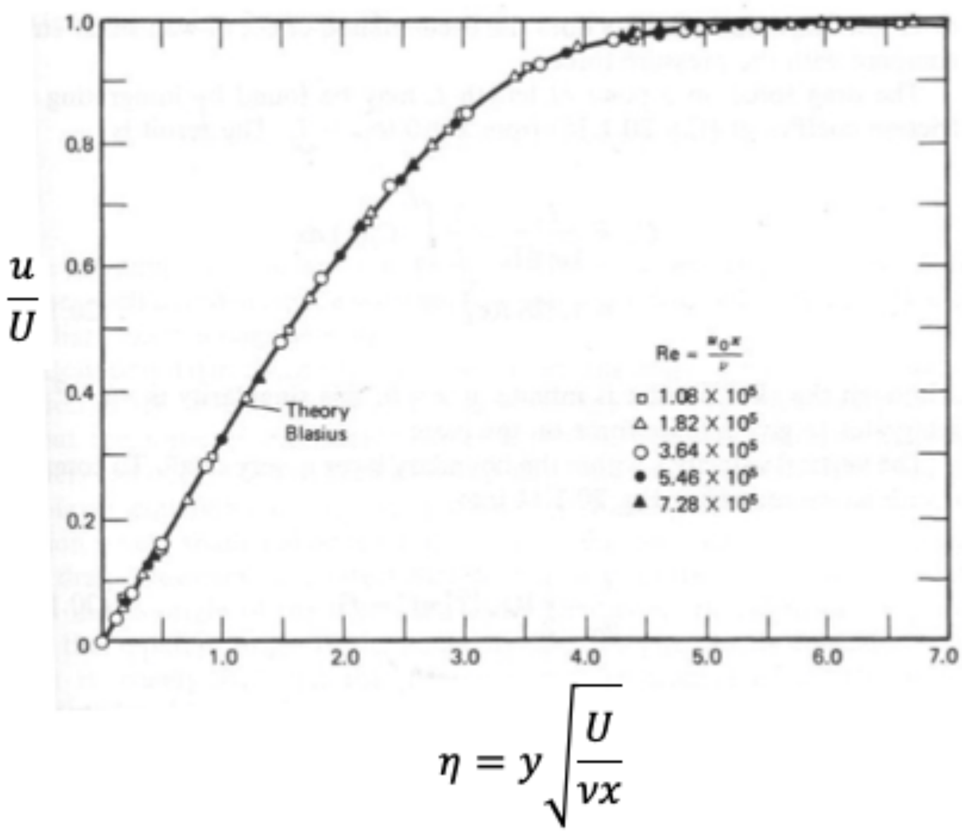


FIGURE 9.6. A plot showing experimental data for the dimensionless horizontal speed in a flat plate, no pressure gradient boundary layer flow plotted against the similarity variable η. The line in the plot is the numerical Blasius solution. This plot is from Panton, R.L., *Incompressible Flow*, 2nd ed., Wiley.

(2) The boundary layer thickness, δ, is found from the numerical solution to occur at $\eta = 3.5$,

$$F'(\eta = 3.5) = \frac{u}{U}(\eta = 3.5) = 0.99. \tag{9.61}$$

The boundary later thickness is then,

$$\eta = 3.5 = \delta\sqrt{\frac{U}{2\nu x}} \implies \frac{\delta}{x} = \frac{5.0}{\mathrm{Re}_x^{1/2}}. \tag{9.62}$$

(3) The displacement and momentum thicknesses can also be found numerically using their definitions,

$$\frac{\delta_D}{x} = \frac{1.72}{\mathrm{Re}_x^{1/2}}, \quad \frac{\delta_M}{x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}. \tag{9.63}$$

(4) The shear stress at the plate surface in dimensionless form is known as the friction coefficient, c_f,

$$c_f := \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{U\mu\left(\frac{d(u/U)}{d\eta}\frac{\partial\eta}{\partial y}\right)\Big|_{y=0}}{\frac{1}{2}\rho U^2} = \frac{\sqrt{2}F''|_{\eta=0}}{\sqrt{\mathrm{Re}_x}} = \frac{0.664}{\mathrm{Re}_x^{1/2}}. \tag{9.64}$$

The friction coefficient is a ratio of the shear stress to the dynamic pressure in the flow.

