k}} \quad \text{and} \quad \frac{\rho^*}{\rho_0} = \left(1+\frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{1+\frac{k-1}{2}\mathrm{Ma}^2}{1+\frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

$$\dot{m}_{\text{choked}} = \left(1+\frac{k-1}{2}\right)^{\frac{k+1}{2(1-k)}} p_0\sqrt{\frac{k}{RT_0}}A^*$$

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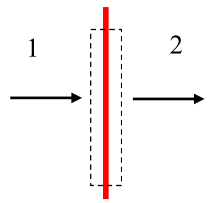
Combine conservation of mass, the linear momentum equation, the 1st Law, the 2nd Law of, the definition of the Mach number, and assume a perfect gas:



$$\mathrm{Ma}_2^2 = \frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - (k-1)}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\mathrm{Ma}_1^2\right]\left[\frac{2k\mathrm{Ma}_1^2 - (k-1)}{(k+1)^2 \mathrm{Ma}_1^2}\right]$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)\mathrm{Ma}_1^2}{(k-1)\mathrm{Ma}_1^2 + 2}$$

$$\frac{p_2}{p_1} = \frac{2k\mathrm{Ma}_1^2}{k+1} - \frac{k-1}{k+1}$$

$$\frac{T_{02}}{T_{01}} = 1$$

$$\frac{p_{02}}{p_{01}} = \frac{A_1^*}{A_2^*} = \frac{\rho_{02}}{\rho_{01}} = \left[\frac{\frac{k+1}{2}\mathrm{Ma}_1^2}{1 + \frac{k-1}{2}\mathrm{Ma}_1^2}\right]^{\frac{k}{k-1}} \left[\frac{2k\mathrm{Ma}_1^2}{k+1} - \frac{k-1}{k+1}\right]^{\frac{1}{1-k}}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

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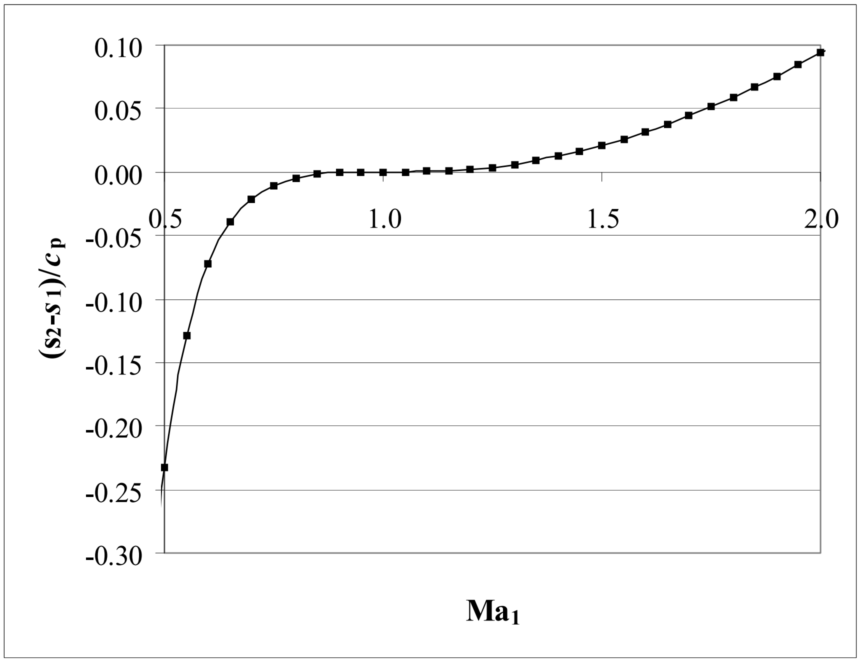

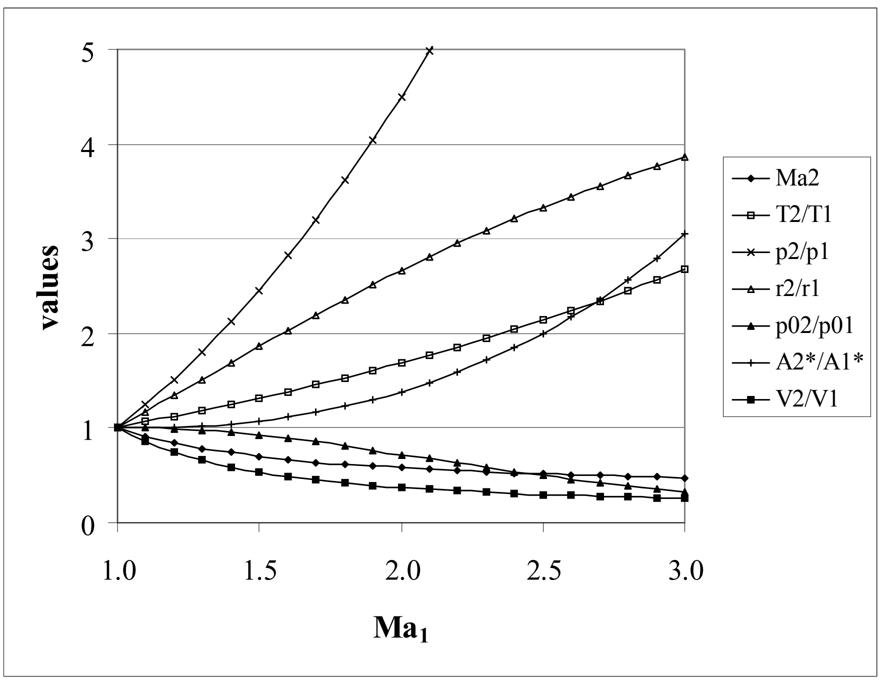

The plot shows the following relations:

