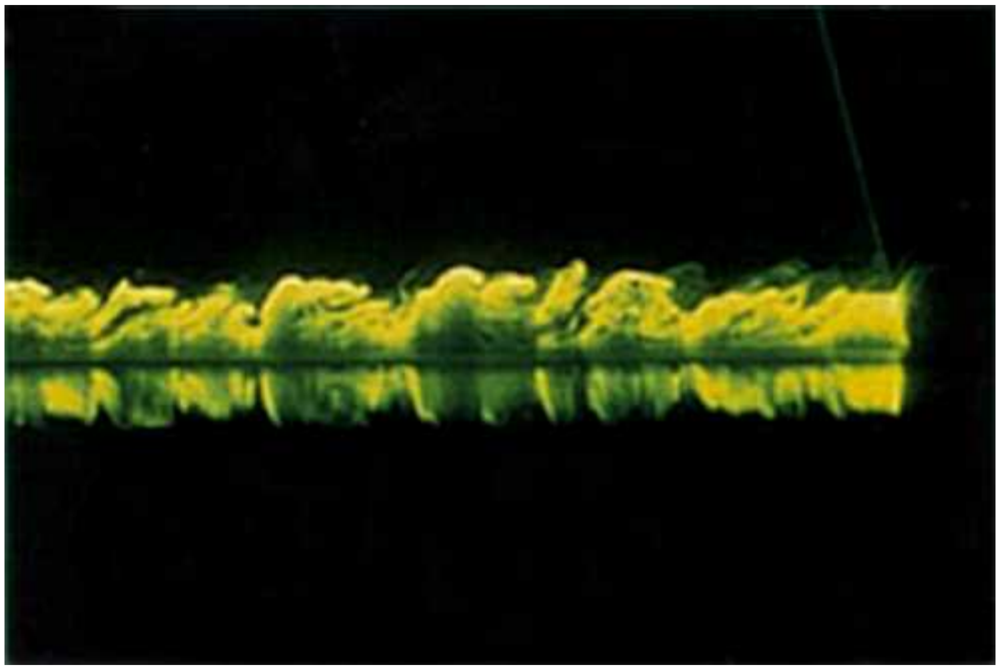
Boundary Layers – Turbulent Boundary Layers



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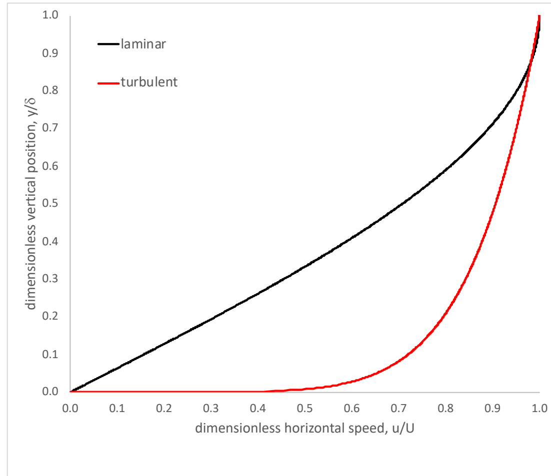
1/7th power law velocity profile for flow over a flat plate with no pressure gradient:

$$\frac{u}{U} = \begin{cases} \left(\frac{y}{\delta}\right)^{1/7} & 0 \le \frac{y}{\delta} < 1 \\ 1 & \frac{y}{\delta} \ge 1 \end{cases}$$



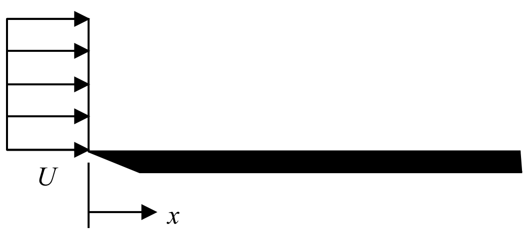
Experimental shear stress correlation:

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \approx 0.020 Re_\delta^{-1/6}$$