(5) The drag coefficient, c_D, defined as the dimensionless drag acting on the plate between $x = 0$ and $x = L$, is found by integrating the shear force over the plate area,

$$c_D := \frac{\int_{x=0}^{x=L} \tau_w dx}{\frac{1}{2}\rho U^2 L} = \frac{1.328}{\text{Re}_L^{1/2}} \quad \text{(where } c_D \text{ is the drag coefficient per unit depth).} \tag{9.65}$$

Note that although the boundary layer assumptions break down near the leading edge of the plate ($\text{Re}_x \not\gg 1$), the distance over which this is the case is small in comparison to the typical lengths of interest. This discrepancy is generally neglected in engineering applications.

(6) This solution is only valid for laminar boundary layers. As a rule of thumb, the transition from laminar flow to turbulent flow occurs at: $\text{Re}_x \approx 500,000$.

(7) Recall that the boundary layer equations on which the Blasius solution is based are valid only when $\text{Re}_x \gg 1$. In practice, it has been found that the Blasius solution is accurate when $\text{Re}_L > 1000$. For $1 \leq \text{Re}_L \leq 1000$, the following relation developed by Imai (1957) is more appropriate,

$$c_D = \frac{1.328}{\text{Re}_L^{1/2}} + \frac{2.3}{\text{Re}_L}. \tag{9.66}$$

(8) In summary, for laminar flow over a flat plate with no pressure gradient, the "exact" Blasius solution is,

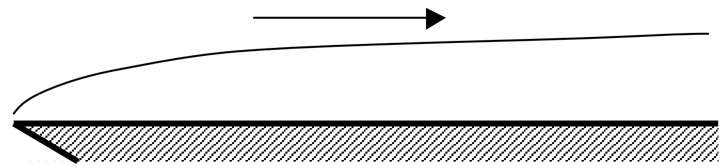
$$\frac{\delta}{x} = \frac{5.0}{\text{Re}_x^{1/2}} \qquad \frac{\delta_D}{x} = \frac{1.72}{\text{Re}_x^{1/2}} \qquad \frac{\delta_M}{x} = \frac{0.664}{\text{Re}_x^{1/2}} \tag{9.67}$$

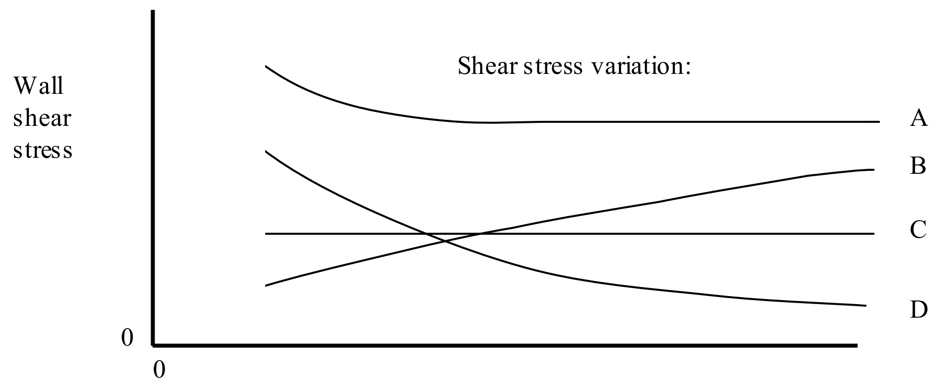
$$c_f = \frac{0.664}{\text{Re}_x^{1/2}} \qquad c_D = \frac{1.328}{\text{Re}_L^{1/2}} \tag{9.68}$$

$$\text{Re}_x < 500,000 \tag{9.69}$$

Air flows over a flat plate as shown below and forms a laminar boundary layer on the surface of the plate. Circle the letter of the statement that best represents the variation of the shear stress (force per unit area) at the wall with distance along the plate.

A. Shear stress variation A
B. Shear stress variation B
C. Shear stress variation C
D. Shear stress variation D





SOLUTION:

The friction coefficient for a laminar boundary layer is given by,

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\mathrm{Re}_x^{1/2}} . \tag{1}$$

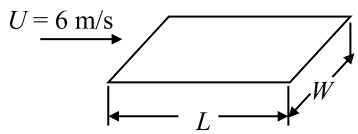
Re-arranging to solve for τ_w in terms of x gives,

$$\tau_w = \tfrac{1}{2}\rho U^2 \frac{0.664}{\left(Ux/\nu\right)^{1/2}} = \frac{\frac{1}{2}\cdot 0.664\rho \nu^{1/2} U^{3/2}}{x^{1/2}} . \tag{2}$$

Thus, as x increases, the wall shear stress will approach a value of zero as the inverse of the square root of x. Hence, the correct answer is curve D.

