Compressible Flow – Isentropic Flow with Area Change

Compressible Flow – Isentropic Flow with Area Change

1D, steady, <u>adiabatic</u> flow of a perfect gas with no work other than pressure work

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{-1} \qquad \frac{T^*}{T_0} = \left(1 + \frac{k-1}{2}\right)^{-1}$$

$$\frac{c}{c_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{-\frac{1}{2}} \qquad \frac{c^*}{c_0} = \left(1 + \frac{k-1}{2}\right)^{-\frac{1}{2}}$$

1D, steady, <u>isentropic</u> flow of a perfect gas with no work other than pressure work

$$\frac{p}{p_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{1-k}} \qquad \frac{p^*}{p_0} = \left(1 + \frac{k-1}{2}\right)^{\frac{k}{1-k}} \qquad \left.\frac{p^*}{p_0}\right|_{\substack{\text{for air}\\(k=1.4)}} = 0.5283$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{1}{1-k}} \qquad \frac{\rho^*}{\rho_0} = \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

From conservation of mass for a 1D, steady flow:

$$\rho VA = \text{constant} \Rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

From Bernoulli's equation (linear momentum equation) for a gas (neglect gravitational forces):

$$\frac{dp}{\rho} + VdV = 0$$

Combine conservation of mass with Bernoulli's equation and the definition of the speed of sound ($c^2 = \left(\partial p/\partial \rho\right)_s$) and the Mach number (Ma = V/c):

$$\left(\mathrm{Ma}^2 - 1\right)\frac{dV}{V} = \frac{dA}{A}$$

Compressible Flow – Isentropic Flow with Area Change

From Conservation of Mass for a 1D, steady flow, combined with the isentropic relations:

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{1+\frac{k-1}{2}\mathrm{Ma}^2}{1+\frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$