$T_2 > T_1$ and $T_2/T_1 \uparrow$ as $\mathrm{Ma}_1 \uparrow$

$p_2 > p_1$ and $p_2/p_1 \uparrow$ as $\mathrm{Ma}_1 \uparrow$

$\rho_2 > \rho_1$ and $\rho_2/\rho_1 \uparrow$ as $\mathrm{Ma}_1 \uparrow$

$V_2 < V_1$ and $V_2/V_1 \downarrow$ as $\mathrm{Ma}_1 \uparrow$

$T_{02} = T_{01}$

$p_{02} < p_{01}$ and $p_{02}/p_{01} \downarrow$ as $\mathrm{Ma}_1 \uparrow$

$\rho_{02} < \rho_{01}$ and $\rho_{02}/\rho_{01} \downarrow$ as $\mathrm{Ma}_1 \uparrow$

$A_2^* > A_1^*$ and $A_2^*/A_1^* \uparrow$ as $\mathrm{Ma}_1 \uparrow$

$\mathrm{Ma}_2 \downarrow$ as $\mathrm{Ma}_1 \uparrow$ Furthermore, $\lim_{\mathrm{Ma}_1 \to \infty} (\mathrm{Ma}_2) = \left(\frac{\gamma - 1}{2\gamma}\right)^{1/2}$

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Normal shock table for air (k = 1.4)

Ma_1	Ma_2	p_2/p_1	p_{02}/p_{01}	T_2/T_1	ρ_2/ρ_1
1.00	1.000	1.000	1.000E+00	1.000	1.000
1.02	0.981	1.047	1.000E+00	1.013	1.033
1.04	0.962	1.095	9.999E-01	1.026	1.067
1.06	0.944	1.144	9.998E-01	1.039	1.101
1.08	0.928	1.194	9.994E-01	1.052	1.135
1.10	0.912	1.245	9.989E-01	1.065	1.169
1.12	0.897	1.297	9.982E-01	1.078	1.203
1.14	0.882	1.350	9.973E-01	1.090	1.238
1.16	0.868	1.403	9.961E-01	1.103	1.272
1.18	0.855	1.458	9.946E-01	1.115	1.307
1.20	0.842	1.513	9.928E-01	1.128	1.342
1.22	0.830	1.570	9.907E-01	1.141	1.376
1.24	0.818	1.627	9.884E-01	1.153	1.411
1.26	0.807	1.686	9.857E-01	1.166	1.446
1.28	0.796	1.745	9.827E-01	1.178	1.481
1.30	0.786	1.805	9.794E-01	1.191	1.516
1.32	0.776	1.866	9.758E-01	1.204	1.551
1.34	0.766	1.928	9.718E-01	1.216	1.585
1.36	0.757	1.991	9.676E-01	1.229	1.620
1.38	0.748	2.055	9.630E-01	1.242	1.655
1.40	0.740	2.120	9.582E-01	1.255	1.690
1.42	0.731	2.186	9.531E-01	1.268	1.724

Ma_1	Ma_2	p_2/p_1	p_{02}/p_{01}	T_2/T_1	ρ_2/ρ_1
2.00	0.577	4.500	7.209E-01	1.688	2.667
2.02	0.574	4.594	7.115E-01	1.704	2.696
2.04	0.571	4.689	7.022E-01	1.720	2.725
2.06	0.567	4.784	6.928E-01	1.737	2.755
2.08	0.564	4.881	6.835E-01	1.754	2.783
2.10	0.561	4.978	6.742E-01	1.770	2.812
2.12	0.558	5.077	6.649E-01	1.787	2.840
2.14	0.555	5.176	6.557E-01	1.805	2.868
2.16	0.553	5.277	6.464E-01	1.822	2.896
2.18	0.550	5.378	6.373E-01	1.839	2.924
2.20	0.547	5.480	6.281E-01	1.857	2.951
2.22	0.544	5.583	6.191E-01	1.875	2.978
2.24	0.542	5.687	6.100E-01	1.892	3.005
2.26	0.539	5.792	6.011E-01	1.910	3.032
2.28	0.537	5.898	5.921E-01	1.929	3.058
2.30	0.534	6.005	5.833E-01	1.947	3.085
2.32	0.532	6.113	5.745E-01	1.965	3.110
2.34	0.530	6.222	5.658E-01	1.984	3.136
2.36	0.527	6.331	5.572E-01	2.002	3.162
2.38	0.525	6.442	5.486E-01	2.021	3.187
2.40	0.523	6.553	5.401E-01	2.040	3.212
2.42	0.521	6.666	5.317E-01	2.059	3.237