A thin flat plate 55 by 110 cm is immersed in a 6 m/s stream of SAE 10 oil at 20 °C. Compute the total skin friction drag if the stream is parallel to (a) the long side and (b) the short side.

SOLUTION:



$\nu_{\text{SAE 10 oil}} = 1.20*10^{-4}\ \text{m}^2/\text{s}$
$\rho_{\text{SAE 10 oil}} = 870\ \text{kg/m}^3$

Determine the Reynolds number at the trailing edge of the plate to see if it's laminar.

$$\text{Re}_L = \frac{UL}{\nu} \quad \text{(The flow is considered laminar if } Re < 1*10^6.\text{)} \tag{1}$$

When L = 1.10 m then Re_L = 55,200 ⇒ laminar flow. When L = 0.55 m then Re_L = 27,500 ⇒ laminar flow.

Now determine the drag on the plate using the drag coefficient, c_D, for laminar flat plate flow (the Blasius solution).

$$D = \left(\tfrac{1}{2}\rho U^2\right)\underbrace{\left(2LW\right)}_{\text{top and bottom faces}} c_D$$

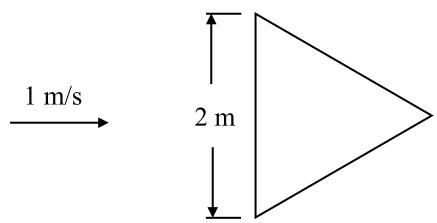
$$\therefore D = \left(\tfrac{1}{2}\rho U^2\right)\left(2LW\right)\left(\frac{1.328}{\text{Re}_L^{1/2}}\right) \tag{2}$$

When L = 1.10 m, W = 0.55 m, Re_L = 55,200, and $\boxed{D = 107\ \text{N}}$.

When L = 0.55 m, W = 1.10 m, Re_L = 27,500, and $\boxed{D = 152\ \text{N}}$.

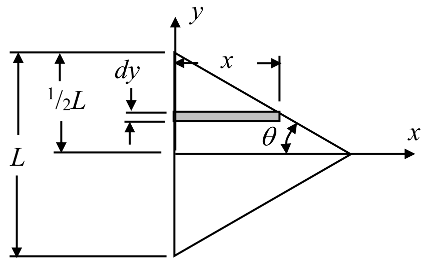
Note that the drag is greater when the short side is aligned with the flow. Why? Because from Eqn. (2) we observe that the drag varies with $\sqrt{L}$ but is proportional to W. Hence the drag will increase more rapidly with increasing width than with increasing length.

A thin equilateral triangle plate is immersed parallel to a 1 m/s stream of air at standard conditions. Estimate the skin friction drag on this plate.



SOLUTION:

Determine the drag acting on thin strips as shown in the figure below. The total drag will be the sum of each of these individual drag forces.



Since the plate is symmetric about the x-axis, consider only the $y > 0$ portion of the plate. The equation of the line defining the downstream edge of the plate is:

$$y = \left(-\tan\theta\right)x + \tfrac{1}{2}L \quad \text{where } 0 \le x \le L\cos\theta \;\; (L = 2\text{ m},\ \theta = 30^\circ) \tag{1}$$

Re-arrange to solve for x in terms of y:

$$x = \frac{\tfrac{1}{2}L - y}{\tan\theta} \tag{2}$$

Determine if the flow transitions to turbulence at any point on the plate.

$$\mathrm{Re}_{\mathrm{crit}} = \frac{Ux_{\mathrm{crit}}}{\nu} = 500{,}000 \;\Rightarrow\; x_{\mathrm{crit}} = 500{,}000\frac{\nu}{U} \tag{3}$$

Using the given data:

ν = 1.5*10^{-5} m^2/s
U = 1 m/s
$\Rightarrow$ x_{crit} = 7.5 m

Since the longest strip length is only $L\cos\theta$ = (2 m)cos(30º) = 1.73 m, the flow over the entire plate will be laminar. Hence, we can use the Blasius solution relations to model the boundary layer characteristics. The drag acting on a single strip of length x and width dy is:

$$dD = c_D \tfrac{1}{2}\rho U^2 x\,dy$$
$$= \frac{1.328}{\mathrm{Re}_x^{1/2}}\tfrac{1}{2}\rho U^2 x\,dy \tag{4}$$

$$\therefore dD = 0.664\sqrt{\nu}\,\rho U^{3/2}x^{1/2}dy \tag{5}$$

The total drag acting on the plate (including the top half, bottom half, and front and back surfaces) is:

$$D = 4\int_{y=0}^{y=\frac{1}{2}L} dD = 4\int_{y=0}^{y=\frac{1}{2}L} 0.664\sqrt{\nu}\rho U^{3/2}x^{1/2}dy$$

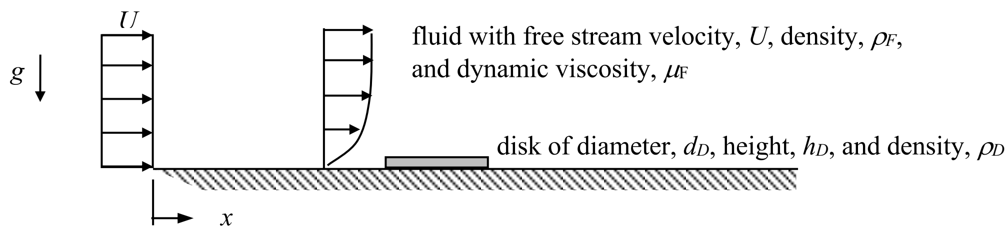
$$= 4\int_{y=0}^{y=\frac{1}{2}L} 0.664\sqrt{\nu}\rho U^{3/2}\left(\frac{\frac{1}{2}L - y}{\tan\theta}\right)^{1/2} dy \qquad \text{(making use of Eqn. (2))} \qquad (6)$$

$$\boxed{\therefore D = 0.626\rho(UL)^{3/2}\sqrt{\frac{\nu}{\tan\theta}}} \qquad (7)$$

Using the given data:

ρ = 1.23 kg/m^3
U = 1 m/s
L = 2 m
ν = 1.5*10^{-5} m^2/s
θ = 30º
$\Rightarrow$ $\boxed{D = 1.1*10^{-2}\text{ N}}$

Consider a thin disk of density, ρ_D, diameter, d_D, and height, h_D, resting on a submerged flat plate as shown in the figure below. Flowing over the plate is a fluid of density, ρ_F, and dynamic viscosity, μ_F, with a free stream velocity, U. There are no pressure gradients in the flow.



Assume the flow upstream of the plate is uniform, but then results in a boundary layer when the fluid contacts the plate. The effective static friction coefficient between the disk and the plate is μ (for simplicity assume that the static and dynamic friction coefficients are equal). For the following questions, assume the following:

d_D = 2 mm
h_D = 0.5 mm
ρ_D = 2500 kg/m^3
ρ_F = 1000 kg/m^3
μ_F = 1.0e-3 kg/(m·s)
U = 1.5 m/s
μ = 0.3
g = 9.81 m/s^2

a. Determine the effective friction force acting to hold the disk in place.
b. If the disk is released at the leading edge of the plate, at what distance from the leading edge will the disk come to rest? (Neglect the inertia of the disk, *i.e.* treat the disk movement in a quasi-static manner).
c. Neglecting the flow over the disk, at what distance from the leading edge will the boundary layer separate? Justify your answer.

SOLUTION:

The disk will remain stationary when the shear force acting on it, F_S, is less than the friction force holding the disk in place, F_F, *i.e.* the disk will remain stationary when

$$F_S \le F_F \quad \text{(the disk's inertia has been neglected)} \tag{1}$$

The friction force is the friction coefficient multiplied by the effective weight.

$$\boxed{F_F = \mu\left(\rho_D - \rho_F\right)\tfrac{\pi}{4} d_D^2 h_D g} \tag{2}$$

Using the given data, $\boxed{F_F = 6.9\text{e-}6\text{ N}}$.