Use the KMIE <u>assuming δ(x = 0) = 0</u>:

$$\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}} \qquad \frac{\delta_D}{x} \approx \frac{0.02}{Re_x^{1/7}} \qquad \frac{\delta_M}{x} \approx \frac{0.016}{Re_x^{1/7}} \qquad c_f \approx \frac{0.027}{Re_x^{1/7}} \qquad c_D \approx \frac{0.031}{Re_L^{1/7}}$$

Eq. (76)



Using a different (and less accurate, but used by Fox and McDonald) experimental shear stress correlation:

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \approx 0.0466 Re_\delta^{-1/4}$$

$$\frac{\delta}{x} \approx \frac{0.382}{Re_x^{1/5}} \qquad \frac{\delta_D}{x} \approx \frac{0.0478}{Re_x^{1/5}} \qquad \frac{\delta_M}{x} \approx \frac{0.0371}{Re_x^{1/5}} \qquad c_f \approx \frac{0.0594}{Re_x^{1/5}} \qquad c_D \approx \frac{0.0742}{Re_L^{1/5}}$$

Eq. (82)

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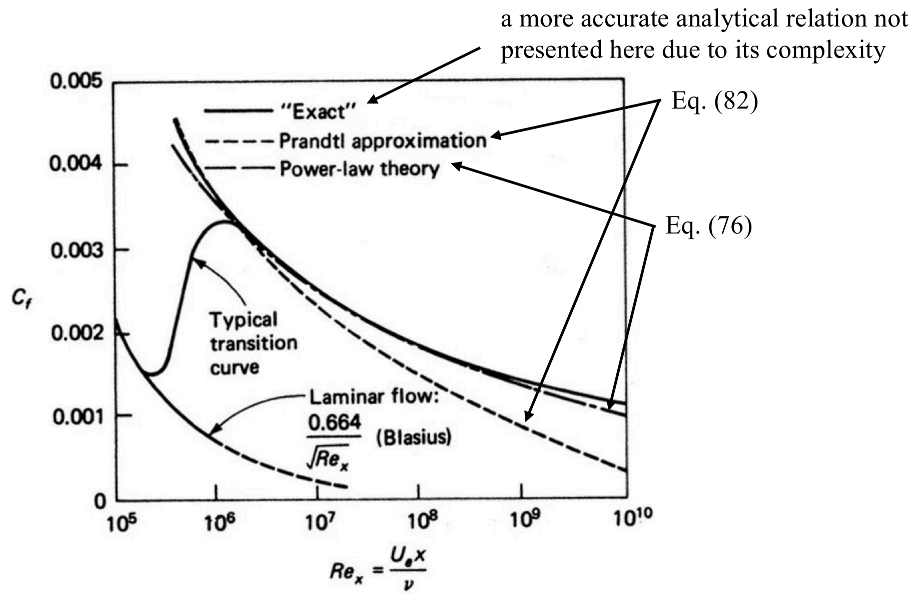

$$\delta_D = \delta^* = \int_0^\delta (1 - \frac{u}{U})dy \qquad \delta_M = \theta = \int_0^\delta \frac{u}{U}(1 - \frac{u}{U})dy \qquad \frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$$

Parameter	**Laminar (Re < 500,000)**	**Turbulent (Re > 500,000)**
99% thickness, δ	$\frac{\delta}{x} = \frac{5.0}{Re_x^{1/2}}$	$\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$
displacement thickness, δ_D or δ^*	$\frac{\delta_D}{x} = \frac{1.72}{Re_x^{1/2}}$	$\frac{\delta_D}{x} = \frac{0.0478}{Re_x^{1/5}}$
momentum thickness, δ_M or Θ	$\frac{\delta_M}{x} = \frac{0.664}{Re_x^{1/2}}$	$\frac{\delta_M}{x} = \frac{0.0371}{Re_x^{1/5}}$
friction coefficient, C_f	$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{Re_x^{1/2}}$	$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}}$
drag coefficient, C_D	$C_D = \frac{D}{\frac{1}{2}\rho U^2 LW} = \frac{1.328}{Re_L^{1/2}}$	$C_D = \frac{D}{\frac{1}{2}\rho U^2 LW} = \frac{0.0742}{Re_L^{1/5}}$