The shear force acting on the disk due to the flowing fluid is (approximately*) the shear stress multiplied by the disk's projected area (when viewed from above):

$$F_S = \tau_w \tfrac{\pi}{4} d_D^2 \tag{3}$$

The shear stress may be estimated using the boundary layer relations. First assume that the boundary layer is laminar (*i.e.* $Re_x < 500{,}000$) so that the friction coefficient is:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_F U^2} = \frac{0.664}{Re_x^{1/2}} \Rightarrow \tau_w = \frac{1}{2}\rho_F U^2 \frac{0.664}{Re_x^{1/2}} \quad (4)$$

where

$$Re_x = \frac{\rho_F U x}{\mu_F} \quad (5)$$

Combining Eqns. (1) - (5) gives:

$$\left(\frac{1}{2}\rho_F U^2 \frac{0.664}{Re_x^{1/2}}\right)\frac{\pi}{4}d_D^2 = \mu\left(\rho_D - \rho_F\right)\frac{\pi}{4}d_D^2 h_D g \quad \text{(when the shear and friction forces just balance)} \quad (6)$$

$$Re_x = \left[0.332\frac{\rho_F U^2}{\mu\left(\rho_D - \rho_F\right)h_D g}\right]^2 \quad (7)$$

Using the given data:

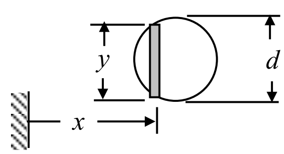
$Re_x = 1.2e5$ (laminar flow assumption ok)

$\Rightarrow$ $\boxed{x = 0.076 \text{ m}}$

Since the flow does not have an adverse pressure gradient (in fact the flow has no pressure gradient), the boundary layer will not separate.

* A more accurate method of calculating the fluid force on the disk is to integrate the shear stress over the disk's surface.

$$F_S = \int_{x=x-\frac{1}{2}d}^{x=x+\frac{1}{2}d} \tau_w \, dA = \int_{x=x-\frac{1}{2}d}^{x=x+\frac{1}{2}d} \underbrace{\frac{1}{2}\rho_F U^2 \frac{0.664}{Re_x^{1/2}}}_{=\tau_w} \underbrace{y dx}_{=dA} \quad (8)$$



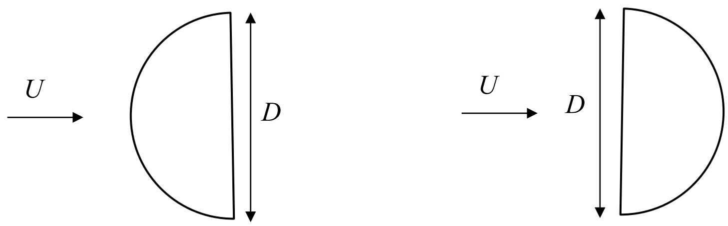
where

$$y = 2\sqrt{\frac{1}{4}d^2 - x^2} \quad (9)$$

Determine the expression for the drag force acting on the thin, semi-circular plate immersed in a flow as shown below, assuming,

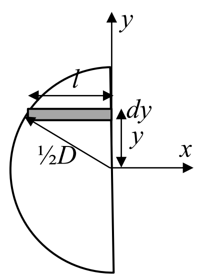
a. the "tip" faces the flow, and
b. the base faces the flow.

Assume the flow is laminar over the entire plate. You need not solve any integrals you encounter.



(a) (b)

SOLUTION:



The drag force on both sides of a thin strip of the plate is,

$$dD = 2c_D \tfrac{1}{2}\rho U^2 l\,dy = 2\frac{1.328}{\mathrm{Re}_l^{1/2}}\tfrac{1}{2}\rho U^2 l\,dy = 1.328\sqrt{\frac{\nu}{Ul}}\rho U^2 l\,dy = 1.328\nu^{1/2}\rho U^{3/2} l^{1/2}dy \;, \quad (1)$$

where

$$l = \left(\tfrac{1}{4}D^2 - y^2\right)^{1/2} \;, \quad (2)$$

Thus, the total drag on the plate is,

$$D = \int_{y=-\frac{1}{2}D}^{y=\frac{1}{2}D} dD = \int_{y=-\frac{1}{2}D}^{y=\frac{1}{2}D} 1.328\nu^{1/2}\rho U^{3/2}\left(\tfrac{1}{4}D^2 - y^2\right)^{1/4}dy = 1.328\nu^{1/2}\rho U^{3/2}\int_{y=-\frac{1}{2}D}^{y=\frac{1}{2}D}\left(\tfrac{1}{4}D^2 - y^2\right)^{1/4}dy \;, \quad (3)$$

$$\boxed{D = 1.328\nu^{1/2}\rho U^{3/2}\int_{y=-\frac{1}{2}D}^{y=\frac{1}{2}D}\left(\tfrac{1}{4}D^2 - y^2\right)^{1/4}dy}. \quad (4)$$

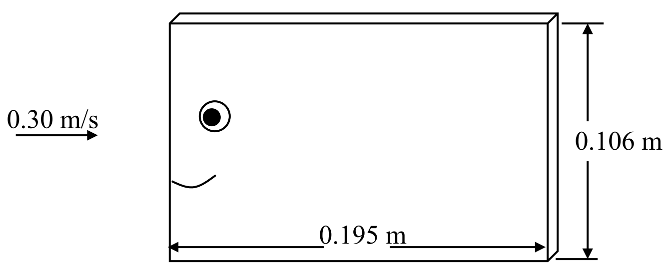
The drag force on the plate will be exactly the same if the plate is reversed since the drag on each strip is identical.

An engineer studies swimming fish in order to develop bio-inspired designs for boat and submarine hulls. These studies focus on a fish known as a "scup" (Stenotomus chrysops), shown in the following photo. The speed of the swimming fish, the body area on one side of the fish, and the length of fish from head to tail are given in the table adjacent to the photograph.

swimming speed =	0.3 m/s
body area on one side =	0.0207 m^2
body length =	0.195 m
salt water density =	1026 kg/m^3
salt water kinematic viscosity =	$1.19*10^{-6}$ m^2/s

a. For the given conditions, determine if the flow over the entire length of the fish is laminar, turbulent, or a combination of the two. Show your work supporting your answer.

b. Estimate the total drag acting on the fish, assuming skin friction drag dominates (a good assumption based on experiments) and that the fish shape may be modeled as a thin, flat rectangular plate as shown below.



c. Calculate the power required for the fish to swim at the given conditions.

SOLUTION:

The Reynolds number based on the fish's length is:

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{(0.30\ \text{m/s})(0.195\ \text{m})}{(1.19*10^{-6}\ \text{m}^2/\text{s})} = 49,200. \tag{1}$$

Since the Reynolds number is less than 500,000, the boundary layer over the entire fish is laminar.

The total drag acting on the (rectangular) fish may be found using the drag coefficient for a laminar boundary layer flow over a flat plate (the Blasius solution),

$$D_{2\text{ sides}} = 2D_{1\text{ side}} = 2c_D \tfrac{1}{2}\rho U^2 LW, \tag{2}$$

where L is the length of the fish (L = 0.195 m) and H is the height of the fish (H = 0.106 m), and c_D for a laminar flat plate flow is,

$$c_D = \frac{1.328}{\mathrm{Re}_L^{1/2}}, \tag{3}$$

where the Reynolds number based on the fish's length was found in Eq. (1).

Using the given data,

$\mathrm{Re}_L = 49{,}200$

$\Rightarrow c_D = 5.99 * 10^{-3}$

Note that experimental measurements provided by Anderson *et al*. (2001) found that the drag coefficient for these conditions is $4.4 * 10^{-3}$. Hence, the prediction from our simplified model has a relative error of about 36%.

$\rho = 1026 \text{ kg/m}^3$

$U = 0.30 \text{ m/s}$

$L = 0.195 \text{ m}$

$H = 0.106 \text{ m}$

$\Rightarrow D_{1\text{ side}} = 5.72 * 10^{-3} \text{ N}$

$$\Rightarrow \boxed{D_{2\text{ sides}} = 1.14 * 10^{-2} \text{ N}} \tag{4}$$

The power required for the fish to swim at a speed of $U = 0.30$ m/s given the drag found in Eq. (4) is,

$$P = D_{2\text{ sides}} U \Rightarrow \boxed{P = 3.43 * 10^{-3} \text{ W} = 3.43 \text{ mW}} \tag{5}$$

A good resource on boundary layer characteristics over swimming fish is:

Anderson, E.J., McGillis, W.R., and Grosenbaugh, M.A., 2001, "The boundary layer of swimming fish," *The Journal of Experimental Biology*, Vol. 204, pp. 81 – 102.

Air, with a density of 1.23 kg/m^3 and a kinematic viscosity of 2.5*10^{-6} m^2/s, enters a long horizontal ventilation duct of circular cross-section (radius of 0.25 m) with a velocity of 1.0 m/s. At the entrance it is assumed that this velocity is uniform over the entire cross-section. However, as the flow proceeds down the duct a thin laminar boundary develops on the inside wall of the duct.

If we first assume that this is like the boundary layer on a flat plate and that the velocity away from the boundary layer remains at 1.0 m/s, find the displacement thickness in meters at a distance *x* (in meters) from the entrance.

Having calculated this displacement thickness we recognize that the velocity outside the boundary layer cannot remain precisely constant at 1 m/s. Using the above calculated displacement thickness, find the uniform velocity outside the boundary layer at a point 200 m from the entrance. What is the pressure difference between the entrance and this point 200 m from the entrance? Describe in words how you might now proceed to a more accurate boundary layer calculation which takes this pressure gradient into account.

SOLUTION:

The displacement thickness, δ_D, for a laminar boundary layer is given by the Blasius solution to be:

$$\frac{\delta_D}{x} = \frac{1.72}{\sqrt{\text{Re}_x}}$$

$$\boxed{\delta_D = 1.72\sqrt{\frac{\nu x}{U}}} \tag{1}$$

Using the given data:

ν = 2.5*10^{-6} m^2/s

U = 1 m/s

$\boxed{\delta_D = (2.7*10^{-3}\ \text{m}^{1/2})\, x^{1/2}}$

From conservation of mass and the definition of the displacement thickness, the velocity in the flow outside of the boundary layer, U, is:

$$\rho U_0 \pi R^2 = \rho U \pi \left(R - \delta_D\right)^2$$

$$\boxed{U = U_0 \frac{R^2}{\left(R-\delta_D\right)^2} = U_0 \frac{1}{\left(1 - \delta_D/R\right)^2}} \tag{2}$$

where U_0 is the uniform fluid velocity at the pipe entrance. Using the given data:

U_0 = 1 m/s

R = 0.25 m

x = 200 m

$\delta_{D,x=200\text{ m}}$ = 3.9*10^{-2} m

$\boxed{U_{x=200\text{ m}} = 1.4\text{ m/s}}$

Apply Bernoulli's equation (neglecting elevation differences) in the outer, irrotational flow region.

$$p_0 + \tfrac{1}{2}\rho U_0^2 = p + \tfrac{1}{2}\rho U^2$$

$$p - p_0 = \Delta p = \tfrac{1}{2}\rho\left(U_0^2 - U^2\right) \tag{3}$$

Using the given data:

ρ = 1.23 kg/m^3

U_0 = 1 m/s

$U_{x=200\text{ m}}$ = 1.4 m/s

$\boxed{\Delta p_{x=200\text{ m}} = -0.59\text{ N/m}^2}$

For a more accurate estimate, one can iterate on the displacement thickness and core flow velocity until a converged solution is reached.

$$\delta_{D,n} = 1.72\sqrt{\frac{\nu x}{U_n}}$$

$$U_n = U_0 \frac{1}{\left(1 - \delta_{D,n}/R\right)^2}$$

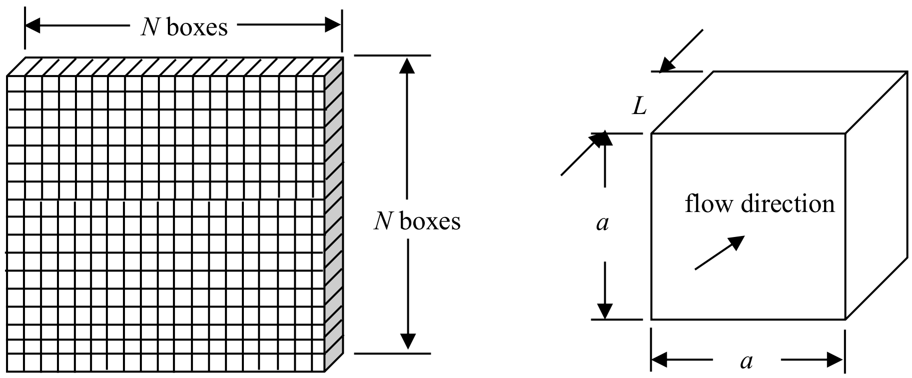
where n is the number of iterations. For example, given the previous data:

ν [m²/s] =	2.50E-06
ρ [kg/m³] =	1.23
x [m] =	200
U_0 [m/s] =	1.0
R[m] =	0.25

n	**$\delta_{D,n}$ [m]**	**U_n [m/s]**	**Δp_n [N/m²]**
1	3.85E-02	1.40	-0.58
2	3.25E-02	1.32	-0.46
3	3.35E-02	1.33	-0.48
4	3.33E-02	1.33	-0.47
5	3.33E-02	1.33	-0.48
6	3.33E-02	1.33	-0.48

Flow straighteners are arrays of narrow ducts placed in wind tunnels to remove swirl and other in-plane secondary velocities. They can be idealized as square boxes constructed by vertical and horizontal plates as shown in the figure. The cross-section of the box is *a* by *a* and the box length is *L*. Assuming laminar flat plate flow and an array of *N* by *N* boxes, derive a formula for:

a. the total drag on the bundle of boxes.
b. the effective pressure drop across the bundle.



SOLUTION:

Determine the drag acting on one wall due to skin friction. From the Blasius solution, the drag coefficient for laminar, flat plate flow is:

$$c_D = \frac{1.328}{\mathrm{Re}_L^{1/2}} \tag{1}$$

where

$$c_D = \frac{D}{\frac{1}{2}\rho U^2 (La)} \tag{2}$$

$$\mathrm{Re}_L = \frac{UL}{\nu} \tag{3}$$

Note that in writing Eqn. (1) we've assumed that there is no pressure gradient in the cell's core flow. This is not exactly correct since there will, in fact, be a pressure gradient due to the growth of the boundary layer along the plate surface and hence an increase in the outer (*i.e.,* cell core) flow velocity (from conservation of mass). However, as a first estimate it is reasonable to assume a constant outer flow velocity and thus a Blasius boundary layer profile. A more precise analysis would account for the growth in the displacement thickness and the resulting increase in the outer velocity.

Using Eqns. (1) and (2), the skin friction drag acting on one wall of a cell is:

$$D_{\substack{\text{one}\\\text{wall}}} = \frac{1.328 \cdot \frac{1}{2}\rho U^2 (La)}{\mathrm{Re}_L^{1/2}} = \frac{0.664 \rho U^2 (La)}{\mathrm{Re}_L^{1/2}} \tag{4}$$

The drag acting on a single cell, which consists of four walls, is:

$$D_{\text{cell}} = 4 D_{\substack{\text{one}\\\text{wall}}} = \frac{2.656 \rho U^2 (La)}{\mathrm{Re}_L^{1/2}} \tag{5}$$

The total drag acting on a grid of NxN cells is:

$$\boxed{D_{\substack{N\text{x}N\\\text{cells}}} = N^2 D_{\text{cell}} = \frac{2.656\rho U^2 (La) N^2}{\text{Re}_L^{1/2}}} \quad (6)$$

where Re$_L$ is given in Eqn. (3).

As stated previously, in deriving Eqn. (1) we've assumed that there is no pressure gradient within the cell which is not entirely correct but should instead be considered a first-cut estimate. Regardless, we can still determine an effective pressure drop across the flow straightener by dividing the drag force acting on the straightener by its area.

$$\boxed{\Delta p = \frac{-D_{N\text{x}N\text{ cells}}}{N^2 a^2}} \text{ (The pressure decreases across the flow straightener.)} \quad (7)$$

A more accurate approach to solving this problem involves iteration. First, determine the velocity in the outer flow region using the displacement thickness, d_D.

$$\dot{m} = \text{constant} = \rho U a^2 = \rho U' \left(a - 2\delta_D\right)^2$$

$$U' = U\left(\frac{a}{a - 2\delta_D}\right)^2 \quad (8)$$

where, from the Blasius solution:

$$\frac{\delta_D}{L} = \frac{1.72}{\text{Re}_L^{1/2}} \quad (9)$$

and

$$\text{Re}_L = \frac{U'L}{\nu} \quad (10)$$

The pressure in the outer layer can be determined using Bernoulli's equation.

$$p + \tfrac{1}{2}\rho U^2 = p' + \tfrac{1}{2}\rho U'^2$$

$$\Delta p = p' - p = \tfrac{1}{2}\rho\left(U^2 - U'^2\right) \quad (11)$$

The iterative procedure is as follows:

1. Assume U ' = 0.
2. Evaluate the displacement thickness using Eqns. (9) and (10).
3. Evaluate the new downstream velocity, U ‘, using Eqn. (8).
4. Repeat steps 2 and 3 until a converged solution for U ‘ occurs.
5. Use Eqn. (11) to determine Δp.